MIRO ŠARE

m-BROJEVI

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ZAGREB, 1970.

Svojim roditeljima
i svojoj braći

Neka bude borba neprestana,
Neka bude što biti ne može,
Na groblju će iznići cvijeće
Za daleko neko pokoljenje!

Njegoš, Gorski vijenac

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Četvrti dio

m - T O P O L O G I J A

PRISTUP

0.1. T U M A Č   S K R A Ć E N I C A

	Skraćenica	Značenje, odnosno primjer upotrebe
1	::	Znači. Čita se.
2	A	A1 :: aksiom 1.
3	D	D3 :: definicija 3.
4	K	K5 :: konvencija 5.
5	T	T7 :: teorem 7.
6	Def.	(X Def.) :: (X je definicija).
7	Sup.	(X Sup.) :: (X je pretpostavka).
8	$\succ\!\!-\!\!-$	(D10,T22 $\succ\!\!-\!\!-$ T23) :: (iz definicije 10 i teorema 22, očit je (dokazuje se) teorem 23), (str.13).
9	(analog)	(D18,D20,T56 (analog T60) $\succ\!\!-\!\!-$ T61) :: (iz D18,D20,T56, analogno dokazu teorema 60, dokazuje se i teorem 61), (str.21).
10	(tot.indukc.)	(T44,D17 (tot.indukc.) $\succ\!\!-\!\!-$ 1) :: (iz T44 i D17 potpunom indukcijom dokazuje se 1), (str.19).
11	(2.pr.indukc.)	(10,11,17 (2.pr.indukc.) $\succ\!\!-\!\!-$ 18) :: (iz 10,11 i 17 po drugom principu* matematičke indukcije, očit je,(dokazuje se) 18), (str.56).

*(S.M.Lane, G.Birkhoff, Algebra 1967, str.42, Th. 8)

12 >---< T192 >——< T193 :: teorem 193 je dual teorema 192. Odnosno, iz dokaza teorema 192 dobiva se dokaz teorema 193 tako, da se svaki m-simbol zamjeni dualnim m-simbolom, a sve ostalo ostavi nepromjenjeno.

13 ⌞——⌝ X ⌞——⌝ Y :: umjesto X, kadgod to ne može dovesti do zabune, pisat će se Y.
(Primjer upotrebe: konvencija 6, str. 199).

0.2. GLOSARIJ SIMBOLA MATEMATIČKE LOGIKE, TEORIJE SKUPOVA I ALGEBRE

1	$\neg$	$\neg x$	::	nije x.	Negacija.
2	&	x & y	::	x i y.	Konjukcija.
3	$\vee$	$x \vee y$	::	x ili y.	Disjunkcija.
4	$\longrightarrow$	$x \longrightarrow y$	::	x povlači y.	Implikacija
5	$\longleftrightarrow$	$x \longleftrightarrow y$	::	x ako i samo ako y.	Ekvivalencija.
6	$\not\longleftrightarrow$	$x \not\longleftrightarrow y$	::	x ili $_{\text{ekskl.}}$ y.	Ekskluzivna disjunkcija.
7	=	x = y	::	x je jednako y.	Identičnost.
8	$\neq$	$x \neq y$	::	x nije jednako y.	Negacija jednakosti.
9	$\in$	$x \in X$	::	x je element od X.	Skupovska pripadnost.
10	$\forall$	$(\forall x \in X)$	::	za svako x u X.	Univerzalni kvantifikator.
11	$\exists$	$(\exists x \in X)$	::	postoji x u X.	Egzistencijalni kvantifikator.
12	$\exists!$	$(\exists! x \in X)$	::	postoji točno jedno x u X.	Kvantifikator jedinstvenosti.
13	0	X = 0	::	X je prazan	Prazni skup.
14	(,)	(x,y).	::	dvojka x,y.	Uređeni par.
15	{ : }	{x: F(x)}. (Suppes, Axiomatic Set Theory, 1960, str.34, Definition Shema 11).			Operator okupljanja.
16	{ }	{a,b,c,..}	::	skup kojem su elementi a,b,c,..	
17	|	k|m	::	m je djeljivo sa k.	
18	$\equiv$	$x \equiv y \pmod n$	::	x i y su kongruentni modulo n. Relacija kongruencije cijelih brojeva.	
19	$\sum$	$\sum_{i=0}^{n} x_i$	::	$x_0 + x_1 + \ldots + x_n$.	
20	$\bar{}$	$\bar{k}$	::	-k. Inverzija realnih brojeva.	

21	$\subseteq$	$X \subseteq Y$ ::	X je sadržano u Y.	Inkluzija.
22	$\subset$	$X \subset Y$ ::	X je pravi podskup od Y.	Prava inkluzija.
23	$\setminus$	$X \setminus Y$ ::	X manje Y.	Diferencija.
24	$\times$	$X \times Y$ ::	X puta Y.	Kartezijev produkt.
25	$\cap$	$X \cap Y$ ::	presjek od X i Y.	Presjek.
26	$\cup$	$X \cup Y$ ::	unija od X i Y.	Unija
		$\bigcup X$ ::	unija svih skupova koji su elementi od X.	
27	$\aleph_0$	Kardinalni broj skupa prirodnih brojeva. Alef nula.		
28	Adom	Adom(F) ::	antidomena relacije F. Područje vrijednosti relacije F.	
29	Dom	Dom(F) ::	domena relacije F. Područje definicije relacije F.	
30	El	El(x,y) ::	eliminanta (rezultanta) polinoma x,y . (B.Waerden, Algebra I, 1960,str. 93) .	
31	inf	inf(x,y) ::	infimum (donja međa) od x i y.	
32	K	K(X) ::	kardinalni broj skupa X.	
33	Λ	(Λs)(F). Churchov lambda operator. (Suppes, Introduction to Logic, 1957, str.243).		
34	N	Skup cijelih brojeva.		
35	N^+	Skup nenegativnih cijelih brojeva.		
36	N_{neg}	Skup negativnih cijelih brojeva.		
37	N_{poz}	Skup pozitivnih cijelih brojeva.		
38	$N(\equiv_n)$	Skup klasa ostataka cijelih brojeva modulo n.		
39	Ξ	Skup komplekenih brojeva.		
40	Re	Skup realnih brojeva.		
41	Re^+	Skup nenegativnih realnih brojeva.		
42	Re_{neg}	Skup negativnih realnih brojeva.		
43	Re_{poz}	Skup pozitivnih realnih brojeva.		

44	S	$X \in S$ :: X je skup simbola. Partitivni skup, skupa svih simbola.
45	$S_{\text{alg.sud.}}$	$x \in S_{\text{alg.sud.}}$ :: x je algebra sudova.
46	S_{bijekc}	$f \in S_{\text{bijekc}}(X,Y)$ :: f je 1-1 funkcija (bijekcija) od X na Y.
47	$S_{\text{Cayley tb.}}$	$x \in S_{\text{Cayley tb.}}(X,f)$ :: x je Cayleyeva tablica binarnog sistema(X,f).
48	$S_{\text{cikl.gr.}}$	$x \in S_{\text{cikl.gr.}}(\varkappa)$ :: za svaki pozitivni cijeli broj $\varkappa$, x je ciklička grupa $\varkappa$-tog reda i za $\varkappa = 0$, x je beskonačna ciklička grupa.
49	S_{funkc}	$f \in S_{\text{funkc}}(X,Y)$ :: f je funkcija od X u Y .
50	$S_{\text{Hasse dgr.}}$	$x \in S_{\text{Hasse dgr.}}(X,r)$ :: x je Hasseov dijagram parcijalnog uređenja skupa X relacijom r. (G. Birkhoff, Lattice Theory, 1961,str.5).
51	$S_{\text{Hurwitz p.}}$	$x \in S_{\text{Hurwitz p.}}$ :: x je Hurwitzov polinom.
52	S_{injekc}	$f \in S_{\text{injekc}}(X,Y)$ :: f je 1-1 funkcija (injekcija) od X u Y.
53	$S_{\text{Klein gr.}}$	$x \in S_{\text{Klein gr.}}$ :: x je četvorna (Kleinova) grupa.
54	S_{part}	$X \in S_{\text{part}}(Y)$ :: X je particija od Y. (Suppes, Axiomatic Set Theory, 1960, str. 53, Def. 35).
55	$S_{\text{Peano s.}}$	$(x,y,z) \in S_{\text{Peano s.}}$ :: (x,y,z) je Peanov sistem i x je skup i y je inicijalni element i z je sljedbenička funkcija.
56	S_{polinom}	$x \in S_{\text{polinom}}(s)$ :: x je polinom u s.
57	S_{relac}	$F \in S_{\text{relac}}(X,Y)$ :: F je relacija od X u Y.
58	S_{surjekc}	$f \in S_{\text{surjekc}}(X,Y)$ :: f je funkcija od X na Y.
59	$S_{\text{rel.ekv.}}$	$F \in S_{\text{rel.ekv.}}(X)$ :: F je relacija ekvivalencije u X.

60	sgn	sgn(k)	::	signum od k. Funkcija predznaka.
61	sup	sup(x,y)	::	supremum (gornja međa) od x i y.

Prvi dio

SISTEM M

U početku bješe riječ
Ev. po Ivanu

1.0. U V O D

1.0.1. m-RIJEČI KAO REPREZENTANTI m-BROJEVA

Sistem M je formalizirana teorija m-brojeva u kojoj kao reprezentanti m-brojeva dolaze m-riječi. m-Riječi su riječi formirane dvoelementnim alfabetom, pretpostavljajući pri tom da je "riječ" u smislu linearne kombinacije(konkatenacije, nadovezivanja) konačnog broja simbola, intuitivno jasan pojam. m-Riječi su prirodni reprezentanti m-brojeva, jer im na najjednostavniji način odražavaju svojstva i strukturu. Gledajući u m-riječima iskustveno poznate i sadržajne objekte, dat će se u ovoj glavi, preko intuitivno jasnih pojmova, kao što su npr. početna slova, broj slova i sličnih, sadržaj glavnim formalnim pojmovima sistema M.

DEFINICIJA 01
m-Riječ je x, ako i samo ako x je konačna, neprazna riječ formirana slovima "a" i "b" i broj slova u x je paran.

PRIMJERI
1. m-Riječi su: aa, ab, ba, bb, aaaaaa, abbaabbbabba, bbba itd.
2. Nisu m-riječi: a, b, aaa, abb, bab, bbbbbbb, abababbaaaa itd.

DEFINICIJA 02
M je skup m-riječi.

DEFINICIJA 03
A = aa i B = bb.

PRIMJERI

3. AB = aabb. 4. BBB = bbbbbb. 5. bABAb = baabbaab.

DEFINICIJA 04

Za svako $x,y \in M$, "nadoveza od x i y" (simb. $x \wedge y$), odnosno "y nadovezano na x" jednako je xy.

PRIMJERI

6. $A \wedge B$ = AB. 8. $B \wedge (B \wedge B)$ = BBB. 10. $ba \wedge ab$ = bAb.
7. $B \wedge A$ = BA. 9. $A \wedge (B \wedge A)$ = ABA.

DEFINICIJA 05

Za svako $x,y \in M$, "x je ekvivalentno sa y" (simb. $x \theta y$), ako i samo ako x i y su identički jednaki, ili za svako $s,t \in \{a,b\}$, y rezultira iz x supstitucijom "tst" sa "t" i obrnuto.

PRIMJERI

11. aaaa θ aa. 13. BB θ B. 15. abab θ ab. 17. bbab θ B.
12. aaba θ aa. 14. aaab θ ab. 16. baba θ ba. 18. bbbb θ B.
19. AAAAABA θ ABA. 20. baAAAAB θ B. 21. BBBabBB θ bb.

DEFINICIJA 06

Za svako $s,t \in \{a,b\}$, "s i t" su dualni (simb. $\bar{s} = t$), ako i samo ako s i t nisu jednaka slova.

PRIMJERI

22. $\bar{a}$ = b. 23. $\bar{b}$ = a. 24. $\bar{\bar{a}}$ = a. 25. $\bar{\bar{b}}$ = b.

1.0.2. ZAVRŠECI I LJUSKA m-RIJEČI

DEFINICIJA 07

Za svako $x \in M$, početak (simb. l(x)), odnosno lijevi završetak od x je slovo kojim x započinje.

PRIMJERI

26. l(aa) = l(A) = a. 27. l(ab) = a. 28. l(ABA) = a.
29. l(B) = l(bb) = b. 30. l(ba) = b.

DEFINICIJA 08

Za svako $x \in M$, svršetak (simb. d(x)), odnosno desni završetak od x je slovo kojim x završava.

PRIMJERI

31. d(A) = a. 33. d(ab) = b. 35. d(ABAAB) = b.
32. d(B) = b. 34. d(ba) = a. 36. d(bABa) = a.

DEFINICIJA 09

Za svako x∈M, ljuska od x (simb. q(x)) je m-riječ koja rezultira kad se svršetak od x nadoveže na početak od x.

PRIMJERI

37. q(A) = A. 39. q(ab) = ab. 41. q(aBa) = A.
38. q(B) = B. 40. q(ba) = ba. 42. q(ABAB) = ab.

1.0.3. SLJEDBENICI m-RIJEČI

DEFINICIJA 010

Za svako x∈M, lijevi sljedbenik od x (simb. Px) je ssx, ako i samo ako s je slovo dualno početku od x.

DEFINICIJA 011

Za svako x∈M, desni sljedbenik od x (simb. xP) jednak je xss, ako i samo ako s je slovo dualno svršetku od x.

PRIMJERI

43. PA = BA. 45. Pab = bbab. 47. AP = AB. 49. ABBP = ABBA.
44. PB = AB. 46. Pba = aaba. 48. BP = BA. 50. BPPP = BABA.
51. abP=abaa. 53. bAbP= baabaa.
52. PPA= ABA. 54. baPPP = babbaabb. 55. PAABA = BAABA.

TEOREM 01

Za svako x,y∈M, (xP)y θ x(Py).

PRIMJERI

56. A(PA) = ABA = (AP)A. 57. A(PB) θ AAB θ AB θ ABB θ (AP)B.
58. ab(Pba) θ abaaba θ (abP)ba θ A.
59. ab(Pab) θ abbbab θ abab θ abaaab θ (abP)ab θ ab.
60. ba(Pba) θ baaaba θ baba θ babbba θ (baP)ba θ ba.

DEFINICIJA 012

Za svako x∈M, lijevi prethodnik od x (simb. P'x) je s̄sx, ako i samo ako s je slovo jednako početku od x.

DEFINICIJA 013

Za svako $x \in M$, desni prethodnik od x (simb. xP') je $xs\bar{s}$, ako i samo ako s je slovo jednako svršetku od x.

PRIMJERI

61. $P'B \; \theta \; abB \; \theta \; ab$. 62. $P'ABA \; \theta \; baBA \; \theta \; BA$. 63. $P'ba = aBa$.
64. $P'ab = bAb$. 65. $ABP' \; \theta \; A$. 66. $BAP' \; \theta \; B$.
67. $P'P'ab = aBAb$. 68. $abP'P' = aBAb$. 69. $PP'A = A$.
70. $P'P'P'ABAB \; \theta \; B$. 71. $baP'P = ba$.

TEOREM 02

Za svako $x,y \in M, (xP')y \; \theta \; x(P'y)$.

PRIMJERI

72. $A(P'A) \; \theta \; (AP')A \; \theta \; A$. 73. $A(P'B) \; \theta \; (AP')B \; \theta \; ab$.
74. $abP'ba \; \theta \; abba$.

1.0.4. DUALNE I KOMPLEMENTARNE m-RIJEČI

DEFINICIJA 014

Za svako $x \in M$, dual od x (simb. Dx ili $\bar{x}$) je m-riječ koja rezultira iz x zamjenom svakog slova dualnim slovom.

PRIMJERI

75. $Dabbaaa = \bar{a}\bar{b}\bar{b}\bar{a}\bar{a}\bar{a} = baabbb$. 76. $DA = B$. 77. $DB = A$.
78. $Dab = ba$. 79. $Dba=ab$. 80. $DABAA = BABB$.

TEOREM 03

Za svako $x \in M$, $Dx \; \theta \; PxP' \; \theta \; P'xP$.

PRIMJERI

81. $PaaP' \; \theta \; bbaaab \; \theta \; bb \; \theta \; \bar{a}\bar{a}$. 82. $P'baP \; \theta \; abbabb \; \theta \; ab \; \theta \; \bar{b}\bar{a}$.

DEFINICIJA 015

Za svako $x \in M$, komplement od x (simb. Kx) je sxt, ako i samo ako s je slovo dualno početku od x i t je slovo dualno svršetku od x.

PRIMJERI

83. $KA = bAb$. 85. $KbAb = abaaba \; \theta \; A$. 87. $Kba = abab \; \theta \; ab$.
84. $KB = aBa$. 86. $KaBa = babbab \; \theta \; B$. 88. $Kab = baba \; \theta \; ba$.

1.0.5. SERIJSKO I PARALELNO NADOVEZIVANJE m-RIJEČI

DEFINICIJA 016

Za svako $x,y \in M$, serijska nadoveza od x i y (simb. $x \smile y$) je m-riječ koja rezultira iz x i y primjenom slijedećih pravila (pri čemu x i y smiju biti i prazne riječi):

(i) $x \smile Ay = Ax \smile y$

(ii) $xB \smile y = x \smile yB$

(iii) $xA \smile By = xABy$

(iv) $x \theta x_1$ & $y \theta y_1 \rightarrow x \smile y \theta x_1 \smile y_1$.

PRIMJERI

89. A ⌣ AB = AA ⌣ B = AAB. 90. aBa ⌣ aBa θ aBa ⌣ ABbaaBa θ AaBa ⌣ BbaaBa θ aBaA ⌣ BbABa θ aBaABbABa θ aBABa.
91. ABA ⌣ BA = ABABA. 92. aBa ⌣ ba θ aBaA ⌣ Bba θ aBaABba θ aBa.
93. AB ⌣ AB = AA ⌣ BB = AABB. 94. ab ⌣ ab θ abbaB ⌣ aabaab θ aaabba ⌣ baabbb θ abbaaa ⌣ bbbaab θ abbaaabbbaab θ aBAb.
95. ab ⌣ ba θ aBaAB ⌣ Bba θ aBaA ⌣ BbaB θ aBaABbaB θ ab.

DEFINICIJA 017

Za svako $x,y \in M$, paralelna nadoveza od x i y (simb. $x \frown y$) je m-riječ koja rezultira iz x i y primjenom slijedećih pravila (pri čemu x i y smiju biti i prazne riječi):

(i) $x \frown By = Bx \frown y$

(ii) $xA \frown y = x \frown yA$

(iii) $xB \frown Ay = xBAy$

(iv) $x \theta x_1$ & $y \theta y_1 \rightarrow x \frown y \theta x_1 \frown y_1$.

PRIMJERI

96. BA ⌢ AB = B ⌢ ABA = BABA. 97. A ⌢ A = AA. 98. B ⌢ A = BA.
99. BA ⌢ A = B ⌢ AA = BAA. 100. B ⌢ B = BB.
101. A ⌢ B θ AabBA ⌢ BAabB θ BAabB ⌢ AabBA θ BAabBAabBA θ BA.

1.0.6. KVAZI UREĐENJE m-RIJEČI

DEFINICIJA 018

Za svako $x,y \in M$, $x \, r \, y$, ako i samo ako početak od x ne dolazi alfabetski ranije od početka od y i svršetak od x ne dolazi alfa - betski iza svršetka od y.

PRIMJERI

102. ba r ba. 103. ba r A. 104. ba r B. 105. ba r ab.
106. A r A . 107. A r ab. 108. B r ab. 109. ab r AB.
110. AB r ab.

1.0.7. KANONSKE m-RIJEČI I NJIHOVA DULJINA

DEFINICIJA 019

Kanonska m-riječ je x, ako i samo ako x je m-riječ, koja u skupu ekvivalentnih m-riječi ima minimalan broj slova.

TEOREM 04

Za svako $x \in M$, x je kanonska m-riječ, ako i samo ako x sadrži samo jedno m-slovo ili su joj svaka dva susjedna m-slova međusobno dualna (m-slova su aa, bb, ab, ba, A i B).

PRIMJERI

111. aa, bb, A, B, ab, ba, AB, BA, ABA, BAB, abba, baab, abbaab i baabbaabbaab su kanonske m-riječi.
112. AA, BABB, baba i bbbb nisu kanonske m-riječi.

TEOREM 05

Ako su x i y kanonske m-riječi, onda $x \ \theta \ y$ povlači $x = y$.

DEFINICIJA 020

Za svako $x \in M$, kanonska forma od x je y, ako i samo ako y je kanonska m-riječ i $y \ \theta \ x$.

PRIMJERI

113. Kanonska forma od AA je A.
114. Kanonska forma od babaab je baab.

DEFINICIJA 021

m_t-Slova su aa (resp.A) i bb (resp. B). m_o-Slova, odnosno m-nule su ba i ab, pri čemu se ba, resp. ab zove s-nula, resp. p-nula. m-Jedinice su BA i AB, pri čemu se BA, resp. AB zove s-jedinica, resp. p-jedinica.

DEFINICIJA 022

Za svako $x \in M$, pozitivna m-riječ je x, ako i samo ako kanonska forma od x sadrži bar jedno m_t-slovo. M_{poz} je skup pozitivnih m-riječi.

DEFINICIJA 023

Za svako $x \in M$, nepozitivna m-riječ je x, ako i samo ako kanonska forma od x sadrži bar jedno m_o-slovo. M_{nepoz} je skup nepozitivnih m-riječi.

TEOREM 06

Za svako $x \in M$, x je pozitivna m-riječ ili x je nepozitivna m-riječ, ali ne oboje.

DEFINICIJA 024

Za svako $x \in M$, x je nenegativna m-riječ, ako i samo ako x je pozitivna m-riječ ili x je m-nula. M_{neneg} je skup nenegativnih m-riječi.

DEFINICIJA 025

Za svako $x \in M$, x je negativna m-riječ, ako i samo ako x nije nenegativna m-riječ. M_{neg} je skup negativnih m-riječi.

PRIMJERI

115. {AAA, bbaaaa, ABBAba, B, baB} $\subset M_{poz}$.
116. {ba, ab, baba, aBABa, abaBBB} $\subset M_{nepoz}$.
117. {AA, bbbaa, ba, ab, Bbababa } $\subset M_{neneg}$.
118. {aBa, bAb, aBAb, bABABABABa } $\subset M_{neg}$.

DEFINICIJA 026

Ako se a valuira sa 0 i b sa 1, onda za svako $x \in M$, defekt od x (simb. h(x)) jednako je valuaciji svršetka od x manje valuacija početka od x.

PRIMJERI

119. h(aa) = 0. 120. h(BA) = 0 - 1 = -1. 121. h(ab) = 1.
122. h(BAB) = 0. 123. h(ABBAB) = 1.

DEFINICIJA 027

(i) Za svako $x \in M_{poz}$, s-duljina od x jednaka je broju s-jedinica (BA) u kanonskoj formi od x.

(ii) Za svako $x \in M_{nepoz}$, s-duljina od x jednaka je negativnom broju p-nula (ab) u kanonskoj formi od x.

(iii) s-duljina od x simbolički se označava sa $\lambda_s(x)$.

DEFINICIJA 028

(i) Za svako $x \in M_{poz}$, p-duljina od x jednaka je broju p-jedinica (AB) u kanonskoj formi od x.

(ii) Za svako $x \in M_{nepoz}$, p-duljina od x jednaka je negativnom broju s-nula (ba) u kanonskoj formi od x.

(iii) p-duljina od x simbolički se označava sa $\lambda_p(x)$.

DEFINICIJA 029

(i) Za svako $x \in M_{poz}$, o-duljina od x jednaka je broju m_t-slova u kanonskoj formi od x.

(ii) Za svako $x \in M_{nepoz}$, o-duljina od x jednaka je 1-n, ako i samo ako n je broj m_o-slova u kanonskoj formi od x.

(iii) o-duljina od x simbolički se označava sa $\lambda(x)$.

PRIMJERI

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s-, p- i o-Duljina prvih osam, po rastućoj o-duljini, pozitivnih m-riječi			
x	$\lambda_s(x)$	$\lambda_p(x)$	$\lambda(x)$
A	0	0	1
B	0	0	1
AB	0	1	2
BA	1	0	2
ABA	1	1	3
BAB	1	1	3
ABAB	1	2	4
BABA	2	1	4

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s-;p- i o-Duljina prvih osam, po padajućoj o-duljini, nepozitivnih m-riječi			
x	$\lambda_s(x)$	$\lambda_p(x)$	$\lambda(x)$
ab	-1	0	0
ba	0	-1	0
abba	-1	-1	-1
baab	-1	-1	-1
abbaab	-2	-1	-2
baabba	-1	-2	-2
abbaabba	-2	-2	-3
baabbaab	-2	-2	-3

TEOREM 07

Za svako $x \in M$, $h(x) = \lambda_p(x) - \lambda_s(x)$.

1.1. U T E M E L J E N J E   S I S T E M A   M

DEFINICIJA 1

$$(\forall W\in S)(\Psi(W) = \{(G^*,(G,f)): G^*\in S \;\&\; G\in S \;\&$$

$$(f\in S_{funkc}(((W \cup G^*) \times (W \cup G^*)), G^*) \;\&$$

$$(s,t\in W \longrightarrow f(s,t)\in G \;\&\; (x,y\in G \longrightarrow f(x,y)\in G) \;\&$$

$$((H\in S \;\&\; H \subseteq G \;\&\; (s,t\in W \rightarrow f(s,t)\in H) \;\&\; (x,y\in H \rightarrow f(x,y)\in H)) \longrightarrow H = G) \;\&$$

$$(G^* = G \cup \{z: (s\in W \;\&\; x\in G) \longrightarrow z = f(s,x)\}) \;\&$$

$$(x,y,z\in G^* \longrightarrow f(f(x,y),z) = f(x,f(y,z))) \;\&$$

$$(r,s,t\in W \longrightarrow (f(f(r,s),f(r,t)) = f(r,t) \;\&\; f(f(r,s),f(t,s)) = f(r,s))) \;\&$$

$$((s,t\in W \;\&\; x,y\in G \;\&\; (f(s,x) = f(t,y) \vee f(x,t) = f(y,t))) \longrightarrow s = t) \;\&$$

$$((s,t\in W \;\&\; x,y\in G \;\&\; f(x,f(s,t)) = f(y,f(s,t)) \longrightarrow (x = y \vee f(x,f(s,t)) = x \vee f(y,f(s,t)) = y) \;\&$$

$$((s,t\in W \;\&\; x,y\in G \;\&\; f(f(s,t),x) = f(f(s,t),y)) \longrightarrow (x = y \vee f(f(s,t),x) = x \vee f(f(s,t),y) = y) \;\&$$

$$((p,r,s,t\in W \;\&\; x\in G \;\&\; f(x,x) = f(f(p,r),f(s,t))) \longrightarrow (x = f(p,t) \;\&\; (p = s \vee r = t)))\}).$$

A K S I O M   1

$$(W\in S \;\&\; \kappa(W) \le 2) \longrightarrow (\exists! X)(X\in\Psi(W)).$$

DEFINICIJA 2

$$(i)(M^* = G^* \;\&\; M = G \;\&\; \wedge = f) \longleftrightarrow ((G^*,(G,f))\in \Psi(\{a,b\}) \;\&\; \kappa(\{a,b\}) = 2).$$

$$(ii)(\text{Sistem } M)_{\{a,b\}} = (M, \wedge).$$

D2,A1,D1 $\succ\!\!-\!\!-$

TEOREM 1

$\Psi(\{a,b\}) = \{(M^*,(M,\wedge)\}$ & $M^* \in S$ & $M \subseteq M^*$ &

$\wedge \in S_{surjekc}(((\{a,b\} \cup M^*) \times (\{a,b\} \cup M^*)), M^*)$.

D2 $\succ\!\!-\!\!-$

TEOREM 2

$\neg\, a = b$.

DEFINICIJE 3-4

3 $(\forall x,y \in M^*)(xy = x \wedge y)$.

4 $\Omega = \{x\colon s,t \in \{a,b\} \rightarrow x = s \wedge t\}$.

D3,D4 $\succ\!\!-\!\!-$

TEOREM 3

$\Omega = \{aa,\ ab,\ ba,\ bb\}$.

T1,D1,D3,T3 $\succ\!\!-\!\!-$

TEOREMI 4-12

4 $\Omega \subseteq M$.

5 $x,y \in M \longrightarrow xy \in M$.

6 $((H \in S$ & $\Omega \subseteq H$ & $H \subseteq M$ & $(x,y \in H \rightarrow xy \in H)) \longrightarrow H = M)$.

7 $(\forall x,y,z \in M^*)(x(yz) = (xy)z)$.

8 $(\forall r,s,t \in \{a,b\})(rsrt = rt$ & $rsts = rs)$.

9 $((\forall p,r,s,t \in \{a,b\})(\forall x,y \in M)((prx = sty \vee xrp = yts \longrightarrow p = s)$.

10 $(\forall u \in \Omega)(\forall x,y \in M)(ux = uy \longrightarrow (x = y \vee ux = x \vee uy = y))$.

11 $(\forall u \in \Omega)(\forall x,y \in M)(xu = yu \longrightarrow (x = y \vee xu = x \vee yu = y))$.

12 $(\forall p,r,s,t \in \{a,b\})(\forall x \in M)(xx = prst \longrightarrow (x = pt$ & $(p = s \vee r = t)))$.

TEOREM 13
$(\forall u\in\Omega)(uu = u)$.

D o k a z

(1) $u\in\Omega$	Sup.
(2) $(\exists s,t\in\{a,b\})(u = st)$	1,T3
(3) $ust = stst$	2
(4) $uu = stst$	3,2
(5) $uu = st$	4,T8

1,2,5 $\succ\!\!-\!\!-$ T13.

TEOREM 14
$(\forall p,r,s,t\in\{a,b\})(pr = st \;\rightarrow\; p = s \;\&\; r = t)$.

D o k a z

(1) $p,r,s,t\in\{a,b\} \;\&\; pr = st$	Sup.
(2) $prrr = sttt$	1,T8
(3) $p = s \;\&\; r = t$	2,T9

1,3 $\succ\!\!-\!\!-$ T14.

DEFINICIJA 5
$(\forall s,t\in\{a,b\})(\bar{s} = t \;\leftrightarrow\; \neg\, s = t)$.

D5,T2 $\succ\!\!-\!\!-$ T15,T16.

TEOREM 15
$\bar{a} = b$.

TEOREM 16
$\bar{b} = a$.

D5,T15,T16 $\succ\!\!-\!\!-$

TEOREM 17
$(\forall s\in\{a,b\})(\bar{\bar{s}} = s)$.

DEFINICIJA 6
$A = aa$.

DEFINICIJA 7
$B = bb$.

DEFINICIJA 8
$\Omega_t = \{x\colon s\in\{a,b\} \;\&\; x = ss\}$.

D6-D8 >——

TEOREM 18

$\Omega_t = \{aa,bb\} = \{A,B\}$.

DEFINICIJA 9

$\Omega_o = \{x:\ s\in\{a,b\}\ \ \&\ \ x = s\bar{s}\}$.

D9,T15,T16 >——

TEOREM 19

$\Omega_o = \{ab,ba\}$.

T3,T18,T19 >——

TEOREM 20

$\Omega_t \cup \Omega_o = \Omega\ \ \&\ \ \Omega_t \cap \Omega_o = 0$.

1.2. O P E R A T O R I   I Z L J U Š T E N J A

1.2.1. ZAVRŠECI m-BROJA

TEOREM 21

$(\forall x\in M)(\exists!\, s\in\{a,b\})(ssx = x)$.

D o k a z

(1) $u\in\Omega$	Sup.
(2) $(\exists s,t\in\{a,b\})(u = st)$	1,D4
(3) $ssu = sst$	2
(4) $(\forall u\in\Omega)(\exists s\in\{a,b\})(ssu = u)$.	1,2,3,T8
(5) $G = \{x:\ x\in M\ \ \&\ \ (\exists s\in\{a,b\})(ssx = x)\}$.	Def.
(6) $\Omega \subseteq G$	4,5
(7) $x\in G\ \ \&\ \ y\in G$	Sup.
(8) $(\exists s\in\{a,b\})(ssx = x)$	7,5
(9) $(ssx)y = xy$	8,T1
(10) $ss(xy) = xy$	9,T7
(11) $x,y\in G \rightarrow xy\in G$	5,7,10
(12) $G = M$.	5,6,11,T6
(13) $(\forall x\in M)(\exists s\in\{a,b\})(ssx = x)$.	5,12
(14) $(\exists x\in M)(\exists s,t\in\{a,b\})(ssx = x\ \ \&\ \ ttx = x)$	Sup.
(15) $ssx = ttx$	14
(16) $s = t$	15,T9
(17) $(\forall x\in M)(\exists!\, s\in\{a,b\})(ssx = x)$. Q.E.D.	13,14,16

DEFINICIJA 10

$(\forall x \in M)(l(x) = s \longleftrightarrow s \in \{a,b\} \;\&\; ssx = x).$

TEOREM 22

$l \in S_{surjekc}(M, \{a,b\}).$

D o k a z

(1) $l \in S_{funkc}(M, \{a,b\}).$	D10,T21
(2) $aa = aaaa \;\&\; bb = bbbb$	T8
(3) $l(aa) = a \;\&\; l(bb) = b$	2,D10
(4) $\{a,b\} \subseteq Adom(l)$	3,T4

1,4, $\succ$—— T22.

D10,T22 $\succ$——

TEOREM 23

$(\forall x \in M)(\forall s \in \{a,b\})(l(x) = s \longleftrightarrow ssx = x).$

TEOREM 24

$(\forall s,t \in \{a,b\})(l(st) = s).$

D o k a z

(1) $s,t \in \Omega$	Sup.
(2) $st = sst$	1,T8
(3) $l(st) = s$	2,T23

1,3 $\succ$—— T24.

T23 $\succ$——

TEOREM 25

$(\forall x \in M)(l(x)l(x)x = x).$

TEOREM 26

$(\forall x,y \in M)(l(xy) = l(x)).$

D o k a z

(1) $x,y \in M$	Sup.
(2) $x = l(x)l(x)x$	1,T25
(3) $xy = (l(x)l(x)x)y$	2
(4) $xy = l(x)l(x)(xy)$	3,T7
(5) $l(xy) = l(x).$	4,T22,T23

1,5 $\succ$—— T26.

D4,T8,T1,T7,T6,T9 (analog T21) $\rightarrowtail$
TEOREM 27
$(\forall x\in M)(\exists! s\in\{a,b\})(xss = x)$.

DEFINICIJA 11
$(\forall x\in M)(d(x) = s \;\leftrightarrow\; s\in\{a,b\} \;\&\; xss = x)$.

D11,T27,T8,T4 (analog T22) $\rightarrowtail$
TEOREM 28
$d\in S_{surjekc}(M,\{a,b\})$.

D11,T28 $\rightarrowtail$
TEOREM 29
$(\forall x\in M)(\forall s\in\{a,b\})(d(x) = s \;\leftrightarrow\; xss = x)$.

T8,T29 (analog T24) $\rightarrowtail$
TEOREM 30
$(\forall s,t\in\{a,b\})(d(st) = t)$.

T29 $\rightarrowtail$
TEOREM 31
$(\forall x\in M)(xd(x)d(x) = x)$.

T31,T7,T28,T29 (analog T26) $\rightarrowtail$
TEOREM 32
$(\forall x,y\in M)(d(xy) = d(y))$.

DEFINICIJA 12
$(\forall x\in M)(\forall s\in\{a,b\})(\bar{l}(x) = s \;\leftrightarrow\; \bar{s} = l(x))$.

DEFINICIJA 13
$(\forall x\in M)(\forall s\in\{a,b\})(\bar{d}(x) = s \;\leftrightarrow\; \bar{s} = d(x))$.

D12,D13,T15,T16 $\rightarrowtail$ T33,T34.
TEOREM 33
$(\forall x\in M)(\bar{l}(x) = a \;\leftrightarrow\; l(x) = b \;\&\; \bar{l}(x) = b \;\leftrightarrow\; l(x) = a)$.

TEOREM 34
$(\forall x\in M)(\bar{d}(x) = a \;\leftrightarrow\; d(x) = b \;\&\; \bar{d}(x) = b \;\leftrightarrow\; d(x) = a)$.

TEOREM 35

$(\neg \exists z\in M)(\forall x\in M)(zx = x)$.

D o k a z

(1) $(\exists z\in M)(\forall x\in M)(zx = x)$	Sup.
(2) $\bar{1}(z)1(z)\in M$	1,T22,T33,T3,T4
(3) $z\bar{1}(z)1(z) = \bar{1}(z)1(z)$	2,1
(4) $1(z) = \bar{1}(z)$	3,T24,T26
(5) $a = b$	4,T22,T33

1,5,T2 $\succ\!\!-$ T35

T28,T34,T3,T4,T30,T32,T2 (analog T35) $\succ\!\!-$

TEOREM 36

$(\neg \exists z\in M)(\forall x\in M)(xz = x)$.

1.2.2. LJUSKA m-BROJA

DEFINICIJA 14

$(\forall x\in M)(q(x) = 1(x)d(x))$.

D14,T3,T24,T30 $\succ\!\!-$

TEOREM 37

$(\forall u\in \Omega)(q(u) = u)$.

T22,T28,T37 $\succ\!\!-$

TEOREM 38

$q\in S_{surjekc}(M,\Omega)$.

T26,T32 $\succ\!\!-$

TEOREM 39

$(\forall x,y\in M)(q(xy) = 1(x)d(y))$.

TEOREM 40

$(\forall x\in M)(q(x)x = x)$.

D o k a z

(1) $x\in M$	Sup.
(2) $q(x)x = 1(x)d(x)x$	1,D14
(3) $\quad = 1(x)d(x)1(x)1(x)x$	2,T25,T7
(4) $\quad = 1(x)1(x)x$	3,T8
(5) $\quad = x$	4,T25

1, .5 $\succ\!\!-$ T40.

D14,T31,T7,T8(analog T40) $\succ\!\!-$

TEOREM 41

$(\forall x \in M)(xq(x) = x)$.

TEOREM 42

$(\forall x \in M)(l(x)\bar{l}(x)x = x)$.

D o k a z

(1)	$x \in M$	Sup.
(2)	$l(x)\bar{l}(x)x = l(x)\bar{l}(x)l(x)l(x)x$	1,T25,T7
(3)	$= l(x)l(x)x$	2,T8
(4)	$= x$.	3,T25

1,2,4 $\succ\!\!-$ T42.

T31,T7,T8 (analog T42) $\succ\!\!-$

TEOREM 43

$(\forall x \in M)(x\bar{d}(x)d(x) = x)$.

1.3. O P E R A T O R I   N A S L J E Đ I V A N J A

1.3.1. LIJEVI I DESNI SLJEDBENIK m-BROJA

DEFINICIJA 15

$(\forall x \in M)(P_l(x) = Px = \bar{l}(x)\bar{l}(x)x)$.

DEFINICIJA 16

$(\forall x \in M)(P_d(x) = xP = x\bar{d}(x)\bar{d}(x))$.

TEOREM 44

$(\forall x,y \in M)(P(xy) = (Px)y)$.

D o k a z

(1)	$x,y \in M$	Sup.
(2)	$P(xy) = \bar{l}(xy)\bar{l}(xy)(xy)$	1,D15
(3)	$= \bar{l}(x)\bar{l}(x)(xy)$	2,D12,T26,T33
(4)	$= (Px)y$	3,D15,T7

1,4 $\succ\!\!-$ T44.

D15,D13,T32,T34,T7 (analog T44) $\succ\!\!-$

TEOREM 45

$(\forall x,y \in M)(xy)P = x(yP))$.

TEOREM 46

$(\forall x \in M) \neg (Px = x)$.

D o k a z

(1) $(\exists x \in M)(Px = x)$	Sup.
(2) $\bar{1}(x)\bar{1}(x)x = x$	1,D15
(3) $\bar{1}(x)\bar{1}(x)x = 1(x)1(x)x$	2,T25
(4) $\bar{1}(x) = 1(x)$	3,T9
(5) $a = b$	4,T33

1,5,T2 $\succ\!\!-$ T46.

D16,T31,T9,T34 (analog T46) $\succ\!\!-$

TEOREM 47

$(\forall x \in M) \neg (xP = x)$.

TEOREM 48

$P_1 \in S_{bijekc}(M,M)$.

D o k a z

(1) $P_1 \in S_{funkc}(M,M)$.	D15,T5
(2) $(\exists x,y \in M)(Px = Py)$	Sup.
(3) $\bar{1}(x)\bar{1}(x)x = \bar{1}(y)\bar{1}(y)y$	2,D15
(4) $\bar{1}(x) = \bar{1}(y)$	3,T9
(5) $x = y \quad \vee \quad Px = x \quad \vee \quad Py = y$	3,4,T10,D15
(6) $x = y$	2,5,T46
(7) $(\forall x,y \in M)(Px = Py \rightarrow x = y)$.	2,6
(8) $y \in M$	Sup.
(9) $y = 1(y)1(y)y$	8,T25
(10) $\quad = 1(y)1(y)\bar{1}(y)1(y)y$	9,T8
(11) $\quad = (P\bar{1}(y)1(y))y$	10,D15,D12
(12) $\quad = P(\bar{1}(y)1(y)y)$	11,T44
(13) $y \in M \rightarrow (\exists x \in M)(y = Px)$	8,12,T5
(14) $M \subseteq Adom(P_1)$	13
(15) $Adom(P_1) = M$.	1,14

1,7,15 $\succ\!\!-$ T48.

D16,T5,T9,T10,T45,T31,T8,D13 (analog T48) $\succ\!\!-$

TEOREM 49

$P_d \in S_{bijekc}(M,M)$.

TEOREM 50

$(\forall x,y\in M)(x(Py) = (xP)y).$

D o k a z

(1)	$x,y\in M \ \& \ d(x) = l(y)$	Sup.
(2)	$x(Py) = x(\bar{d}(x)\bar{d}(x)y)$	1,D15,D12
(3)	$x(Py) = (x\bar{d}(x)\bar{d}(x))y$	2,T7
(4)	$x(Py) = (xP)y$	3,D16
(5)	$(\forall x,y\in M)(d(x) = l(y) \rightarrow x(Py) = (xP)y).$	1,4
(6)	$x,y\in M \ \& \ \neg(d(x) = l(y))$	Sup.
(7)	$x(Py) = x(d(x)d(x)y)$	6,D15,T33
(8)	$= (xd(x)d(x))y$	7,T7
(9)	$= xy$	8,T31
(10)	$= x(l(y)l(y)y)$	9,T25
(11)	$= x(\bar{d}(x)\bar{d}(x)y)$	10,6,T34
(12)	$= (x\bar{d}(x)\bar{d}(x))y$	11,T7
(13)	$= (xP)y$	12,D15
(14)	$(\forall x,y\in M)(\neg(d(x) = l(y)) \rightarrow x(Py) = (xP)y).$	6,13

5,14 $\succ\!\!-$ T50.

PRIMJERI

126 ab(Pab) = abbbab = abab = ab = abaaab = (abP)ab.

127 ba(Pba) = ba = (baP)ba. 128 A(PA) = ABA = (AP)A.

129 ab(Pba) = abaaba = (abP)ba. 130 A(PB) = AAB = ABB = A(PB).

DEFINICIJA 17

$(\forall x\in M)(P_l^o(x) = P^o x = x \ \& \ (\forall n\in N^+)(P_l^{n+1}(x) = P^{n+1}x = P^n Px)).$

DEFINICIJA 18

$(\forall x\in M)(P_d^o(x) = xP^o = x \ \& \ (\forall n\in N^+)(P_d^{n+1}(x) = xP^{n+1} = xPP^n)).$

D17 $\succ\!\!-$

TEOREM 51

$(\forall x\in M)(P_l^1(x) = P^1 x = Px).$

D18 $\succ\!\!-$

TEOREM 52

$(\forall x\in M)(P_d^1(x) = xP^1 = xP).$

D17,T48,T51 (tot.indukc.) $\succ\!\!-$

TEOREM 53

$(\forall n\in N^{+})(P_{1}^{n}\in S_{bijekc}(M,M))$.

D18,T49,T52 (analog T53) $\succ\!\!-$

TEOREM 54

$(\forall n\in N^{+})(P_{d}^{n}\in S_{bijekc}(M,M))$.

PRIMJERI

131 $P^{1}AB = P^{o}PAB = P^{o}BAB = BAB$. 132 $P^{2}B = P^{1}PB = PAB = BAB$.

133 $baP^{3} = baPPP = babbPP = BPP = BAP = BAB = P^{3}ab$.

DEFINICIJA 19

$(\forall x\in M)(\forall n\in N^{+})(P_{1}^{-n}(x) = P^{-n}x = y \longleftrightarrow P^{n}y = x)$.

DEFINICIJA 20

$(\forall x\in M)(\forall n\in N^{+})(P_{d}^{-n}(x) = xP^{-n} = y \longleftrightarrow yP^{n} = x)$.

DEFINICIJA 21

$(\forall x\in M)(P_{1}'(x) = P'x = P^{-1}x)$.

DEFINICIJA 22

$(\forall x\in M)(P_{d}'(x) = xP' = xP^{-1})$.

D19,T53 $\succ\!\!-$

TEOREM 55

$(\forall n\in N)(P_{1}^{n}\in S_{bijekc}(M,M))$.

D20,T54 $\succ\!\!-$

TEOREM 56

$(\forall n\in N)(P_{d}^{n}\in S_{bijekc}(M,M))$.

TEOREM 57

$(\forall x,y\in M)(\forall n\in N)(P^{n}(xy) = (P^{n}x)y)$.

D o k a z

T44,D17 (tot.indukc.) $\succ\!\!-$

(1) $(\forall x,y\in M)(\forall n\in N^{+})(P^{n}(xy) = (P^{n}x)y)$.

(2)	$x,y\in M$ & $n\in N^{+}$	Sup.
(3)	$(\exists z\in M)(x = P^{n}z)$	2,T55
(4)	$xy = (P^{n}z)y$	2,3

(5) $xy = P^n(zy)$ 4,1
(6) $P^{-n}(xy) = zy$ 5,D19
(7) $P^{-n}(xy) = (P^{-n}x)y$ 6,3,D19
1,2,7 $\succ$—— T57.

T45,D18,T56,D20 (analog T57) $\succ$——
TEOREM 58
$(\forall x,y\in M)(\forall n\in N)((xy)P^n = x(yP^n))$.

TEOREM 59
$(\forall x,y\in M)(\forall n\in N)(x(P^n y) = (xP^n)y)$.

D o k a z
T50,D17,D18 (tot.indukc.) $\succ$——
(1)$(\forall x,y\in M)(\forall n\in N^+)(x(P^n y) = (xP^n)y)$.

(2) $x,y\in M$ & $n\in N^+$ Sup.
(3)$(\exists w,z\in M)(xP^{-n} = w$ & $P^{-n}y = z)$ 2,T55,T56
(4)$(xP^{-n})y = wy$ & $x(P^{-n}y) = xz$ 2,3
(5)$(xP^{-n})y = w(P^n z)$ & $x(P^{-n}y) = (wP^n)z$ 4,3,D19
(6)$(xP^{-n})y = x(P^{-n}y)$ 5,1
1,2,6 $\succ$—— T59.

TEOREM 60
$(\forall x\in M)(\forall m,n\in N)(P^m P^n x = P^{m+n}x)$.

D o k a z
D17 (tot.indukc.) $\succ$——
(1)$(\forall x\in M)(\forall m,n\in N^+)(P^m P^n x = P^{m+n}x)$.
1,D19 $\succ$——
(2)$(\forall x\in M)(\forall m,n\in N)(sgn(m) = sgn(n) \rightarrow P^m P^n x = P^{m+n}x)$.

(3) $x\in M$ & $m,n\in N$ & $\neg(sgn(m) = sgn(n))$ & $P^m P^n x = y$. Sup.
3,D19 $\succ$——
(4) $P^n x = P^{-m}y$
3 $\succ$——
(5)$(\exists k\in N)((n+k=-m \lor n=-m+k)$ & $(sgn(k)=sgn(n)=sgn(-m)))$
4,5 $\succ$——
(6) $P^n x = P^{n+k}y \lor P^{-m+k}x = P^{-m}y$
2,5,6,T55 $\succ$——
(7) $x = P^k y \lor P^k x = y$

7,5,D19 >——

(8) $y = P^{n+m}x$

3,8 >——

(9) $(\forall x\in M)(\forall m,n\in N)(\neg(\mathrm{sgn}(m) = \mathrm{sgn}(n)) \rightarrow P^mP^nx = P^{m+n}x)$.

2,9 >—— T60.

D18,D20,T56 (analog T60) >——

TEOREM 61

$(\forall x\in M)(\forall m,n\in N)(xP^mP^n = xP^{m+n})$.

TEOREM 62

$(\forall x\in M)(\forall n\in N)(d(P^nx) = d(x))$.

D o k a z

(1) $x\in M$	Sup.
(2) $d(Px) = d(\bar{I}(x)\bar{I}(x)x)$	1,D15
(3) $(\forall x\in M)(d(Px) = d(x))$.	1,2,T32
3,D17 (tot.indukc.) >——	
(4) $(\forall x\in M)(\forall n\in N^+)(d(P^nx) = d(x))$.	
(5) $x\in M \;\&\; n\in N^+ \;\&\; P^{-n}x = y$	Sup.
(6) $x = P^ny$	5,D19
(7) $d(x) = d(y)$	6,4
(8) $(\forall x\in M)(\forall n\in N)(d(P^{-n}x) = d(x))$.	5,7
4,8 >—— T62.	

D16,T26,D19 (analog T62) >——

TEOREM 63

$(\forall x\in M)(\forall n\in N)(l(xP^n) = l(x))$.

TEOREM 64

$(\forall x\in M)(P'x = \bar{I}(x)l(x)x)$.

D o k a z

(1) $x\in M$	Sup.
(2) $(\exists y\in M)(x = Py)$	1,T53
(3) $x = \bar{I}(y)\bar{I}(y)y$	2,D15
(4) $l(x) = \bar{I}(y)$	3,T26,T24
(5) $\bar{I}(x)l(x)x = l(y)\bar{I}(y)\bar{I}(y)\bar{I}(y)y$	2,4,T33
(6) $\quad = l(y)\bar{I}(y)y$	4,T8
(7) $\quad = y$	5,T42
(8) $P^{-1}x = \bar{I}(x)l(x)x$	2,7,D19
1,8,D21 >—— T64.	

T54,D16,T30,T32,T34,T8,T43,D20,D22 (analog T64) $\succ\!\!-\!\!-$

TEOREM 65

$(\forall x\in M)(xP' = xd(x)\bar{d}(x)).$

PRIMJERI

134 $P'ba = abba = aBa.$

135 $P'A = baA = ba.$

136 $P'ab = baab = bAb.$

137 $P'B = abB = ab.$

138 $baP' = baab = bAb.$

139 $AP' = Aab = ab.$

140 $abP' = abba = aBa.$

141 $BP' = Bba = ba.$

142 $P'AAABAA = BAA.$

143 $BABBBBP' = BA.$

144 $A(P'B) = AabB = ab = AabB = (AP')B.$

145 $(BP')B = BbaB = B(P'B) = Bab = B.$

146 $P^{-2}BA = P'P'BA = P'abBA = baabBA = baaa = ba.$

TEOREM 66

$(\forall x\in M)(\forall u\in\Omega)(ux = x \;\vee\; ux = Px \;\vee\; ux = P'x).$

D o k a z

(1) $(\exists x,y\in M)(\exists u\in\Omega)(y = ux).$ Sup.

(2) $y = l(x)l(x)x \;\vee\; y = \bar{l}(x)\bar{l}(x)x \;\vee$
$y = \bar{l}(x)l(x)x \;\vee\; y = l(x)\bar{l}(x)x$ 1,D4,T22

(3) $y = x \;\vee\; y = Px \;\vee\; y = P'x \;\vee\; y = x$ 2,T25,D15,T64,T42

1,3, $\succ\!\!-\!\!-$ T66.

D4,T28,T31,D16,T65,T43 (analog T66) $\succ\!\!-\!\!-$

TEOREM 67

$(\forall x\in M)(\forall u\in\Omega)(xu = x \;\vee\; xu = xP \;\vee\; xu = xP').$

1.3.2. q-SLJEDBENIK m-BROJA

DEFINICIJE 23-24

23 $(\forall x\in M)(\forall n\in N)(Q^n x = P^{2n}x).$

24 $(\forall x\in M)(\forall n\in N)(xQ^n = xP^{2n}).$

DEFINICIJE 25-28

25 $(\forall x\in M)(Qx = Q^1x).$

26 $(\forall x\in M)(xQ = xQ^1).$

27 $(\forall x\in M)(Q'x = Q^{-1}x).$

28 $(\forall x\in M)(xQ' = xQ^{-1}).$

D23,D24,D17,D18 $\succ\!\!-\!\!-$

TEOREM 68

$(\forall x\in M)(Q^0x = x = xQ^0)$.

D23,D19 $\succ\!\!-\!\!-$

TEOREM 69

$(\forall x,y\in M)(\forall n\in N)(Q^nx = y \;\rightarrow\; x = Q^{-n}y)$.

D24,D20 $\succ\!\!-\!\!-$

TEOREM 70

$(\forall x,y\in M)(\forall n\in N)(xQ^n = y \;\rightarrow\; x = yQ^{-n})$.

TEOREM 71

$(\forall x\in M)(\forall n\in N)(Q^nx = xQ^n)$.

D o k a z

D23,D17 $\succ\!\!-\!\!-$ 1,2

(1) $QA = PPA = PBA = ABA = ABP = APP = AQ$.

(2) $QB = BQ$.

D23,D17,T8 $\succ\!\!-\!\!-$ 3,4

(3) $Qba = PPba = Paaba = bbaaba = bbaa = babbaa = baPP = baQ$.

(4) $Qab = abQ$.

1,2,3,4,T3 $\succ\!\!-\!\!-$

(5) $(\forall u\in\Omega)(Qu = uQ)$.

(6)	$G = \{x\colon x\in M \;\&\; Qx = xQ\}$.	Def.
(7)	$\Omega \subseteq G$	6,5
(8)	$x\in G \;\&\; y\in G$	Sup.
(9)	$Q(xy) = (Qx)y$	8,D23,T58
(10)	$= (xQ)y$	9,8,6
(11)	$= x(Qy)$	10,T59
(12)	$= x(yQ)$	11,8,6
(13)	$= (xy)Q$	12,D23,T57
(14)	$x,y\in G \;\rightarrow\; xy\in G$	8,13,11
(15)	$G = M$	6,7,14,T6
(16)	$(\forall x\in M)(Qx = xQ)$.	6,15
(17)	$H = \{n\colon n\in N^+ \;\&\; (\forall x\in M)(Q^nx = xQ^n)\}$	Def.
(18)	$0\in H$	17,T68
(19)	$n\in H$	Sup.

(20)	$(\forall x \in M)(Q^{n+1}x = QxQ^n)$	19,D23,T60
(21)	$= xQ^{n+1})$	20,16,T61
(22)	$n \in H \rightarrow (n+1) \in H$	19,21,17

17,18,22 (tot.indukc.) >——

(23)	$H = N^+$	
(24)	$(\forall x \in M)(\forall n \in N^+)(Q^n x = xQ^n)$.	23,17
(25)	$(\forall x \in M)(\forall n \in N^+)(\exists y \in M)(Q^{-n}x = y)$	D23,T55
(26)	$(x = Q^n y)$	25,D19
(27)	$(x = yQ^n)$	26,24
(28)	$(xQ^{-n} = y)$	27,T70
(29)	$(\forall x \in M)(\forall n \in N^+)(Q^{-n}x = xQ^{-n})$.	28,25

24,29 >—— T71.

D23,D24,T55-T61,T71 >—— T72-T76.

TEOREM 72

$(\forall n \in N)(Q^n \in S_{bijekc}(M,M))$.

TEOREM 73

$(\forall x,y \in M)(\forall n \in N)(Q^n(xy) = (Q^n x)y \ \& \ (xy)Q^n = x(yQ^n))$.

TEOREM 74

$(\forall x,y \in M)(\forall n \in N)(x(Q^n y) = (xQ^n)y)$.

TEOREM 75

$(\forall x \in M)(\forall m,n \in N)(Q^m Q^n x = Q^{m+n}x \ \& \ xQ^{m+n} = xQ^m Q^n)$.

TEOREM 76

$(\forall x,y \in M)(\forall n \in N)(Q^n xy = xQ^n y \ \& \ xyQ^n = xQ^n y)$.

PRIMJERI

147 Qba = bbaaba = BA. 148 Q'ba = baabba = bABa. 149 QA = ABA.
150 Qab = aabbab = AB. 151 Q'ab = abbaab = aBAb. 152 QB = BAB.
153 QBA = BQA = BAQ. 154 Q'BA = ba. 155 ab = Q'AB.

TEOREM 77

$(\forall x \in M)(\forall n \in N)(\neg(P^{2n+1}x = xP^{2n+1}))$.

D o k a z

(1)	$(\exists x \in M)(\exists n \in N)(P^{2n+1}x = xP^{2n+1})$	Sup.
(2)	$P^{2n}(Px) = (xP^{2n})P$	1,T60
(3)	$Q^n(Px) = (xQ^n)P$	2,D23

(4) $Q^n(Px) = Q^n(xP)$	3,T71,T73
(5) $Px = xP$	4,T72
(6) $\bar{l}(x)\bar{l}(x)x = x\bar{d}(x)\bar{d}(x)$	5,D15,D16
(7) $\bar{l}(x) = l(x)$ & $d(x) = \bar{d}(x)$	6,T26,T24,T32,T30
(8) $a = b$	7,T33,T34

1,8,T2 $\succ\!\!-\!\!-$ T77.

TEOREM 78

$(\forall x\in M)(\forall n\in N)(q(Q^n x) = q(x))$.

D o k a z

(1) $(\forall x\in M)(\forall n\in N)(q(Q^n x)$	$= l(Q^n x)d(Q^n x))$	D14
(2)	$= l(xQ^n)d(Q^n x))$	1,T71
(3)	$= l(x)d(x))$	2,T62,T63

3,D14 $\succ\!\!-\!\!-$ T78.

TEOREM 79

$(\forall x\in M)(\exists n\in N)(x = Q^n q(x))$.

D o k a z

T37,T68 $\succ\!\!-\!\!-$

(1) $(\forall u\in Q)(u = Q^o q(u))$.

T3,D23 $\succ\!\!-\!\!-$

(2) $(\forall u,v\in Q)(\exists k\in N)(uv = Q^k l(uv)d(uv))$.

(3) $G = \{x:\ x\in M\ \ \&\ \ (\exists n\in N)(x = Q^n q(x))\}$	Def.
(4) $Q \subseteq G$	3,1
(5) $x\in G$ & $y\in G$	Sup.
(6) $(\exists p,r\in N)(x = Q^p q(x)$ & $y = Q^r q(y))$	5,3
(7) $xy = Q^{p+r}q(x)q(y)$	6,T75
(8) $xy = Q^{p+r+k}l(q(x)q(y))d(q(x)q(y))$	7,2,T75
(9) $xy = Q^{p+r+k}l(x)d(y)$	8,T26,T32,D14,T24,T30
(10) $xy = Q^{p+r+k}q(xy)$	9,D14,T26,T32
(11) $x,y\in G \rightarrow xy\in G$	5,10,3
(12) $G = M$	3,4,11,T6

3,12 $\succ\!\!-\!\!-$ T79.

TEOREM 80

$(\forall x\in M)(\exists n\in N)(x = P^{n}\,\bar{d}(x)d(x) = l(x)\bar{l}(x)P^{n})$.

D o k a z

T3,D15,D16 $\succ\!\!-\!\!-$

(1)$(\forall u\in \Omega)(\exists k\in N)(u = P^{k}\,\bar{d}(u)d(u) = l(u)\bar{l}(u)P^{k})$.

(2)$(\forall x\in M)(\exists k,m\in N)(x=Q^{m}P^{k}\,\bar{d}(x)d(x)=l(x)\bar{l}(x)P^{k}Q^{m})$ 1,T79,T71

(3) $x = P^{2m+k}\,\bar{d}(x)d(x) = l(x)\bar{l}(x)P^{2m+k}$ 2,D23,T60,T61

2,3 $\succ\!\!-\!\!-$ T80.

TEOREM 81

$(\forall x,y\in M)(xy = yx \longleftrightarrow q(x) = q(y))$.

D o k a z

(1)$(\exists x,y\in M)(xy = yx)$ Sup.

(2) $l(x) = l(y)$ & $d(y) = d(x)$ 1,T26,T32

(3) $q(x) = q(y)$ 2,D14

(4)$(\forall x,y\in M)(xy = yx \rightarrow q(x) = q(y))$. 3,1

(5)$(\exists x,y\in M)(q(x) = q(y))$ Sup.

(6)$(\exists m,n\in N)(x = Q^{m}q(x)$ & $y = Q^{n}q(y))$ 5,T79

(7) $xy = Q^{m+n}q(x)q(y)$ 6,T75,T76

(8) $xy = Q^{n}q(y)Q^{m}q(x)$ 7,5,T75,T76

(9)$(\forall x,y\in M)(q(x) = q(y) \rightarrow xy = yx)$. 8,6,5

4,9 $\succ\!\!-\!\!-$ T81.

1.4. ČETVORNA GRUPA PERMUTACIJA m-BROJEVA

1.4.1. OPERATOR DUALNOSTI

DEFINICIJA 29
$(\forall x \in M)(Dx = \bar{x} = PxP')$.

TEOREM 82
$(\forall x \in M)(DDx = x)$.

(1) $x \in M$	Sup.
(2) $DDx = D(PxP')$	1,D29
(3) $\quad = PPxP'P'$	2,D29
(4) $\quad = QxQ'$	3,D23,D24
(5) $\quad = x$.	4,T71,T75,D27,T68

1,5 $\succ\!\!-$ T82.

T82 $\succ\!\!-$
TEOREM 83
$(\forall x,y \in M)(Dx = y \rightarrow Dy = x)$.

TEOREM 84
$D \in S_{bijekc}(M,M)$.

D o k a z
D29,T48,T56,T1 $\succ\!\!-$
(1) $D \in S_{funkc}(M,M)$.

(2) $x \in M$	Sup.
(3) $(\exists y \in M)(Dx = y)$	2,1
(4) $DDx = Dy$	3
(5) $(\forall x \in M)(\exists y \in M)(x = Dy)$	2,3,4,T82
(6) $Adom(D) = M$.	1,5
(7) $x,y \in M$ & $Dx = Dy$	Sup.
(8) $DDx = DDy$	7
(9) $(\forall x,y \in M)(Dx = Dy \rightarrow x = y)$.	7,8,T82

1,6,9 $\succ\!\!-$ T84.

D29,D15,T65 $\succ\!\!\!-\!\!-$

TEOREM 85

$(\forall x \in M)(Dx = \bar{l}(x)\bar{l}(x)xd(x)\bar{d}(x))$.

PRIMJERI

156 Daa=bbaaab=bbab=bb.

157 Dbb=aabbba=aaba=aa.

158 DAB=bbABba=BA.

159 Dba=aabaab=ab.

160 Dab=bbabba=ba.

161 DaBAb=bbaBAbba=bABa.

TEOREM 86

$(\forall s,t \in \{a,b\})(D(st) = \bar{s}\bar{t})$.

D o k a z

T15,T16 $\succ\!\!\!-\!\!-$

$(1)(\forall s,t \in \{a,b\})(s = t \;\vee\; \bar{s} = t)$.

(2)	$s,t \in \{a,b\}$				Sup.
(3)	$D(st)$	$= \bar{s}\bar{s}stt\bar{t}$			2,T85
(4)	$D(st)$	$= \bar{s}\bar{s}sss\bar{s}$	v	$D(st) = \bar{s}\bar{s}s\bar{s}\bar{s}s$	3,1
(5)		$= \bar{s}\bar{s}s\bar{s}$	v	$= \bar{s}\bar{s}\bar{s}s$	4,T8
(6)		$= \bar{s}\bar{s}$	v	$= \bar{s}s$	5,T8
(7)		$= \bar{s}\bar{t}$	v	$= \bar{s}\bar{t}$	6,1,T17

2,7 $\succ\!\!\!-\!\!-$ T86.

TEOREM 87

$(\forall x,y \in M)(D(xy) = (Dx)(Dy))$.

D o k a z

(1)	$x,y \in M$		Sup.
(2)	$D(xy)$	$= PxyP'$	1,D29
(3)		$= (PxP')(PyP')$	2,T50,T60,T61
(4)		$= (Dx)(Dy)$	3,D29

1,4 $\succ\!\!\!-\!\!-$ T87.

PRIMJERI

162 DAB = BA.

163 DBA = AB.

164 DBBABAAB = AABABBA.

165 DaBABAb = bABABa .

166 Dabba = baab.

167 Dbaab = abba.

TEOREM 88

$(\forall x \in M)(Dx = P'xP)$.

D o k a z

(1) $x \in M$	Sup.
(2) $Dx = P'P(Dx)P'P$	1,T60,T61
(3) $\quad = P'(DDx)P$	2,D29
(4) $\quad = P'xP$	1,3,T82

1,4 ≻—— T88.

T88,T64 ≻——

TEOREM 89

$(\forall x \in M)(Dx = \bar{l}(x)l(x)x\bar{d}(x)\bar{d}(x))$.

PRIMJERI

168. Daa=baaabb=bb. 169. Dba=abbabb=ab. 170. DAB=baABab=BA.

TEOREM 90

$(\forall x \in M)(l(\bar{x}) = \bar{l}(x) \;\&\; d(\bar{x}) = \bar{d}(x))$.

D o k a z

(1) $x \in M$	Sup.
(2) $Dx = \bar{l}(x)\bar{l}(x)xd(x)\bar{d}(x)$	1,T85
(3) $l(Dx) = \bar{l}(x) \;\&\; d(Dx) = \bar{d}(x)$	1,2,T26,T32

1,3,D29 ≻—— T90.

PRIMJERI

171. $l(\bar{A})$=b. 172. $d(\bar{B})$=a. 173. $l(\bar{b}\bar{a})$=a. 174. $d(\bar{a}\bar{b})$=a.

DEFINICIJA 30

$(\forall x \in M)(\bar{q}(x) = Dq(x))$.

PRIMJERI

175. $\bar{q}(A)$=B. 176. $\bar{q}(B)$=A. 177. $\bar{q}(ba)$=ab. 178. $\bar{q}(ab)$=ba.
179. $\bar{q}(BAB)$=A.

TEOREM 91

$(\forall x \in M)(\bar{q}(x) = \bar{l}(x)\bar{d}(x))$.

D o k a z

(1) $x \in M \;\&\; l(x) = s \;\&\; d(x) = t$	Sup.
(2) $\bar{q}(x) = D(st)$	1,D30,D14
(3) $\quad = \bar{s}\bar{t}$	2,T86
(4) $\quad = \bar{l}(x)\bar{d}(x)$	1,3,T33,T34

1,4 ≻—— T91.

TEOREM 92

$(\forall x \in M)(q(\bar{x}) = \bar{q}(x))$.

D o k a z

(1)	$x \in M$	Sup.
(2)	$q(Dx) = l(Dx)d(Dx)$	1,D14
(3)	$= \bar{l}(x)\bar{d}(x)$	2,T90
(4)	$= \bar{q}(x)$	3,T91

1,4 $\succ\!\!-\!\!-$ T92.

TEOREM 93

$(\forall x \in M)(\forall n \in N)(P^n Dx = DP^n x)$.

D o k a z

(1)	$x \in M \ \& \ n \in N$	Sup.
(2)	$P^n Dx = P^n PxP'$	1,D29
(3)	$= P^{n+1} xP'$	2,T60,T51
(4)	$= PP^n xP'$	3,T60,T51
(5)	$= DP^n x$	4,D29

1,5 $\succ\!\!-\!\!-$ T93.

D29,T61,T52 (analog T93) $\succ\!\!-\!\!-$

TEOREM 94

$(\forall x \in M)(\forall n \in N)((Dx)P^n = D(xP^n))$.

D23,D24,T93,T94 $\succ\!\!-\!\!-$

TEOREM 95

$(\forall x \in M)(\forall n \in N)(DQ^n x = Q^n Dx = (Dx)Q^n = D(xQ^n))$.

1.4.2. OPERATOR KOMPLEMENTARNOSTI

TEOREM 96

$(\forall s,t\in\{a,b\})(\forall x\in M)(sxt\in M)$.

D o k a z

T3,T5 $\succ\!\!-\!\!-$

(1)$(\forall u\in\Omega)(\forall s,t\in\{a,b\})(sut\in M)$.

(2) $G = \{x\colon x\in M \;\&\; (s,t\in\{a,b\} \rightarrow sxt\in M)\}$	Def.
(3) $\Omega \subseteq G$	2,1
(4) $x\in G \;\&\; y\in G$	Sup.
(5)$(\forall s,t\in\{a,b\})(sxd(x)\in M \;\&\; d(x)yt\in M)$	4,2,T22,T28
(6) $sxd(x)d(x)yt\in M$	5,T5
(7) $sxyt\in M$	6,T31
(8) $x,y\in G \rightarrow xy\in G$	4,7
(9) $G = M$.	2,3,8,T6

2,9 $\succ\!\!-\!\!-$ T96.

DEFINICIJA 31

$(\forall x\in M)(Kx = \bar{l}(x)x\bar{d}(x))$.

PRIMJERI

180 Kab = baba = ba.

181 Kba = abab = ab.

182 Kaa = baab = bAb.

183 Kbb = abba = aBa.

184 KaBa = babbab = B.

185 KbAb = abaaba = A.

186 KAB = bABa.

187 KaBAb= baBAba = BA.

D31,T96 $\succ\!\!-\!\!-$

TEOREM 97

$(\forall x\in M)(Kx\in M)$.

TEOREM 98

$(\forall x\in M)(KKx = x)$.

D o k a z

(1) $x\in M$	Sup.
(2) $KKx = K(\bar{l}(x)x\bar{d}(x))$	1,D31
(3) $\quad = l(x)\bar{l}(x)x\bar{d}(x)d(x)$	2,D31
(4) $\quad = x$	3,T42,T43

1,4 $\succ\!\!-\!\!-$ T98.

T98 $\succ\!\!\!-\!\!-$

TEOREM 99

$(\forall x\in M)(Kx = y \;\rightarrow\; Ky = x).$

TEOREM 100

$K\in S_{bijekc}(M,M).$

D o k a z

D31,T97 $\succ\!\!\!-\!\!-$

(1) $K\in S_{funkc}(M,M).$	
(2) $x\in M$	Sup.
(3) $(\exists y\in M)(Kx = y)$	2,1
(4) $KKx = Ky$	3
(5) $Adom(K) = M.$	1,2,4,T98
(6) $(\forall x,y\in M)(Kx = Ky \;\rightarrow\; x = y).$	T98

1,5,6 $\succ\!\!\!-\!\!-$ T100.

D31,T26,T32 $\succ\!\!\!-\!\!-$

TEOREM 101

$(\forall x\in M)(l(Kx) = \bar{l}(x) \;\&\; d(Kx) = \bar{d}(x)).$

D14,T101,T90 $\succ\!\!\!-\!\!-$

TEOREM 102

$(\forall x\in M)(q(Kx) = \bar{q}(x)).$

TEOREM 103

$(\forall x\in M)(\forall n\in N)(P^nKx = KP^{-n}x).$

D o k a z

(1) $x\in M$	Sup.
(2) $PKx = P\bar{l}(x)x\bar{d}(x)$	1,D31
(3) $\quad = l(x)l(x)\bar{l}(x)x\bar{d}(x)$	2,D15,D12
(4) $\quad = l(x)x\bar{d}(x)$	3,T42
(5) $\quad = l(x)\bar{l}(x)l(x)x\bar{d}(x)$	4,T8
(6) $\quad = K\bar{l}(x)l(x)x$	5,D31,D12
(7) $(\forall x\in M)(PKx = KP'x).$	1,6,T64
(8) $(\exists x\in M)(\exists n\in N_{poz})(P^nKx = KP^{-n}x)$	Sup.
(9) $P^{n+1}Kx = PKP^{-n}x$	8,T60
(10) $P^{n+1}Kx = KP^{-(n+1)}x$	9,7,T60

$(11)(\forall x\in M)(\forall n\in N_{poz})(P^nKx=KP^{-n}x \rightarrow P^{n+1}Kx=KP^{-(n+1)}x)$. 8,10

7,11,D21,T51 (tot.indukc.) $\succ\!\!-$
$(12)(\forall x\in M)(\forall n\in N^+)(P^nKx = KP^{-n}x)$.
D31,D15,D12,T42,T8,T64,T60,D21,T51 (analog 12) $\succ\!\!-$
$(13)(\forall x\in M)(\forall n\in N^+)(P^{-n}Kx = KP^nx)$.
12,13 $\succ\!\!-$ T103.

D31,D16,D13,T43,T8,T65,T61,D22,T52 (analog T103) $\succ\!\!-$
TEOREM 104
$(\forall x\in M)(\forall n\in N)((Kx)P^n = K(xP^{-n}))$.

T103,T104,D23,D24,T71 $\succ\!\!-$
TEOREM 105
$(\forall x\in M)(\forall n\in N)(KQ^nx = Q^{-n}Kx = (Kx)Q^{-n} = K(xQ^n))$.

1.4.3. OPERATOR INVERZNOSTI

DEFINICIJA 32
$(\forall x\in M)(Ix = KDx)$.

PRIMJERI

188 Iba = KDba = Kab = baba = ba.
189 Iab = KDab = Kba = abab = ab.
190 Iaa = KDaa = Kbb = abba = aBa.
191 Ibb = KDbb = Kaa = baab = bAb.
192 IaBa = KbAb = abAba = A.
193 IbAb = KaBa = baBab = B.
194 IBA = bABa.

TEOREM 106
$(\forall x\in M)(IIx = x)$.

D o k a z

(1)	$x\in M$	Sup.
(2)	$IIx = KDKDx$	1,D32
(3)	$= KDK\bar{l}(x)\bar{l}(x)xd(x)\bar{d}(x)$	2,T85
(4)	$= KDl(x)\bar{l}(x)\bar{l}(x)xd(x)\bar{d}(x)d(x)$	3,D31,D12,D13
(5)	$= K\bar{l}(x)xd(x)d(x)\bar{d}(x)$	4,T85,T8
(6)	$= K\bar{l}(x)x\bar{d}(x)$	5,T31
(7)	$= KKx$	6,D31
(8)	$= x$	7,T98

1,8 $\succ\!\!-$ T106.

T106 $\succ\!\!-\!\!-$
TEOREM 107
$(\forall x\in M)(Ix = y \rightarrow Iy = x)$.

D32,T84,T100 $\succ\!\!-\!\!-$
TEOREM 108
$I\in S_{bijekc}(M,M)$.

TEOREM 109
$(\forall x\in M)(q(Ix) = q(x))$.

(1)	$x\in M$	Sup.
(2)	$q(Ix) = q(KDx)$	1,D32
(3)	$= \bar{q}(Dx)$	2,T102
(4)	$= q(DDx)$	3,T92,D29
(5)	$= q(x)$	4,T82

1,5 $\succ\!\!-\!\!-$ T109.

TEOREM 110
$(\forall x\in M)(\forall n\in N)(P^n Ix = IP^{-n}x)$.

D o k a z

(1)	$x\in M$ & $n\in N$	Sup.
(2)	$P^n Ix = P^n KDx$	1,D32
(3)	$= KP^{-n}Dx$	2,T103
(4)	$= KDP^{-n}x$	3,T93
(5)	$= IP^{-n}x$	4,D32

1,5 $\succ\!\!-\!\!-$ T110.

D32,T104,T94 (analog T110) $\succ\!\!-\!\!-$
TEOREM 111
$(\forall x\in M)(\forall n\in N)((Ix)P^n = I(xP^{-n}))$.

D23,D24,T110,T111 $\succ\!\!-\!\!-$
TEOREM 112
$(\forall x\in M)(\forall n\in N)(IQ^n x = Q^{-n}Ix = (Ix)Q^{-n} = I(xQ^n))$.

TEOREM 113

$(\forall x \in M)(Ix = DKx)$.

D o k a z

(1)	$x \in M$	Sup.
(2)	$Ix = KDx$	1,D32
(3)	$= K\bar{1}(x)\bar{1}(x)xd(x)\bar{d}(x)$	2,T85
(4)	$= 1(x)\bar{1}(x)\bar{1}(x)xd(x)\bar{d}(x)d(x)$	3,D31
(5)	$= 1(x)\bar{1}(x)\bar{1}(x)xd(x)$	4,T8
(6)	$= 1(x)\bar{1}(x)\bar{1}(x)x\bar{d}(x)d(x)d(x)$	5,T43
(7)	$= 1(x)\bar{1}(x)(Kx)d(x)d(x)$	6,D31
(8)	$= \bar{1}(Kx)1(Kx)(Kx)\bar{d}(Kx)\bar{d}(Kx)$	7,T101
(9)	$= DKx$	8,T89

1,9 $\succ\!\!-$ T113.

1.4.4. GRUPA TRANSFORMACIJA IDENTIČNOSTI, DUALNOSTI, KOMPLEMENTARNOSTI I INVERZNOSTI

DEFINICIJA 33

$\Gamma = \{Q^0, D, K, I\}$.

DEFINICIJA 34

$(\forall F_1, F_2 \in \Gamma)(F_1 \star F_2 = F_1F_2 = \{(x,y): x \in M \;\&\; y = F_1F_2x\})$.

D33,D34,T68 $\succ\!\!-$

TEOREM 114

$(\forall F \in \Gamma)(Q^0F = F \;\&\; FQ^0 = F)$.

D33,T114,T82,T98,T106 $\succ\!\!-$

TEOREM 115

$(\forall F \in \Gamma)(FF = Q^0)$.

D32,D34,T113 $\succ\!\!-$

TEOREM 116

$I = KD = DK$.

TEOREM 117
D = KI = IK.

D o k a z

(1)	KI = KKD	T116
(2)	$= Q^oD$	1,T115
(3)	= D	2,T114
(4)	$= DQ^o$	3,T114
(5)	= DKK	4,T115
(6)	= IK	5,T116

1,3,6 $\succ\!\!\!-$ T117.

T114-T116 (analog T117) $\succ\!\!\!-$
TEOREM 118
K = ID = DI.

D33,D34,T114-T118 $\succ\!\!\!-$
TEOREM 119

✶	Q^o	D	K	I
Q^o	Q^o	D	K	I
D	D	Q^o	I	K
K	K	I	Q^o	D
I	I	K	D	Q^o

$\in S_{\text{Cayley tb.}}(\Gamma, ✶)$.

1.5. m-S T R U K T U R A (OPERACIJE ZBRAJANJA m-BROJEVA)

DEFINICIJA 35

$$F_{\cup} = \{((x,y),z):\ x,y\in M\ \&\ ((d(x)=a\ \&\ l(y)=a\ \&\ z=Axy)\ \vee\ (d(x)=a\ \&\ l(y)=b\ \&\ z=xy)\ \vee\ (d(x)=b\ \&\ l(y)=a\ \&\ z=Q'AxyB)\ \vee\ (d(x)=b\ \&\ l(y)=b\ \&\ z=xyB))\}.$$

D35,T22,T28,T1 ≻—

TEOREM 120

$(\forall x,y\in M)(\exists!z\in M)(((x,y),z)\in F_{\cup})$.

DEFINICIJA 36

$(\forall x,y\in M)(x \cup y = z \ \longleftrightarrow\ ((x,y),z)\in F_{\cup})$.

DEFINICIJA 37

$$F_{\cap} = \{((x,y),z):\ x,y\in M\ \&\ ((d(x)=a\ \&\ l(y)=a\ \&\ z=xyA)\ \vee\ (d(x)=a\ \&\ l(y)=b\ \&\ z=Q'BxyA)\ \vee\ (d(x)=b\ \&\ l(y)=a\ \&\ z=xy)\ \vee\ (d(x)=b\ \&\ l(y)=b\ \&\ z=Bxy))\}.$$

T120 ≻—≺

TEOREM 121

$(\forall x,y\in M)(\exists!z\in M)(((x,y),z)\in F_{\cap})$.

DEFINICIJA 38

$(\forall x,y\in M)(x \cap y = z \ \longleftrightarrow\ ((x,y),z)\in F_{\cap})$.

DEFINICIJA 39

$\mathcal{M} = (M,\ \Omega,\ \cup,\ \cap\ )$.

D36,D35 ≻—

TEOREM 122

$$(\forall x,y\in M)(x \cup y = z \ \longleftrightarrow\ ((d(x)=a\ \&\ l(y)=a\ \&\ z=Axy)\ \vee\ (d(x)=a\ \&\ l(y)=b\ \&\ z=xy)\ \vee\ (d(x)=b\ \&\ l(y)=a\ \&\ z=Q'AxyB)\ \vee\ (d(x)=b\ \&\ l(y)=b\ \&\ z=xyB))).$$

T122 ≻—≺

TEOREM 123

$$(\forall x,y\in M)(x \cap y = z \ \longleftrightarrow\ ((d(x)=b\ \&\ l(y)=b\ \&\ z=Bxy)\ \vee\ (d(x)=b\ \&\ l(y)=a\ \&\ z=xy)\ \vee\ (d(x)=a\ \&\ l(y)=b\ \&\ z=Q'BxyA)\ \vee\ (d(x)=a\ \&\ l(y)=a\ \&\ z=xyA))).$$

PRIMJERI

195 $A \cup B = AB$.

196 $B \cap A = BA$.

197 $A \cup A = AAA = A$.

198 $B \cap B = BBB = B$.

199 $B \cup A = Q'ABAB = AB$.

200 $A \cap B = Q'BABA = BA$.

201 $BA \cup aBAb = ab$.

202 $ab \cup ab = Q'aaababbb = Q'ab = aBAb$.

203 $ba \cap ba = Q'bbbababaaa = Q'ba = bABa$.

204 $AB \cup ba = aabbbabb = aababb = AB$.

205 $ab \cap BA = bbabbbaa = bbabaa = BA$.

206 $BA \cup ab = aabbaaab = aabbab = AB$.

207 $ba \cap AB = baaabbaa = babbaa = BA$.

T122,T8 >——

TEOREM 124

$\cup$	ba	A	B	ab
ba	ba	A	B	ab
A	A	A	AB	ab
B	B	AB	B	ab
ab	ab	ab	ab	Q'ab

$\in S_{\text{Cayley tb.}}(\Omega, \cup)$.

T124 >——<

TEOREM 125

$\cap$	ab	B	A	ba
ab	ab	B	A	ba
B	B	B	BA	ba
A	A	BA	A	ba
ba	ba	ba	ba	Q'ba

$\in S_{\text{Cayley tb.}}(\Omega, \cap)$.

T124,T125 >——

TEOREM 126

$(\forall u,v\in\Omega)((q(u \cup v) = ab \;\&\; q(u \cap v) = ba) \longleftrightarrow v = Du)$.

TEOREM 127

$(\forall x,y\in M)(Ax \cup y = x \cup Ay)$.

D o k a z

(1)	$x,y\in M \;\&\; d(x) = a \;\&\; l(y) = a$	Sup.
(2)	$Ax \cup y = AAxy$	1,T122
(3)	$= Axy$	2,T13
(4)	$= x \cup y$	3,T122
(5)	$= x \cup Ay$	1,4,T25
(6)	$(\forall x,y\in M)(d(x) = a \;\&\; l(y) = a \rightarrow Ax \cup y = x \cup Ay)$.	1,5

(7) $x,y \in M \;\&\; d(x) = a \;\&\; l(y) = b$ Sup.
(8) $Ax \cup y = Axy$ 7,T122
(9) $\quad = AxAy$ 8,7,T31
(10) $\quad = x \cup Ay$ 9,7,T26,T122
(11) $(\forall x,y \in M)(d(x)=a \;\&\; l(y)=b \rightarrow Ax \cup y = x \cup Ay)$. 7,10

(12) $x,y \in M \;\&\; d(x) = b \;\&\; l(y) = a$ Sup.
(13) $Ax \cup y = Q'AAxyB$ 12,T122
(14) $\quad = Q'AxyB$ 13,T13
(15) $\quad = x \cup y$ 14,12,T122
(16) $\quad = x \cup Ay$ 15,12,T25
(17) $(\forall x,y \in M)(d(x)=b \;\&\; l(y)=a \rightarrow Ax \cup y = x \cup Ay)$. 16,12

(18) $x,y \in M \;\&\; d(x) = b \;\&\; l(y) = b$ Sup.
(19) $Ax \cup y = AxyB$ 18,T122
(20) $\quad = AxQ'PPyB$ 19,T60
(21) $\quad = Q'AxAAyB$ 20,18,T59,T76
(22) $\quad = Q'AxAyB$ 21,T13
(23) $\quad = x \cup Ay$ 22,18,T122
(24) $(\forall x,y \in M)(d(x)=b \;\&\; l(y)=b \rightarrow Ax \cup y = x \cup Ay)$. 23,18

6,11,17,24,T22,T28 >—— T127.

T122,T13,T31,T25,T32,T61,T59,T76,T22,T28 (analog T127) >——

TEOREMI 128-129

128 $(\forall x,y \in M)(xB \cup y = x \cup yB)$.

129 $(\forall x,y \in M)(xA \cup By = xABy)$.

T127 >——<
TEOREM 130
$(\forall x,y \in M)(Bx \cap y = x \cap By)$.

T128 >——<
TEOREM 131
$(\forall x,y \in M)(xA \cap y = x \cap yA)$.

T129 >——<
TEOREM 132
$(\forall x,y \in M)(xB \cap Ay = xBAy)$.

PRIMJERI

208 $A \cup A = A \cup Aba = AA \cup ba = AA \cup Bba = AABba = A.$

209 $B \cup A = B \cup Aba = AB \cup ba = AB \cup Bba = A \cup BbaB = AB.$

210 $B \cup B = baB \cup B = ba \cup BB = baA \cup BB = baABB = B.$

211 $aBa \cup aBa = aBaA \cup ABbABa = AaBaA \cup BbABa = aBaABbABa = aBABa.$

212 $ab \cup ba = aBaAB \cup Bba = aBaA \cup BbaB = aBaABB = ab.$

213 $ba \cup ba = baA \cup Bba = ba.$ 214 $AB \cup AB = AA \cup BB = AB.$

215 $BA \cap BA = BB \cap AA = BBAA.$ 216 $AB \cap AB = ABAB.$

217 $AB \cap BA = BAB \cap A = BABAA.$ 218 $BA \cap AB = B \cap ABA = BBABA.$

219 $A \cap B = abA \cap Bab = BabA \cap ab = Bab \cap abA = B \cap A = BA.$

TEOREM 133

$(\forall x \in M)(A \cup x = x \cup A = Ax).$

D o k a z

(1)	$x \in M$	Sup.
(2)	$l(x) = a \quad \vee \quad l(x) = b$	1,T22
(3)	$A \cup x = AAx \quad \vee \quad A \cup x = Ax$	2,T24,T122
(4)	$(\forall x \in M)(A \cup x = Ax).$	1,3,T13
(5)	$x \in M$	Sup.
(6)	$d(x) = a \quad \vee \quad d(x) = b$	5,T28
(7)	$x \cup A = Ax \quad \vee \quad x \cup A = Q'AxAB$	6,T24,T122
(8)	$= Q'AxBAB$	7,6,T31
(9)	$= AxBPPQ'$	8,D16,T71
(10)	$= AxB$	9,T61
(11)	$= Ax$	10,T31
(12)	$(\forall x \in M)(x \cup A = Ax).$	5,11

4,12 >—— T133.

T133 >——<

TEOREM 134

$(\forall x \in M)(B \cap x = x \cap B = Bx).$

T22,T30,T25,T122,T13,T28,T60,T71 (analog T133) >——

TEOREM 135

$(\forall x \in M)(B \cup x = x \cup B = xB).$

T135 >——<

TEOREM 136

$(\forall x \in M)(A \cap x = x \cap A = xA).$

T122,T76,T78 >——

TEOREM 137

$(\forall x,y\in M)(\forall n\in N)(x \cup Q^n y = Q^n(x \cup y) = Q^n x \cup y).$

T137 >——<

TEOREM 138

$(\forall x,y\in M)(\forall n\in N)(x \cap Q^n y = Q^n(x \cap y) = Q^n x \cap y).$

TEOREM 139

$(\forall x,y\in M)(q(x \cup y) = q(q(x) \cup q(y))).$

D o k a z

(1) $x,y\in M$	Sup.
(2) $(\exists m,n\in N)(x = Q^m q(x)\ \ \&\ \ y = Q^n q(y))$	1,T79
(3) $q(x \cup y) = q(Q^m q(x) \cup Q^n q(y))$	1,2
(4) $\qquad\qquad = q(Q^{m+n}(q(x) \cup q(y))$	3,T137,T75
(5) $\qquad\qquad = q(q(x) \cup q(y)).$	4,T78

1,5 >—— T139.

T139 >——<

TEOREM 140

$(\forall x,y\in M)(q(x \cap y) = q(q(x) \cap q(y))).$

TEOREM 141

$(\forall x,y\in M)(x \cup y = y \cup x).$

D o k a z

(1) $(\forall u,v\in \Omega)(u \cup v = v \cup u).$	T124
(2) $x,y\in M$	Sup.
(3) $(\exists m,n\in N)(\exists u,v\in\Omega)(x = Q^m u\ \ \&\ \ y = Q^n v)$	2,T79,T38
(4) $x \cup y = Q^m u \cup Q^n v$	3
(5) $\qquad\quad = Q^m Q^n(u \cup v)$	4,T137
(6) $\qquad\quad = Q^m Q^n(v \cup u)$	5,1
(7) $\qquad\quad = Q^n v \cup Q^m u$	6,T137
(8) $\qquad\quad = y \cup x$	7,3

2,8 >—— T141.

T141 >——<

TEOREM 142

$(\forall x,y\in M)(x \cap y = y \cap x).$

TEOREM 143

$(\forall x,y,z)(x \cup (y \cup z) = (x \cup y) \cup z)$.

D o k a z

(1) $(\forall u,v,w\in\Omega)(u \cup (v \cup w) = (u \cup v) \cup w)$.	T124
(2) $x,y,z\in M$	Sup.
(3) $(\exists k,m,n\in N)(\exists u,v,w\in\Omega)(x=Q^k u \ \& \ y=Q^m v \ \& \ z=Q^n w)$	2,T79,T38
(4) $x \cup (y \cup z) = Q^k u \cup (Q^m v \cup Q^n w)$	3
(5) $\qquad = Q^k Q^m Q^n (u \cup (v \cup w))$	4,T137
(6) $\qquad = Q^k Q^m Q^n ((u \cup v) \cup w)$	5,1
(7) $\qquad = (Q^k u \cup Q^m v) \cup Q^n w$	6,T137
(8) $\qquad = (x \cup y) \cup z$	7,3

2,8 >—— T143.

T143 >——<

TEOREM 144

$(\forall x,y,z\in M)(x \cap (y \cap z) = (x \cap y) \cap z)$.

TEOREM 145

$(\forall x\in M)(ba \cup x = x = x \cup ba)$.

D o k a z

(1) $x\in M$	Sup.
(2) $l(x) = a \quad \vee \quad l(x) = b$	1,T22
(3) $ba \cup x = aabax \quad \vee \quad ba \cup x = bax$	2,T122
(4) $\qquad = aax \quad \vee \quad = bax$	3,T8
(5) $\qquad = x \quad \vee \quad = x$	4,2,T25,T42
(6) $ba \cup x = x = x \cup ba$	5,T141

1,6 >—— T145.

T145 >——<

TEOREM 146

$(\forall x\in M)(ab \cap x = x = x \cap ab)$.

T22,T122,T8,T25,T42 (analog T145) >——

TEOREM 147

$(\forall x\in M)(ab \cup x = Q'AxB)$.

T147 >——<

TEOREM 148

$(\forall x\in M)(ba \cap x = Q'BxA)$.

TEOREM 149

$((k \in M \;\; \& \;\; (\forall x \in M)(x \cup k = x)) \rightarrow k = ba.$

D o k a z

$(1)(\exists k \in M)(\forall x \in M)(x \cup k = x)$	Sup.
$(2)\; ba \cup k = ba$	1,T4,T3
$(3)\; k = ba$	2,T145

1,3 >—— T149.

T149 >——<

TEOREM 150

$((k \in M \;\; \& \;\; (\forall x \in M)(x \cap k = x)) \rightarrow k = ab.$

T137,T145,T146,T138 >——

TEOREM 151

$(\forall x \in M)(\forall n \in N)(x \cup Q^{n}ba = Q^{n}x = x \cap Q^{n}ab).$

T151 >——

TEOREM 152

$(\forall x \in M)(x \cup BA = Qx).$

T152 >——<

TEOREM 153

$(\forall x \in M)(x \cap AB = Qx).$

TEOREM 154

$(\forall x \in M)(x \cup Kx = ab).$

D o k a z

$(1)(\forall u \in \Omega)(u \cup Ku = ab\,).$	T124
$(2)\; x \in M$	Sup.
$(3)(\exists n \in N)(\exists w \in \Omega)(x = Q^{n}w)$	2,T79,T33
$(4)\; x \cup Kx = Q^{n}w \cup KQ^{n}w$	3
$(5)\; = Q^{n}(w \cup KQ^{n}w)$	4,T137
$(6)\; = Q^{n}(w \cup Q^{-n}Kw)$	5,T105
$(7)\; = w \cup Kw$	6,T137,T75,T68
$(8)\; = ab$	7,3,1

2,8 >—— T154.

T154 >——<

TEOREM 155

$(\forall x \in M)(x \cap Kx = ba).$

TEOREM 156

$(\forall x,y\in M)((x \cup y = ab \;\&\; x \cap y = ba) \rightarrow y = Kx)$.

D o k a z

(1) $(\exists x,y\in M)(x \cup y = ab \;\&\; x \cap y = ba)$	Sup.
(2) $q(x \cup y) = ab \;\&\; q(x \cap y) = ba$	1,T37
(3) $q(q(x) \cup q(y)) = ab \;\&\; q(q(x) \cap q(y)) = ba$	2,T139,T140
(4) $q(y) = Dq(x)$	3,T38,T126
(5) $q(y) = q(Kx)$	4,T102,D30
(6) $x \cup y \cup Kx = ab \cup Kx$	1
(7) $ab \cup y = ab \cup Kx$	6,T154
(8) $Q'AyB = Q'A(Kx)B$	7,T147
(9) $AyB = A(Kx)B$	8,T72
(10) $AyB=A\bar{l}(x)x\bar{d}(x)B \;\&\; l(y)=\bar{l}(x) \;\&\; d(y)=\bar{d}(x)$	9,D31,4,T86
(11) $(q(x)=A \;\&\; Py=AbxbB) \vee (q(x)=B \;\&\; yP=AaxaB) \vee$ $(q(x)=ba \;\&\; y=AaxbB) \vee (q(x)=ab \;\&\; PyP=AbxaB)$	10,4,T33,T34
(12) $(q(x)=A \;\&\; y=baAbxb) \vee (q(x)=B \;\&\; y=axaBba) \vee$ $(q(x)=ba \;\&\; y=axb) \vee (q(x)=ab \;\&\; y=baAbxaBba)$	11,D19,D20 T64,T65,T8
(13) $(q(x)=A \;\&\; y=bxb) \vee (q(x)=B \;\&\; y=axa) \vee$ $(q(x)=ba \;\&\; y=axb) \vee (q(x)=ab \;\&\; y=bxa)$	12,T8
(14) $y = Kx$	13,D31

1,14 $\succ\!\!\!-\!\!-$ T156.

TEOREM 157

$(\forall x,y\in M)(D(x \cup y) = Dx \cap Dy)$.

D o k a z

(1) $(\forall u,v\in Q)(D(u \cup v) = Du \cap Dv)$	T124
(2) $x,y\in M$	Sup.
(3) $(\exists m,n\in N)(\exists u,v\in Q)(x = Q^m u \;\&\; y = Q^n v)$	2,T79
(4) $D(x \cup y) = D(Q^m u \cup Q^n v)$	3
(5) $= DQ^m Q^n(u \cup v)$	4,T137
(6) $= Q^m Q^n D(u \cup v)$	5,T95
(7) $= Q^m Q^n(Du \cap Dv)$	6,3,1
(8) $= Q^m Du \cap Q^n Dv$	7,T137
(9) $= DQ^m u \cap DQ^n v$	8,T95
(10) $= Dx \cap Dy$	9,3

2,10 $\succ\!\!\!-\!\!-$ T157.

T157 $\succ\!\!\longrightarrow\!\!\prec$

TEOREM 158

$(\forall x,y\in M)(D(x \cap y) = Dx \cup Dy)$.

1.6. R E L A C I J A r

1.6.1. KVAZI UREĐENJE m-BROJEVA

DEFINICIJA 40

$(\forall s,t\in\{a,b\})((s <_{alf} t) \longleftrightarrow (s = a \;\&\; t = b))$.

D40 $\succ\!\!\longrightarrow$ T159-T161.

TEOREM 159

$a <_{alf} b$.

TEOREM 160

$(\forall s,t\in\{a,b\})((\inf\{s,t\} = a \longleftrightarrow s = a \vee t = a) \;\&$
$(\inf\{s,t\} = b \longleftrightarrow s = b \;\&\; t = b))$.

TEOREM 161

$(\forall s,t\in\{a,b\})((\sup\{s,t\} = b \longleftrightarrow s = b \vee t = b) \;\&$
$(\sup\{s,t\} = a \longleftrightarrow s = a \;\&\; t = a))$.

TEOREM 162

$(\forall x,y\in M)(l(x \cup y) = \inf\{l(x),l(y)\})$.

D o k a z

(1)	$x,y\in M$	Sup.
(2)	$l(x) = l(y) = s \;\vee\; \neg\, l(x) = l(y)$	1,T22
(3)	$l(x \cup y) = s \;\vee\; l(x \cup y) = a$	2,T124,T26
(4)	$l(x \cup y) = \inf\{l(x),l(y)\}$	2,3,T160

1,4 $\succ\!\!\longrightarrow$ T162.

T22,T28,T124,T125,T160,T161 (analog T162) $\succ\!\!\longrightarrow$ T163-T165.

TEOREM 163

$(\forall x,y\in M)(l(x \cap y) = \sup\{l(x),l(y)\})$.

TEOREM 164

$(\forall x,y\in M)(d(x \cup y) = \sup\{d(x),d(y)\})$.

TEOREM 165
$(\forall x,y\in M)(d(x \cap y) = \inf\{d(x),d(y)\})$.

D14,T162,T164 $\succ$——
TEOREM 166
$(\forall x,y\in M)(q(x \cup y) = \inf\{l(x),l(y)\}\sup\{d(x),d(y)\})$.

D14,T163,T165 $\succ$——
TEOREM 167
$(\forall x,y\in M)(q(x \cap y) = \sup\{l(x),l(y)\}\inf\{d(x),d(y)\})$.

T159-T167,T22,T28 $\succ$—— T168-T174.
TEOREM 168
$(\forall x\in M)(q(x \cup x) = q(x))$.

TEOREM 169
$(\forall x\in M)(q(x \cap x) = q(x))$.

TEOREM 170
$(\forall x,y\in M)(q(x \cup (y \cap x)) = q(x))$.

TEOREM 171
$(\forall x,y\in M)(q(x \cap (y \cup x)) = q(x))$.

TEOREM 172
$(\forall x,y\in M)(q(x \cup y) = q(y) \longleftrightarrow q(x \cap y) = q(x))$.

TEOREM 173
$(\forall x,y,z\in M)(q(x \cup (y \cap z)) = q((x \cup y) \cap (x \cup z)))$.

TEOREM 174
$(\forall x,y,z\in M)(q(x \cap (y \cup z)) = q((x \cap y) \cup (x \cap z)))$.

DEFINICIJA 41
$(\forall x,y\in M)(x\ r\ y \longleftrightarrow q(x \cup y) = q(y))$.

DEFINICIJA 42
$(\forall x,y\in M)(x\ \bar{r}\ y \longleftrightarrow q(x \cap y) = q(y))$.

TEOREM 175

$(\forall x,y\in M)(x\ r\ y \leftrightarrow y\ \bar{r}\ x)$.

D o k a z

(1) $x,y\in M$ & $x\ r\ y$ — Sup.
(2) $q(x \cup y) = q(y)$ — 1,D41
(3) $q(x \cap y) = q(x)$ — 2,T172
(4) $(\forall x,y\in M)(x\ r\ y \rightarrow y\ \bar{r}\ x)$. — 1,3,D42
(5) $(\exists x,y\in M)(y\ \bar{r}\ x)$ — Sup.
(6) $q(x \cap y) = q(y)$ — 5,D42
(7) $q(x \cup y) = q(y)$ — 6,T172
(8) $(\forall x,y\in M)(y\ \bar{r}\ x \rightarrow x\ r\ y)$. — 5,7,D41

4,8 $\succ\!\!-$ T175.

TEOREM 176

$(\forall x,y\in M)(x\ r\ y \leftrightarrow (\exists z\in M)(x \cup z = y))$.

D o k a z

(1) $x,y\in M$ & $x\ r\ y$ — Sup.
(2) $q(x \cup y)=q(y)$ & $(\exists m,n\in N)(x \cup y=Q^m q(x \cup y)$ & $y = Q^n q(y))$ — 1,D41,T79
(3) $x \cup y = Q^m q(y)$ — 2
(4) $Q^n(x \cup y) = Q^m Q^n q(y)$ — 3,T75
(5) $x \cup Q^{n-m}y = y$ — 4,2,T69,T137
(6) $(\forall x,y\in M)((x\ r\ y \rightarrow (\exists z\in M)(x \cup z = y))$. — 1,5,T72
(7) $(\exists x,y,z\in M$ & $x \cup z = y$ — Sup.
(8) $x \cup x \cup z = x \cup y$ — 7
(9) $q(x \cup x \cup z) = q(x \cup y)$ — 8
(10) $q(q(x \cup x) \cup q(z)) = q(x \cup y)$ — 9,T139,T143,T37
(11) $q(q(x) \cup q(z)) = q(x \cup y)$ — 10,T168
(12) $q(x \cup z) = q(x \cup y)$ — 11,T139
(13) $q(y) = q(x \cup y)$ — 12,7
(14) $(\forall x,y,z\in M)(x \cup z = y \rightarrow x\ r\ y)$. — 7,13,D41

6,14 $\succ\!\!-$ T176.

T176 $\succ\!\!-\!\!\prec$

TEOREM 177

$(\forall x,y\in M)(x\ \bar{r}\ y \leftrightarrow (\exists z\in M)(x \cap z = y))$.

T176,T145 $\succ\!\!-$

TEOREM 178

$(\forall x \in M)(x\ r\ x).$

TEOREM 179

$(\forall x,y,z \in M)((x\ r\ y\ \&\ y\ r\ z) \rightarrow x\ r\ z).$

D o k a z

(1) $x,y,z \in M\ \&\ x\ r\ y\ \&\ y\ r\ z$	Sup.
(2) $(\exists u,v \in M)(x \cup u = y\ \&\ y \cup v = z)$	1,T176
(3) $x \cup u \cup v = z$	2

1,3,T176 $\succ\!\!-$ T179.

TEOREM 180

$(\forall x,y \in M)(x\ r\ y \rightarrow (\forall z \in M)(x \cup z\ r\ y \cup z\ \&\ x \cap z\ r\ y \cap z)).$

D o k a z

(1) $x,y \in M\ \&\ x\ r\ y$	Sup.
(2) $(\exists w \in M)(x \cup w = y)$	1,T176
(3) $(\forall z \in M)(x \cup z \cup w = y \cup z)$	2
(4) $(\forall x,y \in M)(x\ r\ y \rightarrow (\forall z \in M)(x \cup z\ r\ y \cup z)).$	1,3,T176
(5) $y\ \bar{r}\ x$	1,T175
(6) $(\exists w \in M)(y \cap w = x)$	5,T177
(7) $(\forall z \in M)(y \cap z \cap w = x \cap z)$	6
(8) $y \cap z\ \bar{r}\ x \cap z$	7,T177
(9) $(\forall x,y \in M)(x\ r\ y \rightarrow (\forall z \in M)(x \cap z\ r\ y \cap z)).$	1,8,T175

4,9 $\succ\!\!-$ T180.

TEOREM 181

$(\forall x,y \in M)(x\ r\ y \leftrightarrow q(x)\ r\ q(y)).$

D o k a z

(1) $x,y \in M\ \&\ x\ r\ y$	Sup.
(2) $(\exists z \in M)(x \cup z = y\ \&$ $(\exists m,n \in N)(x = Q^m q(x)\ \&\ y = Q^n q(y))$	1,T176,T79
(3) $Q^m q(x) \cup z = Q^n q(y)$	2
(4) $q(x) \cup Q^{m-n} z = q(y)$	3,T137,T69,T75
(5) $(\forall x,y \in M)(x\ r\ y \rightarrow q(x)\ r\ q(y)).$	1,4,T150
(6) $(\exists x,y \in M)(q(x)\ r\ q(y))$	Sup.

(7)$(\exists z\in M)(q(x) \cup z = q(y)$ &
$(\exists m,n\in N)(x = Q^m q(x)$ & $y = Q^n q(y))$ 6,T176,T79
(8) $Q^{m+n}(q(x) \cup z) = Q^{m+n}q(y)$ 7
(9) $Q^m q(x) \cup Q^{n-m}q(z) = Q^n q(y)$ 8,T137,T69,T75
(10) $x \cup Q^{n-m}q(z) = y$ 9,7
(11)$(\forall x,y\in M)(q(x)\ r\ q(y) \rightarrow x\ r\ y)$. 6,10,T176
5,11 $\succ\!\!-$ T181.

T181 $\succ\!\!-\!\!\prec$
TEOREM 182
$(\forall x,y\in M)(x\ \bar{r}\ y \leftrightarrow q(x)\ \bar{r}\ q(y))$.

TEOREM 183
$(\forall x,y\in M)(x\ r\ y \leftrightarrow (l(y) <_{alf} l(x) \vee l(y) = l(x))$ &
$(d(x) <_{alf} d(y) \vee d(x) = d(y)))$.

D o k a z
(1) $x,y\in M$ & $x\ r\ y$ Sup.
(2) $q(x \cup y) = q(y)$ 1,D41
(3) $l(x \cup y) = l(y)$ & $d(x \cup y) = d(y)$ 2,D14
(4) $\inf\{l(x),l(y)\}=l(y)$ & $\sup\{d(x),d(y)\}=d(y)$ 3,T162,T164
1,4,T162,T164 $\succ\!\!-$
(5)$(\forall x,y\in M)(x\ r\ y \rightarrow ((l(y) <_{alf} l(x) \vee l(y) = l(x))$ &
$(d(x) <_{alf} d(y) \vee d(x) = d(y)))$.
D41,D14,T162,T164 (analog 5) $\succ\!\!-$
(6)$(\forall x,y\in M)((l(y) <_{alf} l(x) \vee l(y) = l(x)$ &
$(d(x) <_{alf} d(y) \vee d(x) = d(y))) \rightarrow x\ r\ y)$.
5,6 $\succ\!\!-$ T183.

D42,D14,T163,T165 (analog T183) $\succ\!\!-$
TEOREM 184
$(\forall x,y\in M)(x\ \bar{r}\ y \leftrightarrow (l(x) <_{alf} l(y) \vee l(x) = l(y))$ &
$(d(y) <_{alf} d(x) \vee d(y) = d(x)))$.

T183,T22,T28,T159 $\succ\!\!-$
TEOREM 185
$(\forall x,y\in M)(x\ r\ y$ & $y\ r\ x \leftrightarrow q(x) = q(y))$.

T183,T159,T22 $\succ\!\!-$ T186-T188.
TEOREM 186
$(\forall x\in M)(ba\ r\ x$ & $x\ r\ ab)$.

TEOREM 187
$(\forall x \in M)(A \; r \; x \iff l(x) = a)$.

TEOREM 188
$(\forall x \in M)(B \; r \; x \iff d(x) = b)$.

T187,T188,T2 $\succ\!\!-\!\!-$
TEOREM 189
$\neg(A \; r \; B) \;\; \& \;\; \neg(B \; r \; A)$.

PRIMJERI
220 $ba \; r \; ABA$. 221 $B \; r \; BAB$. 222 $BAB \; r \; B$. 223 $ab \; \bar{r} \; A$.
224 $ab \; \bar{r} \; B$. 225 $ab \; \bar{r} \; ba$. 226 $ab \; r \; AB$. 227 $BA \; r \; BABA$.
228 $BABA \; r \; BA$.

1.6.2. MODULARNA DISTRIBUTIVNOST OPERACIJA ZBRAJANJA

TEOREM 190
$(\forall x,y,z \in M)(x \cup (y \cap z) = (x \cup y) \cap z \iff x \; r \; z)$.

D o k a z
T183,T24,T30,T159 $\succ\!\!-\!\!-$
(1)$(\forall u,v,w \in \Omega)(u \cup (v \cap w) = (u \cup v) \cap w \iff u \; r \; w)$.
T79,T137,T75,T72 $\succ\!\!-\!\!-$
(2)$(\forall x,y,z \in M)(x \cup (y \cap z) = (x \cup y) \cap z \iff$
$q(x) \cup (q(y) \cap q(z)) = (q(x) \cup q(y)) \cap q(z))$.
2,T138,1,T181 $\succ\!\!-\!\!-$ T190.

T190 $\succ\!\!-\!\!-\!\!\prec$
TEOREM 191
$(\forall x,y,z \in M)(x \cap (y \cup z) = (x \cap y) \cup z \iff x \; \bar{r} \; z)$.

PRIMJERI
229 $A \cup (B \cap A) = (A \cup B) \cap A = ABA$. 230 $A \cup (BA \cap AB) =$
$(A \cup BA) \cap AB = ABA \cap (B \cup A) = (ABA \cap B) \cup A = ABABA$.
231 $AB \cap (B \cup A) = (AB \cap B) \cup A = ABAB$.
232 $A \cup (A \cap B) \neq (A \cup A) \cap B$.
233 $AB \cup (A \cap B) \neq (AB \cup A) \cap B$.

1.7. NERAZLOŽIVOST (ELEMENTARNOST) m-STRUKTURE

1.7.1. s-JEZGRA I p-JEZGRA m-BROJA

DEFINICIJA 43

$(\forall x \in M)(J_s(x) = baxba)$.

DEFINICIJA 44

$(\forall x \in M)(J_p(x) = abxab)$.

PRIMJERI

234	$J_s(A) = ba$.		240	$J_s(ab) = bABa$.		246	$J_s(BAB) = BA$.	
235	$J_p(B) = ab$.		241	$J_p(ba) = aBAb$.		247	$J_p(ABA) = AB$.	
236	$J_s(B) = ba$.		242	$J_s(BA) = BA$.		248	$J_s(aBAb) = bABABa$.	
237	$J_p(A) = ab$.		243	$J_p(AB) = AB$.		249	$J_p(bABa) = aBABAb$.	
238	$J_s(ba) = ba$.		244	$J_s(ABA) = BA$.		250	$J_s(bABa) = bABa$.	
239	$J_p(ab) = ab$.		245	$J_p(BAB) = AB$.		251	$J_p(aBAb) = aBAb$.	

TEOREM 192

$(\forall x \in M)(J_s(x) = (x \cup ab) \cap ba)$.

D o k a z

(1)	$x \in M$	Sup.
(2)	$(x \cup ab) \cap ba = Q'B(x \cup ab)A$	1,T148
(3)	$= Q'BQ'AxBA$	2,T147
(4)	$= P^{-2}BAxBAP^{-2}$	3,T71,D23,D24
(5)	$= P'AxBP'$	4,T64,T65,T8
(6)	$= baxba$	5,T64,T65,T8
(7)	$= J_s(x)$	6,D43

1,7 >—— T192.

T192 >——<

TEOREM 193

$(\forall x \in M)(J_p(x) = (x \cap ba) \cup ab)$.

PRIMJERI

252 $J_s(ABAB) = (ABAB \cup ab) \cap ba = AB \cap ba = BA$.

253 $J_s(BABA) = (BABA \cup ab) \cap ba = ABAB \cap ba = BABA$.

254 $J_p(ABAB) = (ABAB \cap ba) \cup ab = BABA \cup ab = ABAB$.

255 $J_p(BABA) = (BABA \cap ba) \cup ab = BA \cup ab = AB$.

D43,T39 $\succ\!\!-$

TEOREM 194

$(\forall x\in M)(q(J_s(x)) = ba)$.

T194 $\succ\!\!-\!\!\prec$

TEOREM 195

$(\forall x\in M)(q(J_p(x)) = ab)$.

1.7.2. s-RASTAV I p-RASTAV m-BROJA

TEOREM 196

$(\forall x\in M)(q(x) = ba \rightarrow x = ba \cup J_s(x))$.

D o k a z

(1)	$x\in M \ \&\ q(x) = ba$	Sup.
(2)	$J_s(x) = baxba$	1,D43
(3)	$= x$	2,1,T42,T43

1,3,T145 $\succ\!\!-$ T196.

T196 $\succ\!\!-\!\!\prec$

TEOREM 197

$(\forall x\in M)(q(x) = ab \rightarrow x = ab \cap J_p(x))$.

TEOREM 198

$(\forall x\in M)(q(x) = A \rightarrow x = A \cup J_s(x))$.

D o k a z

(1)	$x\in M \ \&\ q(x) = A$	Sup.
(2)	$J_s(x) = baxba = bax$	1,D43,T43
(3)	$A \cup J_s(x) = aabax$	2,T133
(4)	$= aax$	3,T8
(5)	$= x$	4,T25

1,5 $\succ\!\!-$ T198.

T198 $\succ\!\!-\!\!\prec$

TEOREM 199

$(\forall x\in M)(q(x) = B \rightarrow x = B \cap J_p(x))$.

T42,T135,T8,T31 (analog T198) $\succ\!\!-\!\!-$

TEOREM 200

$(\forall x \in M)(q(x) = B \;\rightarrow\; x = B \cup J_s(x))$.

T200 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 201

$(\forall x \in M)(q(x) = A \;\rightarrow\; x = A \cap J_p(x))$.

TEOREM 202

$(\forall x \in M)(q(x) = ab \;\rightarrow\; x = AB \cup J_s(x))$.

D o k a z

(1)	$x \in M$ & $q(x) = ab$	Sup.
(2)	$AB \cup J_s(x) = AB \cup baxba$	1,D43
(3)	$= A \cup baxbabb$	2,T128
(4)	$= aabaxbabb$	3,T133
(5)	$= AxB$	4,T8
(6)	$= x$	5,1,T25,T31

1,6 $\succ\!\!-\!\!-$ T202.

T202 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 203

$(\forall x \in M)(q(x) = ba \;\rightarrow\; x = BA \cap J_p(x))$.

TEOREM 204

$(\forall x \in M)(\exists ! u \in M)(x = u \cup J_s(x))$.

D o k a z

T196,T198,T200,T202,T38 $\succ\!\!-\!\!-$

(1) $(\forall x \in M)(\exists u \in M)(x = u \cup J_s(x))$.

(2)	$(\exists x,u,w \in M)(x = u \cup J_s(x)$ & $x = w \cup J_s(x))$,	Sup.
(3)	$(\exists n \in N)(J_s(x) = Q^n ba)$	2,T79,T194
(4)	$u \cup Q^n ba = w \cup Q^n ba$	3,2
(5)	$Q^n(u \cup ba) = Q^n(w \cup ba)$	4,T137
(6)	$u = w$	5,T75,T145

1,2,6 $\succ\!\!-\!\!-$ T204.

T204 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 205

$(\forall x \in M)(\exists ! u \in M)(x = u \cap J_p(x))$.

DEFINICIJA 45

$(\forall x,y\in M)(q_s(x) = u \;\longleftrightarrow\; x = u \cup J_s(x)).$

DEFINICIJA 46

$(\forall x,y\in M)(q_p(x) = u \;\longleftrightarrow\; x = u \cap J_p(x)).$

D45 >——

TEOREM 206

$(\forall x\in M)(x = q_s(x) \cup J_s(x)).$

T206 >——<

TEOREM 207

$(\forall x\in M)(x = q_p(x) \cap J_p(x)).$

T206,T207 >——

TEOREM 208

$(\forall x\in M)(q_s(x) \cup J_s(x) = q_p(x) \cap J_p(x)).$

DEFINICIJA 47

$\Omega_s = \{u\colon\ x\in M \;\&\; u = q_s(x)\}.$

DEFINICIJA 48

$\Omega_p = \{u\colon\ x\in M \;\&\; u = q_p(x)\}.$

D47,D45,T196,T198,T200,T202 >——

TEOREM 209

$\Omega_s = \{ba,\ A,\ B,\ AB\}.$

T209 >——<

TEOREM 210

$\Omega_p = \{ab,\ B,\ A,\ BA\}.$

T209,T183 >——

TEOREM 211

$(\forall u,w\in\Omega_s)(u \cup w = z \;\longleftrightarrow\; z\in\Omega_s \;\&\; u\ r\ z \;\&\; w\ r\ z \;\&\; (\forall x\in M)(u\ r\ x \;\&\; w\ r\ x \;\rightarrow\; z\ r\ x)).$

T211 >——<

TEOREM 212

$(\forall u,w\in\Omega_p)(u \cap w = z \;\longleftrightarrow\; z\in\Omega_p \;\&\; u\ \bar{r}\ z \;\&\; w\ \bar{r}\ z \;\&\; (\forall x\in M)(u\ \bar{r}\ x \;\&\; w\ \bar{r}\ x \;\rightarrow\; z\ \bar{r}\ x)).$

T209,D43 $\succ\!\!-\!\!-$

TEOREM 213

$(\forall u\in\Omega_s)(J_s(u) = ba).$

T213 $\succ\!\!-\!\!\prec$

TEOREM 214

$(\forall u\in\Omega_p)(J_p(u) = ab).$

TEOREM 215

$(\forall u\in\Omega_s)(q_s(u) = u).$

D o k a z

(1) $u\in\Omega_s$	Sup.
(2) $u = q_s(u) \cup J_s(u)$	1,T206
(3) $u = q_s(u) \cup ba$	2,1,T213

1,3,T145 $\succ\!\!-\!\!-$ T215.

T215 $\succ\!\!-\!\!\prec$

TEOREM 216

$(\forall u\in\Omega_p)(q_p(u) = u).$

T196-T202,D45,D46,T3,T209,T210 $\succ\!\!-\!\!-$

TEOREM 217

$$\begin{aligned}(\forall x\in M)((q(x) = ba &\longleftrightarrow q_s(x) = ba \longleftrightarrow q_p(x) = BA)\ \&\\ (q(x) = A &\longleftrightarrow q_s(x) = A \longleftrightarrow q_p(x) = A\ )\ \&\\ (q(x) = B &\longleftrightarrow q_s(x) = B \longleftrightarrow q_p(x) = B\ )\ \&\\ (q(x) = ab &\longleftrightarrow q_s(x) = AB \longleftrightarrow q_p(x) = ab)).\end{aligned}$$

T217 $\succ\!\!-\!\!-$ T218-T220.

TEOREM 218

$(\forall x\in M)(l(q_s(x)) = l(x)\ \ \&\ \ d(q_s(x)) = d(x)).$

TEOREM 219

$(\forall x\in M)(l(q_p(x)) = l(x)\ \ \&\ \ d(q_p(x)) = d(x)).$

TEOREM 220

$(\forall x,y\in M)(q_s(x) = q_s(y) \longleftrightarrow q(x) = q(y) \longleftrightarrow q_p(x) = q_p(y)).$

TEOREM 221

$(\forall x\in M)(\forall F_1,F_2\in\{q,q_s,q_p\})(F_1(F_2(x)) = F_1(x))$.

D o k a z

T37,T215,T216 $\succ\!\!\!-\!\!\!-$

(1) $(\forall x\in M)(\forall F\in\{q,q_s,q_p\})(F(F(x)) = F(x))$.

(2) $x\in M$ & $F_1,F_2\in\{q,q_s,q_p\}$ & $F_1(F_2(x)) = u$	Sup.
(3) $F_1(F_2(x)) = F_1(u)$	2,1
(4) $F_2(F_2(x)) = F_2(u)$	3,T220
(5) $F_2(x) = F_2(u)$	4,1
(6) $F_1(x) = F_1(u)$	5,T220

2,3,6 $\succ\!\!\!-\!\!\!-$ T221.

TEOREM 222

$(\forall x\in M)(\forall n\in N)(Q^n x = x \rightarrow n = 0)$.

D o k a z

(1) $(\exists u\in\Omega)(\exists k\in N^+)(Q^{2k+1}u = u)$	Sup.
(2) $(\exists s,t\in\{a,b\})(Q^{2k+1}st = st)$	1,T3
(3) $Q^{2k}st = stQ'$	2,T71,T70,D28
(4) $Q^{2k}st = stt\bar{t}\bar{t}t$	3,T65
(5) $Q^{2k}ttst = ttstt\bar{t}\bar{t}t$	4,T76
(6) $Q^{2k}tt = t\bar{t}\bar{t}t$	5,T8
(7) $Q^k ttQ^k tt = t\bar{t}\bar{t}t$	6,T13,T75,T76
(8) $Q^k tt = tt$ & $t = \bar{t}$	7,T12
(9) $(\forall u\in\Omega)(\forall k\in N^+)\neg(Q^{2k+1}u = u)$.	1,8,T15,T16,T2
(10) $G=\{n: n\in N_{poz}$ & $(\forall u\in\Omega)\neg(Q^n u = u)\}$	Def.
(11) $1\in G$	10,9
(12) $(\exists n\in N_{poz})(\neg n\in G$ & $(\forall m\in N_{poz})(m < n \rightarrow m\in G)$	Sup.
(13) $(\exists k\in N_{poz})(n = 2k)$	12,10,9
(14) $(\exists u\in\Omega)(Q^{2k}u = u)$	13,12,10
(15) $Q^k uQ^k u = uu$	14,T75,T76
(16) $(\exists k\in N_{poz})(\exists u\in\Omega)(k < n$ & $Q^n u = u)$	15,T12,13
(17) $(\forall m,n\in N_{poz})((m < n \rightarrow m\in G) \rightarrow n\in G)$	16,12,10

10,11,17 (2.pr.indukc.) $\succ\!\!\!-\!\!\!-$

(18) $G = N_{poz}$	
(19) $(\forall n\in N^+)(\forall u\in\Omega)(Q^n u = u \rightarrow n = 0)$.	18,10
(20) $(\exists n\in N^+)(\exists u\in\Omega)(Q^{-n}u = u)$	Sup.

(21) $Q^n u = u$ 20,T70

(22)$(\forall u\in\Omega)(\forall n\in N)(Q^n u = u \rightarrow n = 0)$. 20,21,19

(23)$(\exists x\in M)(\exists n\in N)(Q^n x = x)$ Sup.

(24)$(\exists u\in\Omega)(\exists m\in N)(Q^{m+n} u = Q^m u)$ 23,T79,T75

(25) $Q^n u = u$ 24,T75,T72

(26) $n = 0$ 25,22

23,26 >—— T222.

TEOREM 223

$(\forall x\in M)(\exists! n\in N)(x = Q^n q_s(x))$.

D o k a z

(1) $x\in M$ Sup.

(2)$(\exists k\in N)(q_s(x) = Q^k q(q_s(x))$ 1,T79

(3) $= Q^k q(x)$ 2,T221

(4)$(\forall x\in M)(\exists m\in N)(q(x) = Q^m q_s(x))$. 1,3,T69

(5)$(\forall x\in M)(\exists n\in N)(x = Q^n q_s(x))$. T79,4,T75

(6)$(\exists x\in M)(\exists m,n\in N)(x = Q^m q_s(x)\ \&\ x = Q^n q_s(x))$ Sup.

(7) $Q^m q_s(x) = Q^n q_s(x)$ 6

(8)$(\exists k\in N)(Q^{m+k} q(x) = Q^{n+k} q(x))$. 7,4,T69

(9) $Q^{m-n} q(x) = q(x)$ 8,T69,T75

(10) $m = n$. 9,T222

5,6,10 >—— T223.

T223 >——<

TEOREM 224

$(\forall x\in M)(\exists! n\in N)(x = Q^n q_p(x))$.

DEFINICIJA 49

$(\forall x\in M)(\forall n\in N)(\lambda_s(x) = n \leftrightarrow x = Q^n q_s(x))$.

DEFINICIJA 50

$(\forall x\in M)(\forall n\in N)(\lambda_p(x) = n \leftrightarrow x = Q^n q_p(x))$.

T223,D49 >——

TEOREM 225

$(\forall x\in M)(x = Q^{\lambda_s(x)} q_s(x))$.

T225 $\succ\!\!\!-\!\!\!\prec$

TEOREM 226

$(\forall x\in M)(x = Q^{\lambda_p(x)} q_p(x)).$

TEOREM 227

$(\forall x\in M)(\lambda_s(x) = 0 \;\longleftrightarrow\; x\in\Omega_s).$

D o k a z

T225,T68,D47 $\succ\!\!-\!\!-$

$(1)(\forall x\in M)(\lambda (x) = 0 \;\rightarrow\; x\in\Omega\ )$

T225 $\succ\!\!\!-\!\!\!\prec$

TEOREM 226

$(\forall x\in M)(x = Q^{\lambda_p(x)} q_p(x))$.

TEOREM 227

$(\forall x\in M)(\lambda_s(x) = 0 \leftrightarrow x\in\Omega_s)$.

D o k a z

T225,T68,D47 $\succ\!\!-$

(1) $(\forall x\in M)(\lambda_s(x) = 0 \rightarrow x\in\Omega_s)$.

(2) $x\in\Omega_s$ Sup.

(3) $x = Q^{\lambda_s(x)} q_s(x)$ 2,T225

(4) $x = Q^{\lambda_s(x)} x$ 3,T215

(5) $\lambda_s(x) = 0$ 4,T222

1,2,5 $\succ\!\!-$ T227.

T227 $\succ\!\!\!-\!\!\!\prec$

TEOREM 228

$(\forall x\in M)(\lambda_p(x) = 0 \leftrightarrow x\in\Omega_p)$.

DEFINICIJA 51

$(\forall x\in M)(\rho_s(x) = (q_s(x),\lambda_s(x)))$.

DEFINICIJA 52

$(\forall x\in M)(\rho_p(x) = (q_p(x),\lambda_p(x)))$.

D51,T209,T215,D49,T227,T72 $\succ\!\!-$

TEOREM 229

$\rho_s\in S_{bijekc}(M,\Omega_s \times N)$.

T229 $\succ\!\!\!-\!\!\!\prec$

TEOREM 230

$\rho_p\in S_{bijekc}(M,\Omega_p \times N)$.

TEOREM 231

$(\forall x\in M)(\forall n\in N)(q_s(Q^n x) = q_s(x))$.

D o k a z

(1) $x\in M \;\&\; n\in N$	Sup.
(2) $q(Q^n x) = q(x)$	1,T78
(3) $q_s(Q^n x) = q_s(x)$	2,T220

1,3, $\succ\!\!-$ T231.

T231 $\succ\!\!-\!\!\prec$

TEOREM 232

$(\forall x\in M)(\forall n\in N)(q_p(Q^n(x)) = q_p(x))$.

TEOREM 233

$(\forall x\in M)(\forall n\in N)(\lambda_s(Q^n x) = \lambda_s(x) + n)$.

D o k a z

(1) $x\in M \;\&\; n\in N$	Sup.
(2) $Q^n x = Q^{n+\lambda_s(x)} q_s(x)$	1,T225,T75
(3) $\quad = Q^{n+\lambda_s(x)} q_s(Q^n x)$	2,T231
(4) $\lambda_s(Q^n x) = n+\lambda_s(x)$	3,D49

1,4 $\succ\!\!-$ T233.

T233 $\succ\!\!-\!\!\prec$

TEOREM 234

$(\forall x\in M)(\forall n\in N)(\lambda_p(Q^n x) = \lambda_p(x) + n)$.

T225,T75,T231,T233 $\succ\!\!-$

TEOREM 235

$(\forall x,y\in M)(\lambda_s(xy) = \lambda_s(x)+\lambda_s(y)+\lambda_s(q_s(x)q_s(y)))$.

T235 $\succ\!\!-\!\!\prec$

TEOREM 236

$(\forall x,y\in M)(\lambda_p(xy) = \lambda_p(x)+\lambda_p(y)+\lambda_p(q_p(x)q_p(y)))$.

TEOREM 237

$(\forall x,y\in M)(q_s(x \smile y) = q_s(x) \smile q_s(y) \;\&\; \lambda_s(x \smile y) = \lambda_s(x)+\lambda_s(y))$.

D o k a z

(1) $x,y\in M$	Sup.
(2) $x \smile y = Q^{\lambda_s(x)} q_s(x) \smile Q^{\lambda_s(y)} q_s(y)$	1,T225

(3) $x \cup y = Q^{\lambda_s(x)+\lambda_s(y)}(q_s(x) \cup q_s(y))$ 2,T137,T75

(4) $q_s(x \cup y) = q_s(q_s(x) \cup q_s(y))$ &

$\lambda_s(x \cup y) = \lambda_s(q_s(x) \cup q_s(y)) + \lambda_s(x) + \lambda_s(y)$ 3,T231,T233

4,D47,T211,T215,T227 $\succ\!\!-\!\!-$

(5) $q_s(x \cup y) = q_s(x) \cup q_s(y)$ & $\lambda_s(x \cup y) = \lambda_s(x) + \lambda_s(y)$

1,5 $\succ\!\!-\!\!-$ T237.

T237 $\succ\!\!-\!\!\prec$

TEOREM 238

$(\forall x,y \in M)(q_p(x \cap y) = q_p(x) \cap q_p(y)$ & $\lambda_p(x \cap y) = \lambda_p(x) + \lambda_p(y))$.

T237,T227 $\succ\!\!-\!\!-$ T239,T240.

TEOREM 239

$(\forall x,y \in M)(x \cup y = x \rightarrow y \in \Omega_s)$.

TEOREM 240

$(\forall u \in \Omega_s)(\forall x \in M)((u \cup x) \in \Omega_s \rightarrow x \in \Omega_s)$.

T239 $\succ\!\!-\!\!\prec$

TEOREM 241

$(\forall x,y \in M)(x \cap y = x \rightarrow y \in \Omega_p)$.

T240 $\succ\!\!-\!\!\prec$

TEOREM 242

$(\forall u \in \Omega_p)(\forall x \in M)((u \cap x) \in \Omega_p \rightarrow x \in \Omega_p)$.

T206,T237,T227 $\succ\!\!-\!\!-$

TEOREM 243

$(\forall x \in M)(\lambda_s(J_s(x)) = \lambda_s(x))$.

T243,T194,T225 $\succ\!\!-\!\!-$

TEOREM 244

$(\forall x \in M)(J_s(x) = Q^{\lambda_s(x)}ba)$.

T243 $\succ\!\!-\!\!\prec$

TEOREM 245

$(\forall x \in M)(\lambda_p(J_p(x)) = \lambda_p(x))$.

T244 $\succ\!\!-\!\!\prec$

TEOREM 246

$(\forall x\in M)(J_p(x) = Q^{\lambda_p(x)}ab)$.

TEOREM 247

$(\forall x,y\in M)(J_s(x \cup y) = J_s(x) \cup J_s(y))$.

D o k a z

(1)	$x,y\in M$	Sup.
(2)	$J_s(x \cup y) = Q^{\lambda_s(x \cup y)}ba$	1,T225,T243,T194
(3)	$= Q^{\lambda_s(x)}ba \cup Q^{\lambda_s(y)}ba$	2,T237,T145,T137
(4)	$= J_s(x) \cup J_s(y)$	3,T244

1,4 $\succ\!\!-$ T247.

T247 $\succ\!\!-\!\!\prec$

TEOREM 248

$(\forall x,y\in M)(J_p(x \cap y) = J_p(x) \cap J_p(y))$.

DEFINICIJA 53

$(\forall (x_1,x_2),(y_1,y_2)\in(\Omega_s \times N))((x_1,x_2) +_s (y_1,y_2) = (z_1,z_2) \longleftrightarrow$
$(z_1 = x_1 \cup y_1 \ \& \ z_2 = x_2 + y_2))$.

DEFINICIJA 54

$(\forall (x_1,x_2),(y_1,y_2)\in(\Omega_p \times N))((x_1,x_2) +_p (y_1,y_2) = (z_1,z_2) \longleftrightarrow$
$(z_1 = x_1 \cap y_1 \ \& \ z_2 = x_2 + y_2))$.

TEOREM 249

$(\forall x,y\in M)(\rho_s(x \cup y) = \rho_s(x) +_s \rho_s(y))$.

D o k a z

(1)	$x,y\in M$	Sup.
(2)	$\rho_s(x \cup y) = (q_s(x \cup y),\lambda_s(x \cup y))$	1,D51
(3)	$= ((q_s(x) \cup q_s(y)),(\lambda_s(x)+\lambda_s(y)))$	2,T237
(4)	$= (q_s(x),\lambda_s(x)) +_s (q_s(y),\lambda_s(y))$	3,T211,D53
(5)	$= \rho_s(x) +_s \rho_s(y)$	4,D51

1,4 $\succ\!\!-$ T249.

T249 $\succ\!\!-\!\!\prec$

TEOREM 250

$(\forall x,y\in M)(\rho_p(x \cap y) = \rho_p(x) +_p \rho_p(y))$.

1.7.3. DEFEKT m-BROJA

DEFINICIJA 55

$(\forall x\in M)(h(x) = \lambda_p(x) - \lambda_s(x))$.

DEFINICIJA 56

$(\forall x\in M)(\bar{h}(x) = \lambda_s(x) - \lambda_p(x))$.

D55,D56 $\succ$——

TEOREM 251

$(\forall x\in M)(\bar{h}(x) = -h(x))$.

TEOREM 252

$(\forall x\in M)(h(x) = 0 \longleftrightarrow l(x) = d(x))$.

D o k a z

(1) $x\in M$ & $\lambda_s(x) = \lambda_p(x) = n$	Sup.
(2) $Q^n q_s(x) = Q^n q_p(x)$	1,T225,T226
(3) $q_s(x) = q_p(x)$	2,T72
(4) $q(x) = A \vee q(x) = B$	3,T217
(5) $l(x) = d(x)$	4,T24,T30
(6) $(\forall x\in M)(h(x) = 0 \rightarrow l(x) = d(x))$.	1,D55,5

T24,T30,T217,T72,T225,T226 (analog 6) $\succ$——

(7) $(\forall x\in M)(l(x) = d(x) \rightarrow h(x) = 0)$.

6,7 $\succ$—— T252.

TEOREM 253

$(\forall x\in M)(h(x) = 1 \longleftrightarrow l(x) = a \ \& \ d(x) = b)$.

D o k a z

(1) $x\in M$ & $\lambda_p(x) = \lambda_s(x) + 1$	Sup.
(2) $Q^{\lambda_s(x)} q_s(x) = Q^{\lambda_s(x)+1} q_p(x)$	1,T225,T226
(3) $q_s(x) = Qq_p(x)$	2,T75,T72
(4) $q_s(x) = AB$ & $q_p(x) = ab$	3,T217
(5) $l(x) = a$ & $d(x) = b$	4,T24,T30
(6) $(\forall x\in M)(h(x) = 1 \rightarrow l(x) = a \ \& \ d(x) = b)$.	1,D55,5

T225,T226,T75,T217,T24,T30,D55 (analog 5) $\succ$——

(7) $(\forall x\in M)(l(x) = a \ \& \ d(x) = b \rightarrow h(x) = 1)$.

6,7 $\succ$—— T253.

T225,T226,T75,T72,T217,T24,T30,D55 (analog T253) >——

TEOREM 254

$(\forall x\in M)(h(x) = -1 \leftrightarrow l(x) = b \ \& \ d(x) = a).$

D55,T252-T254,T22,T28 >—— T255,T256.

TEOREM 255

$h\in S_{surjekc}(M,\{-1,0,1\}).$

TEOREM 256

$$(\forall x\in M)(q(x) = ba \leftrightarrow h(x) = -1 \ \& \ q(x) = ab \leftrightarrow h(x) = 1 \ \& \\ q(x) = A \leftrightarrow h(x) = 0 \ \& \ q(x) = B \leftrightarrow h(x) = 0).$$

T252-T254,T220,D14 >——

TEOREM 257

$(\forall x\in M)(\forall F\in\{q,q_s,q_p\})(h(F(x)) = h(x)).$

TEOREM 258

$(\forall x\in M)(q_s(x) = Q^{h(x)}q_p(x)).$

D o k a z

(1)	$x\in M$	Sup.
(2)	$Q^{\lambda_s(x)}q_s(x) = Q^{\lambda_p(x)}q_p(x)$	1,T225,T226
(3)	$q_s(x) = Q^{\lambda_p(x)-\lambda_s(x)}q_p(x)$	2,T69,T75
(4)	$\quad\quad = Q^{h(x)}q_p(x)$	3,D55

1,4 >—— T258.

T258 >——<

TEOREM 259

$(\forall x\in M)(q_p(x) = Q^{\bar{h}(x)}q_s(x)).$

TEOREM 260

$(\forall x\in M)(q_s(x) = P^{1+h(x)}\bar{d}(x)d(x) = l(x)\bar{l}(x)P^{1+h(x)}).$

D o k a z

(1)	$A = Pba = abP \ \& \ B = Pab = baP \ \&$ $AB = P^2ab = abP^2 \ \& \ ba = P^0ba = baP^0$	D15,D16,T8
(2)	$(\forall u\in Q_s)(u = P^{h(u)+1}\bar{d}(u)d(u) = l(u)\bar{l}(u)P^{h(u)+1})$	1,T209,T256

2,T221,T257 >—— T260.

T260 >——<

TEOREM 261

$(\forall x\in M)(q_p(x) = P^{1+\bar{h}(x)}\bar{d}(x)d(x) = l(x)\bar{l}(x)P^{1+\bar{h}(x)})$.

TEOREM 262

$(\forall x\in M)(h(Dx) = \bar{h}(x))$.

D o k a z

(1) $x\in M$	Sup.
(2) $h(Dx) = h(q(Dx))$	1,T257
(3) $\quad = h(Dq(x))$	2,T92,D30
(4) $\quad = -h(x)$	3,T91,T256

1,4,T251 >—— T262.

TEOREM 263

$(\forall x\in M)(q_s(Dx) = Dq_p(x))$.

D o k a z

(1) $x\in M$	Sup.
(2) $q_s(Dx) = P^{1+h(Dx)}\bar{d}(Dx)d(Dx)$	1,T260
(3) $\quad = P^{1+\bar{h}(x)}d(x)\bar{d}(x)$	2,T262,T90
(4) $\quad = P^{1+\bar{h}(x)}D\bar{d}(x)d(x)$	3,T15,T16,T34,T86
(5) $\quad = DP^{1+\bar{h}(x)}\bar{d}(x)d(x)$	4,T93
(6) $\quad = Dq_p(x)$	1,5,T261

1,6 >—— T263.

T263 >——<

TEOREM 264

$(\forall x\in M)(q_p(Dx) = Dq_s(x))$.

TEOREM 265

$(\forall x\in M)(\lambda_s(Dx) = \lambda_p(x))$.

D o k a z

(1) $x\in M$	Sup.
(2) $Dx = DQ^{\lambda_p(x)}q_p(x)$	1,T226
(3) $\quad = Q^{\lambda_p(x)}Dq_p(x)$	2,T95
(4) $\quad = Q^{\lambda_p(x)}q_s(Dx)$	3,T263
(5) $\lambda_s(Dx) = \lambda_p(x)$	4,D49

1,5 >—— T265.

T265 $\succ\!\!\!-\!\!\!-\!\!\!\prec$

TEOREM 266

$(\forall x\in M)(\lambda_p(Dx) = \lambda_s(x))$.

TEOREM 267

$(\forall x\in M)(J_p(x) = DQ^{h(x)}J_s(x))$.

D o k a z

(1) $x\in M$ Sup.

(2) $J_p(x) = Q^{\lambda_s(x)+h(x)}{}_{ab}$ 1,T246,D55

(3) $= DQ^{\lambda_s(x)+h(x)}{}_{ba}$ 2,T86

(4) $= DQ^{h(x)}J_s(x)$ 3,T75,T244

1,4 $\succ\!\!-\!\!-$ T267.

T267 $\succ\!\!\!-\!\!\!-\!\!\!\prec$

TEOREM 268

$(\forall x\in M)(J_s(x) = DQ^{\bar{h}(x)}J_p(x))$.

TEOREM 269

$(\forall x,y\in M)(\lambda_s(x \frown y) = \lambda_s(x)+\lambda_s(y)+h(x)+h(y)-h(x \frown y))$.

D o k a z

(1) $x,y\in M$ & $x \frown y = z$ Sup.

(2) $z = Q^{\lambda_p(x)+\lambda_p(y)}q_p(z)$ 1,T238,T138

(3) $= Q^{\lambda_p(x)+\lambda_p(y)+\bar{h}(z)}q_s(z)$ 2,T259,T75

(4) $\lambda_s(z) = \lambda_p(x) + \lambda_p(y) + \bar{h}(z)$ 3,D49

(5) $= \lambda_s(x)+\lambda_s(y)+h(x)+h(y)+\bar{h}(z)$ 4,D55

1,5,T251 $\succ\!\!-\!\!-$ T269.

T269 $\succ\!\!\!-\!\!\!-\!\!\!\prec$

TEOREM 270

$(\forall x,y\in M)(\lambda_p(x \smile y) = \lambda_p(x)+\lambda_p(y)+\bar{h}(x)+\bar{h}(y)-\bar{h}(x \smile y))$.

TEOREM 271
$(\forall x \in M)(\lambda_s(Kx) = -\lambda_s(x) - 1)$.

D o k a z
T154 $\succ\!\!-\!\!-$
(1) $(\forall x \in M)(q_s(x) \cup Kq_s(x) = ab)$.
1,T237,T227,T217 $\succ\!\!-\!\!-$
(2) $(\forall x \in M)(\lambda_s(Kq_s(x)) = -1)$.
T225 $\succ\!\!-\!\!-$
(3) $(\forall x \in M)(Kx = KQ^{\lambda_s(x)} q_s(x))$.
3,T105 $\succ\!\!-\!\!-$
(4) $(\forall x \in M)(Kx = Q^{-\lambda_s(x)} Kq_s(x))$.
4,2,T233 $\succ\!\!-\!\!-$ T271.

T271 $\succ\!\!-\!\!-\!\!\prec$
TEOREM 272
$(\forall x \in M)(\lambda_p(Kx) = -\lambda_p(x) - 1)$.

D32,T265,T271 $\succ\!\!-\!\!-$
TEOREM 273
$(\forall x \in M)(\lambda_s(Ix) = -\lambda_p(x) - 1)$.

T273 $\succ\!\!-\!\!-\!\!\prec$
TEOREM 274
$(\forall x \in M)(\lambda_p(Ix) = -\lambda_s(x) - 1)$.

T271,T274 $\succ\!\!-\!\!-$
TEOREM 275
$(\forall x \in M)(\lambda_s(Kx) = \lambda_p(Ix))$.

T275 $\succ\!\!-\!\!-\!\!\prec$
TEOREM 276
$(\forall x \in M)(\lambda_p(Kx) = \lambda_s(Ix))$.

1.7.4. o-RASTAV m-BROJA

TEOREM 277

$(\forall x\in M)(\exists! n\in N)(x = P^n \bar{d}(x)d(x))$.

D o k a z

(1)	$x\in M$	Sup.
(2)	$x = Q^{\lambda_s(x)} P^{h(x)+1}\bar{d}(x)d(x)$	1,T225,T260,T60
(3)	$= P^{2\lambda_s(x)+h(x)+1}\bar{d}(x)d(x)$	2,D23
(4)	$(\forall x\in M)(\exists n\in N)(x = P^n \bar{d}(x)d(x))$.	1,3
(5)	$(\exists x\in M)(\exists k,m\in N)(x=P^k\bar{d}(x)d(x)\ \&\ x=P^m\bar{d}(x)d(x))$	Sup.
(6)	$P^{k-m}(\bar{d}(x)d(x)) = \bar{d}(x)d(x)$	1,D19,T60
(7)	$(\exists g\in N)(k-m = 2g \ \ v\ \ k-m = 2g+1$	6
(8)	$Q^g(\bar{d}(x)d(x)) = \bar{d}(x)d(x) \ \ v$ $Q^g d(x)d(x)\bar{d}(x)d(x) = \bar{d}(x)d(x)$	7,D23,D15,T8
(9)	$g = 0 \ \ v\ \ d(x) = \bar{d}(x)$	8,T222,T78,T26
(10)	$k = m$	7,9,T34,T2

4,5,10 $\succ\!\!-$ T277.

DEFINICIJA 57

$(\forall x\in M)(\forall n\in N)(\lambda(x) = n \ \longleftrightarrow\ x = P^n \bar{d}(x)d(x))$.

D57,T277 $\succ\!\!-$

TEOREM 278

$(\forall x\in M)(x = P^{\lambda(x)}\bar{d}(x)d(x))$.

TEOREM 279

$(\forall x\in M)(\lambda(x) = 0 \ \longleftrightarrow\ x\in\Omega_o)$.

D o k a z

(1)	$x\in M\ \ \&\ \ \lambda(x) = 0$	Sup.
(2)	$x = \bar{d}(x)d(x)$	1,T278
(3)	$x = ba \ \ v\ \ x = ab$	2,T22,T28,T34
(4)	$(\forall x\in M)(\lambda(x) = 0 \ \rightarrow\ x\in\Omega_o$	1,3,T19

T19,T30,T34,D57,D17 $\succ\!\!-$

(5) $(\forall x\in M)(x\in\Omega_o \ \rightarrow\ \lambda(x) = 0$.

4,5 $\succ\!\!-$ T279.

TEOREM 280

$(\forall x\in M)(\lambda(x) = 1 \longleftrightarrow x\in\Omega_t)$.

D o k a z

(1) $x\in M \ \& \ \lambda(x) = 1$	Sup.
(2) $x = P\bar{d}(x)d(x)$	1,T278
(3) $x = d(x)d(x)\bar{d}(x)d(x)$	2,D15,T24,T17
(4) $x = d(x)d(x)$	3,T8
(5) $x = A \ \vee \ x = B$	4,T28
(6) $(\forall x\in M)(\lambda(x) = 1 \rightarrow x\in\Omega_t)$.	1,5,T18

T18,T30,T8,D15,T17,T24,T51,T278 >——

(7) $(\forall x\in M)(x\in\Omega_t \rightarrow \lambda(x) = 1)$.

6,7 >—— T280.

T225,T260,D23,T60,D57 >——

TEOREM 281

$(\forall x\in M)(\lambda(x) = 2\lambda_s(x) + h(x) + 1)$.

T281 >——<

TEOREM 282

$(\forall x\in M)(\lambda(x) = 2\lambda_p(x) + \bar{h}(x) + 1)$.

T281,T282,T251 >——

TEOREM 283

$(\forall x\in M)(\lambda(x) = \lambda_s(x) + \lambda_p(x) + 1)$.

TEOREM 284

$(\forall x\in M)(x = 1(x)\bar{1}(x)P^{\lambda(x)})$.

D o k a z

(1) $x\in M$	Sup.
(2) $x = P^{2\lambda_s(x)+h(x)+1}\bar{d}(x)d(x)$	1,T278,T281
(3) $\quad = P^{2\lambda_s(x)}q_s(x)$	2,T260
(4) $\quad = q_s(x)P^{2\lambda_s(x)}$	3,D23,T71
(5) $\quad = q_s(x)P^{\lambda(x)-h(x)-1}$	4,T281
(6) $\quad = 1(x)\bar{1}(x)P^{\lambda(x)}$	5,T260,T60

1,6 >—— T284.

T278,T284 $\succ\!\!-\!\!-$

TEOREM 285

$(\forall x\in M)(P^{\lambda(x)}\bar{d}(x)d(x) = l(x)\bar{l}(x)P^{\lambda(x)})$

TEOREM 286

$(\forall x\in M)(\forall n\in N)(\lambda(x) = 2n \rightarrow \neg(l(x) = d(x)).$

D o k a z

(1) $x\in M$ & $n\in N$ & $\lambda(x) = 2n$ — Sup.

(2) $l(x)\bar{l}(x)P^{2n} = \bar{d}(x)d(x)P^{2n}$ — 1,T285,D23,T71

(3) $l(x) = \bar{d}(x)$ — 2,T9

1,3,T33,T34 $\succ\!\!-\!\!-$ T286.

TEOREM 287

$(\forall x\in M)(\neg\, l(x) = d(x)) \rightarrow (\exists n\in N)(\lambda(x) = 2n)).$

D o k a z

(1) $x\in M$ & $l(x) = \bar{d}(x)$ — Sup.

(2) $l(x)\bar{l}(x) = \bar{\bar{d}}(x)d(x)$ — 1,T17

(3) $P^{\lambda(x)}\bar{d}(x)d(x) = \bar{d}(x)d(x)P^{\lambda(x)}$ — 2,T285

1,3,T77 $\succ\!\!-\!\!-$ T287.

T278,T60 $\succ\!\!-\!\!-$

TEOREM 288

$(\forall x\in M)(\forall n\in N)(\lambda(P^n x) = n+\lambda(x)).$

T284,T61 $\succ\!\!-\!\!-$

TEOREM 289

$(\forall x\in M)(\forall n\in N)(\lambda(xP^n) = n + \lambda(x)).$

TEOREM 290

$(\forall x\in M)(\lambda(Dx) = \lambda(x)).$

D o k a z

(1) $x\in M$ — Sup.

(2) $\lambda(Dx) = \lambda(PxP')$ — 1,D29

(3) $\quad = 1 + \lambda(x) - 1$ — 2,T288,T289

1,3 $\succ\!\!-\!\!-$ T290.

TEOREM 291
$(\forall x \in M)(\lambda(Kx) = -\lambda(x))$.

D o k a z

(1) $x \in M$	Sup.
(2) $Kx = KP^{\lambda(x)}\bar{d}(x)d(x)$	1,T278
(3) $\quad = P^{-\lambda(x)}d(x)\bar{d}(x)d(x)\bar{d}(x)$	2,T103,D31
(4) $\quad = P^{-\lambda(x)}d(x)\bar{d}(x)$	3,T8
(5) $\quad = P^{-\lambda(x)}\bar{d}(Kx)d(Kx)$	4,T101,T17
(6) $\lambda(Kx) = -\lambda(x)$	5,D57

1,6 $\succ\!\!-\!\!-$ T291.

T113,T290,T291 $\succ\!\!-\!\!-$
TEOREM 292
$(\forall x \in M)(\lambda(Ix) = -\lambda(x))$.

TEOREM 293
$(\forall x,y \in M)(\lambda(xy) = \lambda(x) + \lambda(y) - \lambda(d(x)1(y)))$.

D o k a z

(1) $x,y \in M$	Sup.
(2) $xy = P^{\lambda(x)}\bar{d}(x)d(x)1(x)\bar{1}(x)P^{\lambda(y)}$	1,T278,T284
(3) $\lambda(xy) = \lambda(x)+\lambda(y)+\lambda(\bar{d}(x)d(x)1(y)\bar{1}(y))$	2,T288,T289
(4) $\lambda(xy) = \lambda(x)+\lambda(y)+\lambda(K(d(x)1(y)))$	3,D31
(5) $\quad = \lambda(x)+\lambda(y)-\lambda(d(x)1(y))$	1,4,T291

1,5 $\succ\!\!-\!\!-$ T293.

1.8. PARTICIJE m-BROJEVA OBZIROM NA PREDZNAK DULJINE m-BROJA

DEFINICIJA 58

$M_s^+ = \{x: x\in M \ \& \ 0 \leq \lambda_s(x)\}.$

DEFINICIJA 59

$M_p^+ = \{x: x\in M \ \& \ 0 \leq \lambda_p(x)\}.$

DEFINICIJA 60

$M_s^- = \{x: x\in M \ \& \ \lambda_s(x) < 0\}.$

DEFINICIJA 61

$M_p^- = \{x: x\in M \ \& \ \lambda_p(x) < 0\}.$

D58,D59,T229 $\succ\!\!-\!\!-$

TEOREM 294

$M_s^+ \cup M_s^- = M \ \& \ M_s^+ \cap M_s^- = 0 \ \& \ \neg M_s^+ = 0 \ \& \ \neg M_s^- = 0.$

T294 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 295

$M_p^+ \cup M_p^- = M \ \& \ M_p^+ \cap M_p^- = 0 \ \& \ \neg M_p^+ = 0 \ \& \ \neg M_p^- = 0.$

TEOREM 296

$x\in M_s^+ \ \rightarrow \ Dx\in M_p^+.$

D o k a z

(1) $x\in M_s^+$	Sup.
(2) $0 \leq \lambda_s(x)$	1,D58
(3) $0 \leq \lambda_p(Dx)$	2,T266
(4) $Dx\in M_p^+$	3,D59

1,4 $\succ\!\!-\!\!-$ T296.

T296 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 297

$x\in M_p^+ \ \rightarrow \ Dx\in M_s^+.$

TEOREM 298

$x \in M_s^+ \longleftrightarrow Kx \in M_s^-$.

D o k a z

(1) $x \in M_s^+$ Sup.

(2) $x \cup Kx = ab$ 1,T154

(3) $\lambda_s(x) + \lambda_s(Kx) = -1$ 2,T237

(4) $\lambda_s(Kx) < 0$ 1,3,D58

(5) $x \in M_s^+ \rightarrow Kx \in M_s^-$ 1,5,D60

T154,T237,D58,D60 (analog 5) $\succ\!\!-\!\!-$

(6) $Kx \in M_s^- \rightarrow x \in M_s^+$

5,6 $\succ\!\!-\!\!-$ T298.

T298 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 299

$x \in M_p^+ \longleftrightarrow Kx \in M_p^-$.

TEOREM 300

$x \in M_s^+ \longleftrightarrow Ix \in M_p^-$.

D o k a z

(1) $x \in M_s^+$ Sup.

(2) $Dx \in M_p^+$ 1,T296

(3) $KDx \in M_p^-$ 2,T299

1,3,D32 $\succ\!\!-\!\!-$

(4) $x \in M_s^+ \rightarrow Ix \in M_p^-$

T296,T299 (analog 4) $\succ\!\!-\!\!-$

(5) $Ix \in M_p^- \rightarrow x \in M_s^+$

4,5 $\succ\!\!-\!\!-$ T300.

T300 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 301

$x \in M_p^+ \longleftrightarrow Ix \in M_s^-$.

DEFINICIJA 62

$M_{poz} = M_s^+ \cap M_p^+$.

DEFINICIJA 63

$M_{neg} = M_s^- \cap M_p^-$.

DEFINICIJA 64

$M_{nepoz} = M_s^- \cup M_p^-$.

DEFINICIJA 65

$M_{neneg} = M_s^+ \cup M_p^+$.

D58,D59,D52 $\succ\!\!-$

TEOREM 302

$M_{nepoz} = M \setminus M_{poz}$.

D o k a z

(1) $x \in M \setminus M_{poz}$	Sup.
(2) $\lambda_s(x) < 0 \quad \vee \quad \lambda_p(x) < 0$	1,D58,D59,D62
(3) $x \in (M_s^- \cup M_p^-)$	2,D60,D61
(4) $(M \setminus M_{poz}) \subseteq M_{nepoz}$.	1,3,D64
(5) $x \in M_{nepoz}$	Sup.
(6) $\lambda_s(x) < 0 \quad \vee \quad \lambda_p(x) < 0$	5,D64,D60,D61
(7) $x \in M \setminus M_{poz}$	6,D58,D59,D62
(8) $M_{nepoz} \subseteq (M \setminus M_{poz})$	5,7

4,8 $\succ\!\!-$ T302.

D58-D65 (analog T302) $\succ\!\!-$

TEOREM 303

$M_{neg} = M \setminus M_{neneg}$.

D58-D65,T265,T266 $\succ\!\!-$

TEOREM 304

$(\forall W \in \{M_{poz}, M_{neg}, M_{nepoz}, M_{neneg}\})(x \in W \quad \longleftrightarrow \quad Dx \in W)$.

TEOREM 305

$x \in M_{poz} \quad \longleftrightarrow \quad Kx \in M_{neg}$.

D o k a z

(1) $x \in M_{poz}$	Sup.
(2) $0 \leq \lambda_s(x) \quad \& \quad 0 \leq \lambda_p(x)$	1,D62,D58,D59
(3) $\lambda_s(Kx) < 0 \quad \& \quad \lambda_p(Kx) < 0$	2,T271,T272
(4) $x \in M_{poz} \quad \rightarrow \quad Kx \in M_{neg}$	3,D63,D60,D61
(5) $Kx \in M_{neg}$	Sup.
(6) $\lambda_s(Kx) < 0 \quad \& \quad \lambda_p(Kx) < 0$	5,D63,D60,D61
(7) $\neg(\lambda_s(x)-1 < 0) \quad \& \quad \neg(\lambda_p(x)-1 < 0)$	6,T271,T272
(8) $0 \leq \lambda_s(x) \quad \& \quad 0 \leq \lambda_p(x)$	7
(9) $Kx \in M_{neg} \quad \rightarrow \quad x \in M_{poz}$	5,8,D58,D59,D62

4,9 $\succ\!\!-$ T305.

T113,T304,T305 $\succ\!\!-\!\!-$

TEOREM 306

$x \in M_{poz} \quad \leftrightarrow \quad Ix \in M_{neg}.$

TEOREM 307

$x \in M_{nepoz} \quad \leftrightarrow \quad Kx \in M_{neneg}.$

D o k a z

(1) $x \in M_{neneg}$	Sup.
(2) $0 \leq \lambda_s(x) \quad \vee \quad 0 \leq \lambda_p(x)$	1,D65,D58,D59
(3) $0 \leq (-\lambda_s(Kx)-1) \quad \vee \quad 0 \leq (-\lambda_p(Kx)-1)$	2,T271,T272
(4) $\lambda_s(Kx) < 0 \quad \vee \quad \lambda_p(Kx) < 0$	3
(5) $x \in M_{neneg} \quad \rightarrow \quad Kx \in M_{nepoz}.$	1,4,D64,D60,D61
(6) $x \in M_{nepoz} \quad \& \quad \neg\, Kx \in M_{neneg}$	Sup.
(7) $Kx \in M_{neg}$	6,T303
(8) $x \in M_{poz}$	7,T305
(9) $\neg\, x \in M_{nepoz}$	8,T302
(10) $x \in M_{nepoz} \quad \rightarrow \quad Kx \in M_{neneg}$	6,9

5,10 $\succ\!\!-\!\!-$ T307.

T113,T304,T307 $\succ\!\!-\!\!-$

TEOREM 308

$x \in M_{nepoz} \quad \leftrightarrow \quad Ix \in M_{neneg}.$

TEOREM 309

$(\forall x \in M)(x \in M_{poz} \quad \leftrightarrow \quad 0 < \lambda(x)).$

D o k a z

(1) $x \in M_{poz}$	Sup.
(2) $0 \leq \lambda_s(x) \quad \& \quad 0 \leq \lambda_p(x)$	1,D62,D58,D59
(3) $x \in M_{poz} \quad \rightarrow \quad \lambda(x) > 0.$	1,T283
(4) $x \in M \quad \& \quad 0 < \lambda(x)$	Sup.
(5) $0 \leq \lambda_s(x) \quad \& \quad 0 \leq \lambda_p(x)$	4,T281,T282,T255
(6) $(\forall x \in M)(0 < \lambda(x) \quad \rightarrow \quad x \in M_{poz}).$	4,5,D62,D58,D59

3,6 $\succ\!\!-\!\!-$ T309.

D58-D65,T281-T283,T255 (analog T309) $\succ\!\!-\!\!-$ T310-T312.

TEOREM 310

$(\forall x \in M)(x \in M_{neg} \quad \leftrightarrow \quad \lambda(x) < 0).$

TEOREM 311

$(\forall x \in M)(x \in M_{nepoz} \longleftrightarrow \lambda(x) \leq 0).$

TEOREM 312

$(\forall x \in M)(x \in M_{neneg} \longleftrightarrow 0 \leq \lambda(x)).$

1.9. U R E Đ E N J E m-B R O J E V A

1.9.1. RELACIJE s-MANJE I p-MANJE

DEFINICIJA 66

$(\forall x,y \in M)(x <_s y \longleftrightarrow (\exists z \in M_s^+)(x \cup z = y \ \& \ \neg x = y)).$

DEFINICIJA 67

$(\forall x,y \in M)(x <_p y \longleftrightarrow (\exists z \in M_p^+)(x \cap z = y \ \& \ \neg x = y)).$

DEFINICIJA 68

$(\forall x,y \in M)(x \leq_s y \longleftrightarrow (x <_s y \ \vee \ x = y)).$

DEFINICIJA 69

$(\forall x,y \in M)(x \leq_p y \longleftrightarrow (x <_p y \ \vee \ x = y)).$

D66,D68 >——

TEOREM 313

$(\forall x \in M)(x \leq_s x).$

T313 >——<

TEOREM 314

$(\forall x \in M)(x \leq_p x).$

TEOREM 315

$(\forall x,y \in M)(x \leq_s y \ \& \ y \leq_s x \ \rightarrow \ x = y).$

D o k a z

(1) $x,y \in M \ \& \ x \leq_s y \ \& \ y \leq_s x$	Sup.
(2) $(\exists w,z \in M_s^+)(x \cup w = y \ \& \ y \cup z = x)$	1,D66
(3) $q(x) \ r \ q(y) \ \& \ q(y) \ r \ q(x)$	2,T181
(4) $q(x) = q(y)$	3,T185
(5) $q_s(x) = q_s(y).$	4,T220
(6) $x \cup w \cup z = x$	2
(7) $\lambda_s(x) + \lambda_s(w) + \lambda_s(z) = \lambda_s(x)$	6,T237

(8) $\lambda_s(w) = \lambda_s(z) = 0.$ 7,2,D58

(9) $\lambda_s(x) + \lambda_s(w) = \lambda_s(y)$ 2,T237

(10) $\lambda_s(x) = \lambda_s(y)$ 9,8

(11) $x = y$ 4,10,T229

1,11 $\succ\!\!-\!\!-$ T315

T315 $\succ\!\!-\!\!\prec$

TEOREM 316

$(\forall x,y\in M)(x \leq_p y \;\&\; y \leq_p x \;\rightarrow\; x = y).$

TEOREM 317

$(\forall x,y,z\in M)(x \leq_s y \;\&\; y \leq_s z \;\rightarrow\; x \leq_s z).$

D o k a z

(1) $x,y,z\in M \;\&\; x \leq_s y \;\&\; y \leq_s z$ Sup.

(2) $(\exists u,v\in M_s^+)(x \cup u = y \;\&\; y \cup v = z)$ 1,D68,D66

(3) $x \cup u \cup v = z \;\&\; (u \cup v)\in M_s^+$ 2,D58

1,3,D66,D68 $\succ\!\!-\!\!-$ T317.

T317 $\succ\!\!-\!\!\prec$

TEOREM 318

$(\forall x,y,z\in M)(x \leq_p y \;\&\; y \leq_p z \;\rightarrow\; x \leq_p z).$

D66,D68,T145 $\succ\!\!-\!\!-$

TEOREM 319

$(\forall x,y\in M)(x \leq_s y \;\leftrightarrow\; (\exists z\in M_s^+)(x \cup z = y)).$

T319 $\succ\!\!-\!\!\prec$

TEOREM 320

$(\forall x,y\in M)(x \leq_p y \;\leftrightarrow\; (\exists z\in M_p^+)(x \cap z = y)).$

T145,T319 $\succ\!\!-\!\!-$

TEOREM 321

$(\forall x\in M_s^+)(ba \leq_s x).$

T321 $\succ\!\!-\!\!\prec$

TEOREM 322

$(\forall x\in M_p^+)(ab \leq_p x).$

TEOREM 323

$(\forall x \in M_p^-)(x \leq_p ba)$.

D o k a z

(1) $x \in M_p^-$ Sup.

(2) $x \cap Kx = ba$ & $Kx \in M_p^+$ 1,T155,T299

1,2,T320 $\succ\!\!-$ T323.

T323 $\succ\!\!-\!\!\prec$

TEOREM 324

$(\forall x \in M_s^-)(x \leq_s ab)$.

TEOREM 325

$(\forall x,y \in M)(x \leq_s y \longleftrightarrow (q_s(x)\ r\ q_s(y)\ \&\ \lambda_s(x) \leq \lambda_s(y)))$.

D o k a z

(1) $x,y \in M$ & $x \leq_s y$ Sup.

(2) $(\exists z \in M_s^+)(x \cup z = y)$ 1,T319

(3) $q_s(x) \cup q_s(z) = q_s(y)$ & $\lambda_s(x)+\lambda_s(z) = \lambda_s(y)$ 2,T237

(4) $(\forall x,y \in M)(x \leq_s y \rightarrow q_s(x)\ r\ q_s(y)$ &
$\lambda_s(x) \leq \lambda_s(y))$. 1,2,T176,D58

T319,T237,T176,D58 (analog 4) $\succ\!\!-$

(5) $(\forall x,y \in M)(q_s(x)\ r\ q_s(y)\ \&\ \lambda_s(x) \leq \lambda_s(y) \rightarrow x \leq_s y)$.

4,5 $\succ\!\!-$ T325

T325 $\succ\!\!-\!\!\prec$

TEOREM 326

$(\forall x,y \in M)(x \leq_p y \longleftrightarrow (q_p(x)\ r\ q_p(y)\ \&\ \lambda_p(x) \leq \lambda_p(y)))$.

T319,T325,T176 $\succ\!\!-$

TEOREM 327

$(\forall x,y \in M)(x \leq_s y \longleftrightarrow x\ r\ y\ \&\ \lambda_s(x) \leq \lambda_s(y))$.

T327 $\succ\!\!-\!\!\prec$

TEOREM 328

$(\forall x,y \in M)(x \leq_p y \longleftrightarrow x\ \bar{r}\ y\ \&\ \lambda_p(x) \leq \lambda_p(y))$.

T327,T181,T256,T186-T189 $\succ\!\!-$

TEOREM 329

$(\forall x,y \in M)(x \leq_s y \rightarrow h(x) \leq h(y))$.

T329 $\succ\!\!-\!\!\prec$

TEOREM 330

$(\forall x,y\in M)(x \leq_p y \;\rightarrow\; \bar{h}(x) \leq \bar{h}(y))$.

TEOREM 331

$(\forall x,y\in M)(x <_s y \;\leftrightarrow\; Dx <_p Dy)$.

D o k a z

(1) $x,y\in M \;\&\; x <_s y$ Sup.

(2) $(\exists z\in M_s^+)(x \cup z = y \;\&\; \neg\, x = y)$ 1,D66

(3) $Dx \cap Dz = Dy \;\&\; Dz\in M_p^+ \;\&\; \neg\, Dx = Dy$ 2,T157,T296,T84

(4) $(\forall x,y\in M)(x <_s y \;\rightarrow\; Dx <_p Dy)$. 1,3,D67

D66,T157,T296,T84,D67 (analog 4) $\succ\!\!-$

(5) $(\forall x,y\in M)(Dx <_p Dy \;\rightarrow\; x <_s y)$.

4,5 $\succ\!\!-$ T331.

T331 $\succ\!\!-\!\!\prec$

TEOREM 332

$(\forall x,y\in M)(x <_p y \;\leftrightarrow\; Dx <_s Dy)$.

TEOREM 333

$(\forall x,y\in M)\neg\,(x <_s y \;\&\; y <_p x)$.

D o k a z

(1) $x,y\in M \;\&\; x <_s y \;\&\; y <_p x$ Sup.

1,D68,D69,T327-T330 $\succ\!\!-$

(2) $x\; r\; y \;\&\; \lambda_s(x) \leq \lambda_s(y) \;\&\; h(x) \leq h(y) \;\&$
$y\; \bar{r}\; x \;\&\; \lambda_p(y) \leq \lambda_p(x) \;\&\; \bar{h}(y) \leq \bar{h}(x)$

2,T175,T181,T251 $\succ\!\!-$

(3) $q(x)\; r\; q(y) \;\&\; h(x) = h(y)$

3,T256,T189 $\succ\!\!-$

(4) $q(x) = q(y)$

2,3,D55 $\succ\!\!-$

(5) $\lambda_s(y) \leq \lambda_s(x)$

2,3,5 $\succ\!\!-$

(6) $\lambda_s(x) = \lambda_s(y)$

4,T220,6,T229 $\succ\!\!-$

(7) $x = y$

1,7,D66,D67 $\succ\!\!-$ T333.

T333 >——

TEOREM 334

$(\forall x,y\in M)(x <_s y \ \rightarrow\ \neg(y <_p x)).$

T334 >——<

TEOREM 335

$(\forall x,y\in M)(x <_p y \ \rightarrow\ \neg(y <_s x)).$

TEOREM 336

$(\forall x,y\in M)(x \leq_s y \ \rightarrow\ (\forall z\in M)((x \cup z) \leq_s (y \cup z))).$

D o k a z

(1) $x,y\in M \ \ \&\ \ x \leq_s y$	Sup.
(2) $(\exists w\in M_s^+)(x \cup w = y)$	1,T319
(3) $(\forall z\in M)(x \cup z \cup w = y \cup z)$	2
(4) $(\forall z\in M)(x \cup z \leq_s y \cup z)$	3,T319

1,3 >—— T336.

T336 >——<

TEOREM 337

$(\forall x,y\in M)(x \leq_p y \ \rightarrow\ (\forall z\in M)((x \cap z) \leq_p (y \cap z))).$

DEFINICIJA 70

$$(\forall x,y\in M)(x\ c_s\ y \ \leftrightarrow\ x <_s y \ \&\ (\forall z\in M)((x \leq_s z \ \&\ z \leq_s y) \ \rightarrow\ (x = z \ \ \vee\ \ z = y))).$$

DEFINICIJA 71

$$(\forall x,y\in M)(x\ c_p\ y \ \leftrightarrow\ x <_p y \ \&\ (\forall z\in M)((x \leq_p z \ \&\ z \leq_p y) \ \rightarrow\ (x = z \ \ \vee\ \ z = y))).$$

TEOREM 338

$(\forall x,y\in M)(x\ c_s\ y \ \rightarrow\ (\forall n\in N)(Q^n x\ c_s\ Q^n y)).$

D o k a z

(1) $x,y\in M \ \ \&\ \ x\ c_s\ y$	Sup.
(2) $(\exists z\in M_s^+)(x \cup z = y \ \ \&\ \ \neg x = y).$	1,D70,D66
(3) $(\forall n\in N)(x \cup z \cup Q^n ba = y \cup Q^n ba)$	2
(4) $(\forall n\in N)(Q^n x \cup z = Q^n y)$	3,T151
(5) $(\forall x,y\in M)(x\ c_s\ y \ \rightarrow\ (\forall n\in N)(Q^n x <_s Q^n y)).$	1,2,4,D66
(6) $x,y\in M \ \ \&\ \ x\ c_s\ y \ \ \&$ $(\exists z\in M)(\exists n\in N)(Q^n x \leq_s z \ \ \&\ \ z \leq_s Q^n y)$	Sup.

$(7)(\exists u,w\in M_s^+)(Q^n x \cup u = z \ \& \ z \cup w = Q^n y$ 6,T319

$(8)\ Q^n(x \cup u \cup w) = Q^n y$ 7,T137

$(9)\ x \cup u \cup w = y$ 8,T72

$(10)\ x \leq_s x \cup u \ \& \ x \cup u \leq_s y$ 9,7,T319

$(11)\ x = x \cup u \ \ v \ \ x \cup u = y$ 10,6,D70

$(12)\ Q^n x = z \ \ v \ \ z = Q^n y$ 11,7,T137

5,6,12,D70 $\succ\!\!-\!\!-$ T338.

T338 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 339

$(\forall x,y\in M)(x\ c_p\ y \ \rightarrow\ (\forall n\in N)(Q^n x\ c_p\ Q^n y)).$

TEOREM 340

$(\forall x\in M)(x\ c_s\ Qx).$

D o k a z

$(1)(\forall x\in M)(x <_s Qx).$ D66,T152,T222

$(2)\ x,y\in M \ \& \ x \leq_s y \ \& \ y \leq_s Qx)$ Sup.

$(3)\ q_s(x)\ r\ q_s(y) \ \& \ \lambda_s(x) \leq \lambda_s(y) \ \&$
$q_s(y)\ r\ q_s(x) \ \& \ \lambda_s(y) \leq \lambda_s(x) + 1$ 2,T325,T231,T233

$(4)\ q_s(x) = q_s(y) \ \&\ \lambda_s(x)=\lambda_s(y)\ v\ \lambda_s(y)=\lambda_s(x)+1$ 3,T185,T220

$(5)(\forall x,y\in M)((x \leq_s y \ \& \ y \leq_s Qx) \ \rightarrow$
$(x = y \ \ v \ \ y = Qx)).$ 2,4,T229,T233

1,5,D70 $\succ\!\!-\!\!-$ T340.

T340 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 341

$(\forall x\in M)(x\ c_p\ Qx).$

TEOREM 342

$(\forall x,y\in M)(x\ c_s\ y \longleftrightarrow (y = Qx\ v\ (q_s(x)\ c_s\ q_s(y)\ \&\ \lambda_s(x) = \lambda_s(y)))).$

D o k a z

D70,T325 $\succ\!\!-\!\!-$

$(1)(\forall x,y\in M)(x\ c_s\ y \ \rightarrow\ (q_s(x) = q_s(y) \ \& \ \lambda_s(x) < \lambda_s(y)) \ \ v$
$(q_s(x) <_s q_s(y) \ \& \ \lambda_s(x) = \lambda_s(y)))$.

$(2)\ x,y\in M \ \& \ x\ c_s\ y \ \& \ q_s(x) = q_s(y) = u$ Sup.

2,1 $\succ\!\!-\!\!-$

$(3)(\exists k\in N_{poz})(\lambda_s(x) + k = \lambda_s(y)).$

3,T338 $\succ\!\!-$

(4) $Q^{\lambda_s(x)}u \leq_s Q^{\lambda_s(x)+1}u \;\&\; Q^{\lambda_s(x)+1}u \leq_s Q^{\lambda_s(x)+k}u$

4,2,D70,T69,T75 $\succ\!\!-$

(5) $u = Qu \;\vee\; u = Q^{k-1}u$

5,T222 $\succ\!\!-$

(6) $k - 1 = 0$

6,3

(7) $\lambda_s(y) = \lambda_s(x)+1$

2,7,T225,T75,D25 $\succ\!\!-$

(8) $(\forall x,y\in M)(x\ c_s\ y \;\&\; q_s(x) = q_s(y) \;\rightarrow\; y = Qx)$.

(9) $x,y\in M \;\&\; x\ c_s\ y \;\&\; \lambda_s(x) = \lambda_s(y) = m \;\&$
$(\exists z\in M)(q_s(x) \leq_s z \;\&\; z \leq q_s(y))$ Sup.

9,T336,T151 $\succ\!\!-$

(10) $Q^m q_s(x) \leq_s Q^m z \;\&\; Q^m z \leq_s Q^m q_s(y)$

10,9,D70 $\succ\!\!-$

(11) $Q^m q_s(x) = Q^m z \;\vee\; Q^m z = Q^m q_s(y)$

11,T72 $\succ\!\!-$

(12) $q_s(x) = z \;\vee\; z = q_s(y)$

9,12,1,D70 $\succ\!\!-$

(13) $(\forall x,y\in M)(x\ c_s\ y \;\&\; \lambda_s(x) = \lambda_s(y) \;\rightarrow$
$q_s(x)\ c_s\ q_s(y))$.

1,8,13 $\succ\!\!-$

(14) $(\forall x,y\in M)(x\ c_s\ y \;\rightarrow\; (y = Qx \;\vee\; (q_s(x)\ c_s\ (q_s(y) \;\&$
$\lambda_s(x) = \lambda_s(y))))$.

14,T340,T338 $\succ\!\!-$ T342.

T342 $\succ\!\!-\!\!\prec$

TEOREM 343

$(\forall x,y\in M)(x\ c_p\ y \;\leftrightarrow\; (y=Qx \vee (q_p(x)\ c_p\ q_p(y) \;\&\; \lambda_p(x)=\lambda_p(y))))$.

TEOREM 344

$ba\ c_s\ A$.

D o k a z

T14,T2 $\succ\!\!-$

(1) $\neg(ba = aa)$.

1,D66,D68,T321 $\succ\!\!-\!\!-$

(2) ba $<_s$ A.

(3)($\exists x \in M$)(ba $\leq_s$ x & x $\leq_s$ A) Sup.

(4) ba r $q_s(x)$ & $0 \leq \lambda_s(x)$ &
$q_s(x)$ r A & $\lambda_s(x) \leq 0$ 3,T325,T181,T227

(5)(a $<_{alf}$ d(x) v a = d(x)) &
(d(x) $<_{alf}$ a v d(x) = a) & $\lambda_s(x) = 0$ 4,T183

(6) d(x) = a & $x \in \Omega_s$ 5,D40,T227

(7) x = ba v x = A 6,T209

D70,2,3,7 $\succ\!\!-\!\!-$ T344.

T344 $\succ\!\!-\!\!\prec$

TEOREM 345

ab c_p B.

T14,T2,D66,D68,T321,T325,T181,T227,T183,T209,D70
(analog T344) $\succ\!\!-\!\!-$

TEOREM 346

ba c_s B.

T346 $\succ\!\!-\!\!\prec$

TEOREM 347

ab c_p A.

TEOREM 348

A c_s AB.

D o k a z

T14,T2,D66,T124 $\succ\!\!-\!\!-$

(1) A $<_s$ AB

(2)($\exists x \in M$)(A $\leq_s$ x & x $\leq_s$ AB) Sup.

(3) A r $q_s(x)$ & $0 \leq \lambda_s(x)$
$q_s(x)$ r AB & $\lambda_s(x) \leq 0$ 2,T325,T227

(4)(l(x) $<_{alf}$ a v l(x) = a) &
(a $<_{alf}$ l(x) v l(x) = a) & $\lambda_s(x) = 0$ 3,T183

(5) l(x) = a & $x \in \Omega_s$ 4,D40,T227

(6) x = A v x = AB 5,T209

D70,1,3,6 $\succ\!\!-\!\!-$ T348.

T348 $\succ\!\!-\!\!\prec$

TEOREM 349

B c_p BA.

T14,T2,D66,T124,T325,T227,T183,D40,T209,D70 (analog T348) $\succ\!\!-$

TEOREM 350

B c_s AB.

T350 $\succ\!\!-\!\!\prec$

TEOREM 351

A c_p BA.

T325,T189 $\succ\!\!-$

TEOREM 352

$\neg(A <_s B)$ & $\neg(B <_s A)$.

T352 $\succ\!\!-\!\!\prec$

TEOREM 353

$\neg(B <_p A)$ & $\neg(A <_p B)$.

DEFINICIJA 72

$(\forall x,y\in M)([x,y]_s = \{z: z\in M \ \& \ x \leq_s z \leq_s y\})$.

DEFINICIJA 73

$(\forall x,y\in M)([x,y]_p = \{z: z\in M \ \& \ x \leq_p z \leq_p y\})$.

PRIMJERI

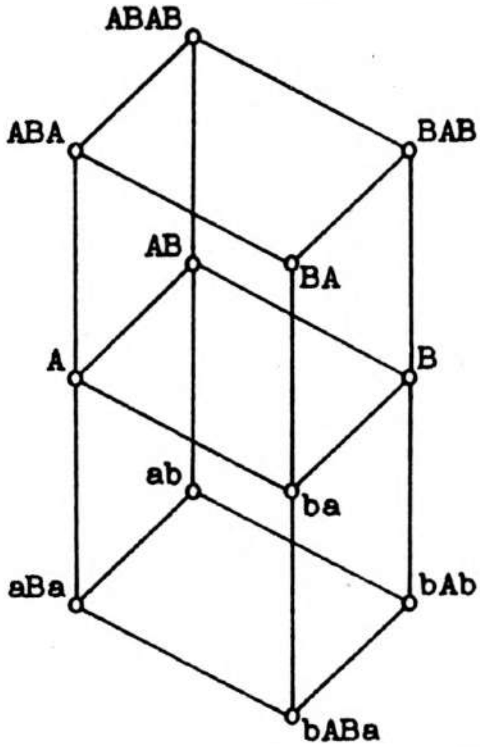
256 $[aBa, ABAB]_s = \{aBa, ab, A, AB, ABA, ABAB\}$.

257 $[A, A]_s = \{A\}$.

258 $[ab, BA]_s = 0$.

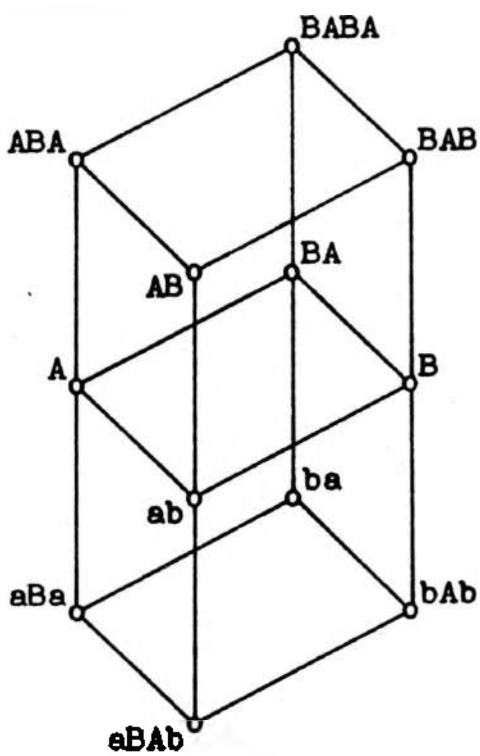
259 $[ab, BA]_p = \{ab, A, B, BA\}$.

T344,T346,T348,T350,T352,D70,T338,T340,T342,D72 >——
TEOREM 354

(

) $\in S_{\text{Hasse dgr.}}\left([bABa, ABAB]_s,\ \leq_s\right)$.

T354 >——<
TEOREM 355

(

) $\in S_{\text{Hasse dgr.}}\left([aBAb, BABA]_p,\ \leq_p\right)$.

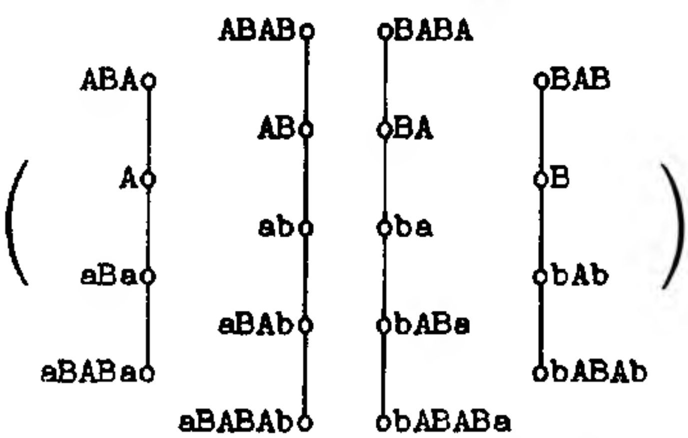
1.9.2. RELACIJA "manje"

DEFINICIJA 74
$(\forall x,y\in M)(x < y \longleftrightarrow x <_s y \ \& \ x <_p y).$

D74,D66,D67 $\succ\!\!-$
TEOREM 356
$< \ = \ <_s \cap <_p.$

DEFINICIJA 75
$(\forall x,y\in M)(x \leq y \longleftrightarrow x < y \ \vee \ x = y).$

T356,T354,T355,D75 $\succ\!\!-$
TEOREM 357



$= X \longrightarrow$

$X\in \mathcal{S}_{\text{Hasse dgr.}}(\{x:\ x\in M \ \& \ -4 \leq \lambda(x) \leq 4\}, \leq).$

D74,D75,T313-T318 $\succ\!\!-$ T358-T360.
TEOREM 358
$(\forall x\in M)(x \leq x).$

TEOREM 359
$(\forall x,y\in M)((x \leq y \ \& \ y \leq x) \rightarrow x = y).$

TEOREM 360
$(\forall x,y,z\in M)((x \leq y \ \& \ y \leq z) \rightarrow x \leq z).$

T356,T325,T326,T185,D55,T256 $\succ\!\!-$
TEOREM 361
$(\forall x,y\in M)(x \leq y \longleftrightarrow q(x) = q(y) \ \& \ \lambda_s(x) \leq \lambda_s(y)).$

T361 $\succ\!\!\!-\!\!\!\prec$

TEOREM 362

$(\forall x,y\in M)(x \leq y \leftrightarrow q(x) = q(y) \ \& \ \lambda_p(x) \leq \lambda_p(y)))$.

T361,T362,T283 $\succ\!\!-$

TEOREM 363

$(\forall x,y\in M)(x \leq y \leftrightarrow q(x) = q(y) \ \& \ \lambda(x) \leq \lambda(y))$.

1.10. RELACIJE KONGRUENCIJE SISTEMA M

DEFINICIJA 76

$(\forall x,y\in M)(\forall n\in N)(x \ \theta_n \ y \leftrightarrow (\exists k\in N^+)(Q^{kn}x = y \ \vee \ x = Q^{kn}y))$.

TEOREM 364

$(\forall x,y\in M)(\forall n\in N)(x \ \theta_n \ y \leftrightarrow q_s(x)=q_s(y) \ \& \ \lambda_s(x) \equiv \lambda_s(y)(\mathrm{mod}\ n))$.

D o k a z

(1) $x,y\in M \ \& \ n\in N \ \& \ x \ \theta_n \ y$	Sup.
(2) $(\exists k\in N^+)(Q^{kn}x = y \ \vee \ x = Q^{kn}y)$	1,D76
(3) $(q_s(Q^{kn}x) = q_s(y) \ \vee \ q_s(x) = q_s(Q^{kn}y)) \ \&$ $(kn+\lambda_s(x) = \lambda_s(y) \ \vee \ \lambda_s(x) = kn + \lambda_s(y))$	2,T233
(4) $q_s(x) = q_s(y) \ \& \ \lambda_s(x) \equiv \lambda_s(y)(\mathrm{mod}\ n)$	3,T231
(5) $(\forall x,y\in M)(\forall n\in N)(x \ \theta_n \ y \rightarrow q_s(x) = q_s(y) \ \&$ $\lambda_s(x) \equiv \lambda_s(y)(\mathrm{mod}\ n))$.	1,4

D76,T231,T233 (analog 5) $\succ\!\!-$

(6) $(\forall x,y\in M)(\forall n\in N)((q_s(x) = q_s(y) \ \&$
$\lambda_s(x) \equiv \lambda_s(y)(\mathrm{mod}\ n)) \rightarrow x \ \theta_n \ y)$.

5,6 $\succ\!\!-$ T364.

T364 $\succ\!\!\!-\!\!\!\prec$

TEOREM 365

$(\forall x,y\in M)(\forall n\in N)(x \ \theta_n \ y \leftrightarrow q_p(x)=q_p(y) \ \& \ \lambda_p(x) \equiv \lambda_p(y)(\mathrm{mod}\ n))$.

T364,T365,T283 $\succ\!\!-\!\!-$

TEOREM 366

$(\forall x,y\in M)(\forall n\in N)(x\ \theta_n\ y \longleftrightarrow q(x) = q(y)\ \ \&\ \ \lambda(x) \equiv \lambda(y)(\mathrm{mod}\ n))$.

PRIMJERI

260 BAB θ_o BBAAB. 261 A θ_1 ABABA; $\neg$ A θ_1 B.

262 ba θ_3 BABABA; bAb θ_3 BABAB.

T366 $\succ\!\!-\!\!-$

TEOREM 367

$(\forall x,y\in M)(\forall n\in N)(x\ \theta_n\ y \longleftrightarrow x\ \theta_{-n}\ y)$.

T364,T229 $\succ\!\!-\!\!-$

TEOREM 368

$(\forall x,y\in M)(x\ \theta_o\ y \longleftrightarrow x = y)$.

T366,T185 $\succ\!\!-\!\!-$

TEOREM 369

$(\forall x,y\in M)(x\ \theta_1\ y \longleftrightarrow x\ r\ y\ \ \&\ \ y\ r\ x)$.

T366 $\succ\!\!-\!\!-$ T370-T372.

TEOREM 370

$(\forall x\in M)(\forall n\in N)(x\ \theta_n\ x)$.

TEOREM 371

$(\forall x,y\in M)(\forall n\in N)(x\ \theta_n\ y \rightarrow y\ \theta_n\ x)$.

TEOREM 372

$(\forall x,y,z\in M)(\forall n\in N)(x\ \theta_n\ y\ \ \&\ \ y\ \theta_n\ z \rightarrow x\ \theta_n\ z)$.

T366,T39,T293 $\succ\!\!-\!\!-$

TEOREM 373

$(\forall x,y,z,w\in M)(\forall n\in N)(x\ \theta_n\ y\ \ \&\ \ z\ \theta_n\ w \rightarrow xz\ \theta_n\ yw)$.

T364,T237 $\succ\!\!-\!\!-$

TEOREM 374

$(\forall x,y,z,w\in M)(\forall n\in N)(x\ \theta_n\ y\ \ \&\ \ z\ \theta_n\ w \rightarrow x \cup z\ \theta_n\ y \cup w)$.

T374 $\succ\!\!-\!\!\prec$

TEOREM 375

$(\forall x,y,z,w\in M)(\forall n\in N)(x\ \theta_n\ y\ \ \&\ \ z\ \theta_n\ w \rightarrow x \cap z\ \theta_n\ y \cap w)$.

T366,T37 $\succ\!\!-\!\!-$

TEOREM 376

$(\forall u,v\in\Omega)(\forall n\in N)(u\ \theta_n\ v \ \rightarrow\ u = v)$.

T181,T366 $\succ\!\!-\!\!-$

TEOREM 377

$(\forall x,y,z,w\in M)(\forall n\in N)((x\ \theta_n\ y\ \ \&\ \ z\ \theta_n\ w\ \ \&\ \ x\ r\ z)\ \rightarrow\ y\ r\ w)$.

T366,T288-T292 $\succ\!\!-\!\!-$

TEOREM 378

$(\forall x,y\in M)(\forall n\in N)(x\ \theta_n\ y \longleftrightarrow (\forall F\in\{P_1,P_d,Q,D,K,I\})(F(x)\ \theta_n\ F(y))$.

DEFINICIJA 77

$(\forall x\in M)(\forall n\in N)(|x|_n = \{y\colon\ y\in M\ \ \&\ \ y\ \theta_n\ x\})$.

DEFINICIJA 78

$(\forall n\in N)(M(\theta_n) = \{X\colon\ x\in M\ \ \&\ \ X = |x|_n\})$.

DEFINICIJA 79

$(\forall n\in N)(\Omega(\theta_n) = \{|ba|_n,\ |ab|_n,\ |A|_n,\ |B|_n\})$.

DEFINICIJA 80

$(\forall n\in N^{+})(\mathscr{M}_n = (M(\theta_n),\ \Omega(\theta_n),\ \wedge\ ,\ \smile\ ,\ \frown\ ,\ r,\ D,\ K,\ I\ ) \longleftrightarrow$

$(\forall\xi\in\{\ \wedge\ ,\ \smile\ ,\ \frown\ ,\ r\ \})(\forall|x|_n,\ |y|_n\in M(\theta_n))(|x|_n\ \xi\ |y|_n = |x\ \xi\ y|_n)\ \&$

$(\forall\psi\in\{D,\ K,\ I\ \})(\forall|x|_n\in M(\theta_n))(\psi|x|_n = |\psi x|_n))$.

1.11. m-S I S T E M

1.11.1. UTEMELJENJE I RED m-SISTEMA

DEFINICIJA 81

$$S_m = \{X: X \subseteq M^* \ \& \ (s,t \in \{a,b\} \longrightarrow st \in X) \ \&$$
$$(x,y \in X \longrightarrow xy \in X) \ \&$$
$$((H \in S \ \& \ H \subseteq X \ \& \ (s,t \in \{a,b\} \rightarrow st \in H) \ \& \ (x,y \in H \rightarrow xy \in H)) \longrightarrow H = X) \ \&$$
$$((\forall p,r,s,t \in \{a,b\})(\forall x,y \in X)((prx = sty \ \vee \ xrp = yts \longrightarrow p = s). \ \&$$
$$((\forall s,t \in \{a,b\})(\forall x,y \in X)(stx = sty \longrightarrow (x = y \ \vee \ stx = x \ \vee \ sty = y)) \ \&$$
$$((\forall s,t \in \{a,b\})(\forall x,y \in X)(xst = yst \longrightarrow (x = y \ \vee \ xst = x \ \vee \ yst = y))\}.$$

DEFINICIJA 82

$S_{m\text{-sist.}}(\{a,b\}) = \{(X, \wedge): X \in S_m\}$.

DEFINICIJA 83

$D4_m$–$D48_m$ su izreke koje se dobivaju iz definicija 4-48 sistema M, tako da se svugdje umjesto M uvrsti X i doda $(\forall X \in S_m)$, ili ukoliko M ne dolazi u dotičnoj definiciji, onda sve ostaje nepromjenjeno.

PRIMJERI

263 $D31_m$ glasi: $(\forall X \in S_m)(\forall x \in X)(Kx = l(x)x\bar{d}(x))$;

$D48_m$ glasi: $(\forall X \in S_m)(Q_p = \{u: x \in X \ \& \ u = q_p(x)\})$.

264 $D4_m$ glasi: $Q = \{x: s,t \in \{a,b\} \longrightarrow x = s \wedge t\}$.

DEFINICIJA 84

$T1_m$–$T11_m$ i $T13_m$–$T221_m$ su izreke koje se dobivaju iz teorema 2-11, resp. 13-221 sistema M, tako da se svugdje umjesto M uvrsti X i doda $(\forall X \in S_m)$, ili ukoliko M ne dolazi u dotičnom teoremu onda sve ostaje nepromjenjeno.

PRIMJERI

265 $T59_m$ glasi: $(\forall X \in S_m)(\forall x, y \in X)(\forall n \in N)(x(P^n y) = (xP^n)y)$.

266 $T8_m$ glasi: $(\forall r, s, t \in \{a, b\})(rsrt = rt \ \& \ rsts = rs)$.

D81-D84, D1-D48, T1-T11, T13-T221 $\succ\!\!-\!\!-$

TEOREM 379

$T1_m$-$T11_m$ i $T13_m$-$T221_m$ su istinite izreke, tj. teoremi koji vrijede za svako $(X, \wedge) \in S_{m\text{-sist.}}(\{a, b\})$.

TEOREM 380

$(\forall X \in S_m)(\forall k \in N)(x \in X \ \& \ Q^k x = x \ \rightarrow \ (\forall n \in N)(\forall z \in X)(Q^{kn} z = z))$.

D o k a z

(1) $k \in N \ \& \ X \in S_m \ \& \ x \in X \ \& \ Q^k x = x$	Sup.
(2) $(\exists g \in N)(\exists s, t \in \{a, b\})(Q^k Q^g st = Q^g st)$	1, T379, $T79_m$, $T38_m$
(3) $Q^k st = st$	2, $T69_m$, $T75_m$
(4) $Q^k sts\bar{s} = sts\bar{s}$	3
(5) $Q^k s\bar{s} = s\bar{s}$	4, $T8_m$
(6) $DQ^k s\bar{s} = Ds\bar{s}$	5
(7) $Q^k \bar{s}s = \bar{s}s$	6, $T95_m$, $T86_m$, $T17_m$
(8) $Q^k \bar{s}s\bar{s}\bar{s} = \bar{s}s\bar{s}\bar{s} \ \& \ Q^k ss\bar{s}s = ss\bar{s}s$	7
(9) $Q^k \bar{s}\bar{s} = \bar{s}\bar{s} \ \& \ Q^k ss = ss$	8, $T8_m$
(10) $(\forall X \in S_m)(\forall k \in N)(x \in X \ \& \ Q^k x = x \ \rightarrow$ $(\forall u \in \Omega)(Q^k u = u))$.	1, 2, 5, 7, 9, $T3_m$

10, T379, $T79_m$, $T75_m$, $T37_m$ $\succ\!\!-\!\!-$ T380.

TEOREM 381

$(\forall G \in S)(\forall X \in S_m)(G_X = \{n: n \in N \ \& \ (\forall x \in X)(Q^n x = x)\}) \ \rightarrow$
$(\exists! k \in N^+)(\forall n \in G_X)(k|n)) \vee G_X = \{0\}$.

D o k a z

(1) "Theorem 6. Any non void set of integers closed under addition and subtraction either consists of zero alone or else contains a least positive element and consists of all the multiples of this integer".(G.Birkhoff - S.M.Lane, A Survey of modern algebra, 1959, str.17).

(2) $(\forall X \in S_m)(G_X = \{n: n \in N \ \& \ (\forall x \in X)(Q^n x = x)\})$	Def.
(3) $(\exists X \in S_m)(\exists g, n \in N)(g \in G_X \ \& \ n \in G_X)$	Sup.
(4) $(\exists x, y \in X)(Q^g x = x \ \& \ Q^n y = y)$.	3, 2
(5) $Q^n x = x$	4, T380
(6) $Q^g Q^n x = x \ \& \ Q^g x = Q^n x$	4, 5

(7) $Q^{g+n}x = x \;\&\; Q^{g-n}x = x$ 6, $T69_m$, $T75_m$

(8) $(\forall X\in S_m)(G_X = \{n: n\in N \;\&\; (\exists x\in X)(Q^n x = x)\} \;\&$
$g,n\in G_X \rightarrow (g+n)\in G_X \;\&\; (g-n)\in G_X)$. 2,3,7

1,8 $\succ\!\!-$ T381.

DEFINICIJA 85

$(\forall X\in S_m)(F_{per}(X) = k$, ako i samo ako k je jedinstveno određeni nenegativni broj iz teorema 381 ili $G_X = 0 \;\&\; k = 0$.

D85, T381 $\succ\!\!-$

TEOREM 382

$(\forall X\in S_m)(\forall k\in N^+)(F_{per}(X) = k \longleftrightarrow ((\exists x\in X)(Q^k x = x) \;\&$
$(\forall n\in N)(\forall y\in X)(Q^n y = y) \rightarrow k|n)))$.

AKSIOM 2

$(\forall k\in N^+)(\exists! X\in S_m)(F_{per}(X) = k)$.

DEFINICIJA 86

$(\forall X\in S_m)(\forall \varkappa\in N^+)(M_\varkappa = X \longleftrightarrow F_{per}(X) = \varkappa)$.

KONVENCIJA 1

$(\forall \varkappa\in N^+)(XM_\varkappa Y) \hookrightarrow (XM_\varkappa Y)$.

D86, T382, A2, T380 $\succ\!\!-$ T383-T386.

TEOREM 383

$(\forall X\in S_m)(F_{per}(X) = 0 \longleftrightarrow (\forall x\in X)(\forall n\in N)(Q^n x = x \rightarrow n = 0))$.

TEOREM 384

$(\forall x\in M_\varkappa)(\forall n\in N)(Q^{n\varkappa} x = x)$.

TEOREM 385

$(\forall x\in M_\varkappa)(\forall m,n\in N)(Q^m x = Q^n x \longleftrightarrow m \equiv n(\mathrm{mod}\ \varkappa))$.

TEOREM 386

$(\forall x\in M_\varkappa)(\forall n\in N)(Q^n x = q(x) \longrightarrow \varkappa|n)$.

T386, $T3_m$, T385 $\succ\!\!-$ T387, T388.

TEOREM 387

$(\forall X\in S_m)(\forall n\in N_{poz})(F_{per}(X) = n \longleftrightarrow \kappa(X) = 4n)$.

TEOREM 388

$(\forall X\in S_m)(F_{per}(X) = 0 \longleftrightarrow \kappa(X) = \aleph_0)$.

1.11.2. IZOMORFIZAM m-SISTEMA n-tog REDA I m-SISTEMA KLASA OSTATAKA m-BROJEVA MODULO n.

TEOREM 389

$(\forall X\in S_m)(F_{per}(X) = 0 \longleftrightarrow (\forall x\in X)(\forall p,r,s,t\in\{a,b\})(xx = prst \rightarrow (x = pt \;\&\; p = s \;\vee\; r = t)))$.

D o k a z

(1) $x\in M_o \;\&\; (\exists p,r,s,t\in\{a,b\})(xx = prst)$	Sup.
(2) $q(x) = pt$	$1,T39_m$
(3) $(\exists n\in N)(x = Q^n pt)$	$2,T79_m$
(4) $Q^{2n}pt = prst$	$3,1,T75_m,T8_m$
(5) $\neg(p = s) \;\&\; \neg(r = t)$	Sup.
(6) $Q^{2n}pt = p\bar{t}\bar{p}t \;\&\; \bar{p} = t \;\vee\; \bar{p} = t$	$4,5,D5_m,1$
(7) $Q^{2n}pp = p\bar{p}\bar{p}p \;\vee\; Q^{2n}p\bar{p} = pp\bar{p}\bar{p}$	6
(8) $Q^{2n}pp = Q'pp \;\vee\; Q^{2n}p\bar{p} = Qp\bar{p}$	$7,T64_m,D15_m,T8_m$
(9) $Q^{2n+1}pp = pp \;\vee\; Q^{2n-1}p\bar{p} = p\bar{p}$	$8,T69_m,T75_m$
(10) $2n+1 = 0 \;\vee\; 2n-1 = 0$	9,1,T383
(11) $n = -\frac{1}{2} \;\vee\; n = \frac{1}{2}$	10
(12) $p = s \;\vee\; r = t$	5,11,3
(13) $Q^{2n}pt = pt$	$4,12,T8_m$
(14) $n = 0$	13,1,T383
(15) $x = pt$	14,3
(16) $(\forall x\in M_o)(\forall p,r,s,t\in\{a,b\})(xx = prst \rightarrow (x = pt \;\&\; ((p = s) \;\vee\; (r = t)))$.	1,12,15

T12,T379,T222,T382 $\succ\!\!-\!\!-$

(17) $(\forall X\in S_m)(\forall x\in X)(\forall p,r,s t\in\{a,b\})(xx = prst \rightarrow (x=pt \;\&\; ((p=s) \;\vee\; (r=t))) \rightarrow F_{per}(X) = 0$.

D86,16,17 $\succ\!\!-\!\!-$ T389.

D2,A1,D82,T389 $\succ\!\!-\!\!-$

TEOREM 390

$M_o = M$.

D80,D86,T364-T378,T379-T386,T389 $\succ\!\!-\!\!-$

TEOREM 391

$(\forall n\in N^+)(\mathscr{M}_n$ izomorfan je sa sistemom $(M_n, Q, \wedge, \smile, \frown, r, D, K, I))$.

1.11.3. m-SISTEM PRVOG REDA

$T379, T3_m, T4_m, D86, T387 \succ\!\!-$

TEOREM 392

$M_1 = \Omega$.

TEOREM 393

$(\forall x, y \in M_1)(x\ r\ y\ \&\ y\ r\ x \rightarrow x = y)$.

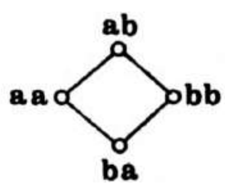
D o k a z

(1) $x, y \in M_1\ \&\ x\ r\ y\ \&\ y\ r\ x$	Sup.
(2) $(\exists m, n \in N)(x = Q^m q(x)\ \&\ y = Q^n q(y))$	$1, T79_m$
(3) $x = q(x)\ \&\ y = q(y)$	$2, 1, T384$
(4) $x = y$	$1, 3, T185_m$

$1, 4 \succ\!\!-$ T393.

$T392, T3_m, T393, T186_m - T189_m \succ\!\!-$

TEOREM 394

$\Bigg($ 

$\Bigg) \in S_{\text{Hasse dgr.}}(M_1, r)$.

$T79_m, T386 \succ\!\!-$

TEOREM 395

$(\forall x \in M_1)(q(x) = x)$.

TEOREM 396

$(\forall x \in M_1)(Kx = Dx)$.

D o k a z

(1) $x \in M_1$	Sup.
(2) $Dq(x) = Dx$	$1, T395$
(3) $q(Kx) = Dx$	$2, T102_m$
(4) $Kx = Dx$	$3, T100_m, T395$

$1, 4 \succ\!\!-$ T396.

$T113_m, T396 \succ\!\!-$

TEOREM 397

$(\forall x \in M_1)(Ix = x)$.

T395,T379,$T168_m$-$T174_m$ $\succ\!\!\!-$
TEOREM 398
$(\forall x,y,z\in M_1)(x \cup x = x \ \& \ x \cap x = x \ \& \ x \cup (y \cap x) = x \ \& \ x \cap (y \cup x) = x \ \& \ x \cup (y \cap z) = (x \cup y) \cap (x \cup z) \ \& \ x \cap (y \cup z) = (x \cap y) \cup (x \cap z) \ \& \ (x \cup y = x \longleftrightarrow x \cap y = x))$.

T398,$T154_m T155_m$ $\succ\!\!\!-$
TEOREM 399
Sistem $(M_1, \cup, \cap, K)$ je četveročlana distributivna rešetka, odnosno četveročlana Boole-ova algebra, odnosno dvogeneratorska slobodna rešetka (free lattice).

1.11.4. MODULI m-SISTEMA

DEFINICIJA 87
$(\forall u\in\Omega)(Mod_u(M_\varkappa) = \{x\colon x\in M_\varkappa \ \& \ q(x) = u\})$.

D87,$T109_m$ $\succ\!\!\!-$
TEOREM 400
$(\forall u\in\Omega)(x\in Mod_u(M_\varkappa) \rightarrow Ix\in Mod_u(M_\varkappa))$.

TEOREM 401
$(\forall\varkappa\in N^+)(\forall u\in\Omega)(Mod_u(M_\varkappa), \wedge)\in S_{cikl.gr.}(\varkappa))$.

D o k a z

(1)$(\forall u\in\Omega)(\forall\varkappa\in N^+)(F_{u,\varkappa} = \{(x,n)\colon x\in Mod_u(M_\varkappa) \ \& \ (\exists n\in N)(x = Q^n q(u))\})$. Def.

(2)$(\exists x\in Mod_u(M_\varkappa))(\exists g,h\in N)((x,g)\in F_{u,\varkappa} \ \& \ (x,h)\in F_{u,\varkappa})$ Sup.

2,1,D87 $\succ\!\!\!-$

(3) $Q^f u = Q^g u$

3,2,T385 $\succ\!\!\!-$

(4)$(\forall x\in Mod_u(M_\varkappa))(\forall g,h\in N)((x,g)\in F_{u,\varkappa} \ \& \ (x,h)\in F_{u,\varkappa} \longrightarrow g \equiv h(\mathrm{mod}\ \varkappa))$.

(5)$(\exists g,h\in N)(\exists x,y\in Mod_u(M_\varkappa))(g \equiv h(\mathrm{mod}\ \varkappa) \ \& \ (x,g)\in F_{u,\varkappa} \ \& \ (y,h)\in F_{u,\varkappa})$ Sup.

5,1,D87 $\succ\!\!\!-$

(6) $x = Q^g u \ \& \ y = Q^h u$

5,6,T385 $\succ\!\!\!-$

(7)$(\forall g,h\in N)(\forall x,y\in Mod_u(M_\varkappa))(g \equiv h(\mathrm{mod}\ \varkappa) \ \& \ (x,g)\in F_{u,\varkappa} \ \& \ (y,h)\in F_{u,\varkappa}) \rightarrow x = y)$.

1,4,7 $\succ\!\!-$

(8)$(\forall u\in\Omega)(\forall \varkappa\in N)(F_{u,\varkappa}\in S_{bijekc}(Mod_u(M_\varkappa),N(\equiv_n)))$.

(9)$(\exists x,y\in Mod_u(M_\varkappa))(\exists g,h\in N)((x,g)\in F_{u,\varkappa}$ & $(y,h)\in F_{u,\varkappa})$. Sup.

9,1,D87 $\succ\!\!-$

(10) $xy = Q^g{}_uQ^h{}_u$

10,T75$_m$,T13$_m$ $\succ\!\!-$

(11) $xy = Q^{g+h}{}_u$

11,9,1,D3$_m$ $\succ\!\!-$

(12)$(\forall u\in\Omega)(\forall \varkappa\in N^+)(\forall x,y\in Mod_u(M_\varkappa))(\forall g,h\in N)((x,y)\in F_{u,\varkappa}$ &
$(y,h)\in F_{u,\varkappa} \rightarrow (x\wedge y,g+h)\in F_{u,\varkappa})$.

1,8,12 $\succ\!\!-$ T401.

D87,T385,T217$_m$,T137$_m$,T75$_m$(analog T401) $\succ\!\!-$

TEOREM 402

$(\forall \varkappa\in N^+)(\forall u\in\Omega)((Mod_u(M_\varkappa),\ \smile\ )\in S_{cikl.gr.}(\varkappa))$.

T402 $\succ\!\!-\!\!\prec$

TEOREM 403

$(\forall \varkappa\in N^+)(\forall u\in\Omega)((Mod_u(M_\varkappa),\ \frown\ )\in S_{cikl.gr.}(\varkappa))$.

TEOREM 404

$(\forall u\in\Omega)(\forall x,y\in Mod_u(M_\varkappa))(xy = x\smile y = x\frown y \leftrightarrow u = A \ \vee\ u = B)$.

D o k a z

(1) $x,y\in Mod_u(M_\varkappa)$ & $xy = x\smile y = x\frown y$ Sup.

1,D86,T79$_m$,T75$_m$,T137$_m$ $\succ\!\!-$

(2)$(\exists m,n\in N)(Q^{m+n}uu = Q^{m+n}(u\smile u) = Q^{m+n}(u\frown u))$

2,T72$_m$,T13$_m$ $\succ\!\!-$

(3) $u = u\smile u = u\frown u$

1,3,T124$_m$,T125$_m$ $\succ\!\!-$

(4)$(\forall x,y\in Mod_u(M_\varkappa))(xy = x\smile y = x\frown y \rightarrow u = A \ \vee\ u = B)$.

(5) $x,y\in Mod_u(M_\varkappa)$ & $(u = A \ \vee\ u = B)$. Sup.

5,T79$_m$,T75$_m$,T137$_m$,T124$_m$,T125$_m$ $\succ\!\!-$

(6) $xy = Q^{n+m}{}_u$ & $x\smile y = Q^{n+m}{}_u$ & $x\frown y = Q^{n+m}{}_u$

5,6,4 $\succ\!\!-$ T404.

1.11.5. IDEALI m-SISTEMA

DEFINICIJA 88

$(\forall u\in\Omega)(s.Id_u(M_\pi) = \{x: x\in M_\pi \ \& \ u\ r\ x\})$.

DEFINICIJA 89

$(\forall u\in\Omega)(p.Id_u(M_\pi) = \{x: x\in M_\pi \ \& \ u\ \bar{r}\ x\})$.

D88,T178$_m$ >——

TEOREM 405

$(\forall u\in\Omega)(u\in s.Id_u(M_\pi))$.

T405 >——<

TEOREM 406

$(\forall u\in\Omega)(u\in p.Id_u(M_\pi))$.

TEOREM 407

$(\forall u\in\Omega)(x,y\in s.Id_u(M_\pi) \ \rightarrow \ (x \frown y)\in s.Id_u(M_\pi))$.

D o k a z

(1) $u\in\Omega \ \& \ x,y\in s.Id_u(M_\pi)$	Sup.
(2) $u\ r\ x \ \& \ u\ r\ y$	1,D88
(3) $(u \frown y)\ r\ (x \frown y) \ \& \ q(u \frown y) = q(u)$	2,T180$_m$,D42$_m$
(4) $q(u \frown y)\ r\ q(x \frown y)$	3,T181$_m$
(5) $q(u)\ r\ q(x \frown y)$	4,3
(6) $u\ r\ (x \frown y)$	5,T181$_m$

1,6,D87 >—— T407.

T407 >——<

TEOREM 408

$(\forall u\in\Omega)(x,y\in p.Id_u(M_\pi) \ \rightarrow \ (x \smile y)\in p.Id_u(M_\pi))$.

TEOREM 409

$(\forall u\in\Omega)(x\in s.Id_u(M_\pi) \ \& \ y\in M_\pi \ \rightarrow \ (x \smile y)\in s.Id_u(M_\pi))$.

D o k a z

(1) $u\in\Omega \ \& \ x\in s.Id_u(M_\pi) \ \& \ y\in M_\pi$	Sup.
(2) $u\ r\ x$	1,D88
(3) $(u \smile y)\ r\ (x \smile y)$	2,1,T180$_m$
(4) $u\ r\ (x \smile y)$	3,T176$_m$

1,4,D87 >—— T409.

T409 $\succ\!\!\!-\!\!\!-\!\!\prec$

TEOREM 410

$(\forall u\in\Omega)((x\in p.Id_u(M_x) \;\&\; y\in M_x \;\rightarrow\; (x \wedge y)\in p.Id_u(M_x)).$

TEOREM 411

$(\forall G\in S)(G \subseteq M_x \;\&\; \neg G = 0 \;\&\; (x,y\in G \;\rightarrow\; (x \wedge y)\in G) \;\&\; (x\in G \;\&\; z\in M \;\rightarrow\; (x \vee z)\in G)) \;\rightarrow\; (\exists u\in\Omega)(G = s.Id_u(M_x))).$

D o k a z

(1) $x\in M_x$	Sup.
(2) $(\exists n\in N)(x = Q^n q(x))$	1,$T79_m$
(3) $Q^n q(x) \vee Q^{-n}ba=q(x) \;\&\; q(x) \vee Q^n ba=x$	2,$T151_m$,$T75_m$
(4) $(\forall x\in M_x)((\exists y\in M_x)(x \vee y = q(x)) \;\&\; (\exists z\in M_x)(q(x) \vee z = x)).$	1,2,3
(5) $(\forall x\in M_x)(x \; r \; q(x) \;\&\; q(x) \; r \; x).$	4,$T176_m$
(6) $u\in\Omega \;\&\; z\in M_x \;\&\; u \; r \; z$	Sup.
(7) $(\exists n\in N)(u \vee z = Q^n(u \vee q(z)))$	6,$T79_m$,$T137_m$
(8) $u \vee z = Q^n q(z)$	7,6,$D41_m$,$T181_m$
(9) $(\forall u\in\Omega)(\forall z\in M_x)(u \; r \; z \;\rightarrow\; u \vee z = z).$	6,7,8
(10) $(\exists G\in S)(G \subseteq M_x \;\&\; \neg G = 0 \;\&\; (x,y\in G \;\rightarrow\; (x \wedge y)\in G) \;\&\; (x\in G \;\&\; z\in M_x \;\rightarrow\; (x \vee z)\in G))$	Sup.
(11) $x\in G \;\rightarrow\; q(x)\in G.$	10,4
(12) $x,y\in G$	Sup.
(13) $(x \wedge y)\in G$	12,10
(14) $q(x \wedge y)\in G \;\&\; q(x \wedge y) \; r \; x \;\&\; q(x \wedge y) \; r \; y$	13,11,5,$T180_m$,$T181_m$
(15) $(\forall x,y\in G)(\exists u\in\Omega)(u \; r \; x \;\&\; u \; r \; y)$	12,14,$T38_m$
(16) $(\exists u\in\Omega)(u\in\Omega \;\&\; G \subseteq s.Id_u(M_x)).$	15,10,D88
(17) $u\in\Omega \;\&\; u\in G \;\&\; z\in s.Id_u(M_x)$	Sup.
(18) $u \; r \; z$	17,D88
(19) $u \vee z = z$	18,9
(20) $z\in G$	19,10
(21) $(\forall u\in\Omega)(u\in G \;\rightarrow\; s.Id_u(M_x) \subseteq G).$	17,20

10,16,21 $\succ\!\!\!-\!\!\!-$ T411.

T411 >——<

TEOREM 412

$(\forall G\in S)((G \subseteq M \;\;\&\;\; \neg G = 0 \;\;\&\;\; (x,y\in G \;\rightarrow\; (x \cup y)\in G \;\;\&$
$(x\in G \;\;\&\;\; z\in M \;\rightarrow\; (x \cap z)\in G)) \;\rightarrow\; (\exists u\in\Omega)(G = p.Id_u(M_\varkappa)))$.

D87,D88,(5) u dokazu T411 >——

TEOREM 413

$(\forall u\in\Omega)(s.Id_u(M_\varkappa) = \bigcup\{X\colon\; w\in\Omega \;\;\&\;\; u\; r\; w \;\;\&\;\; X = Mod_w(M_\varkappa)\})$.

T413 >——<

TEOREM 414

$(\forall u\in\Omega)(p.Id_u(M_\varkappa) = \bigcup\{X\colon\; w\in\Omega \;\;\&\;\; u\; \bar{r}\; w \;\;\&\;\; X = Mod_w(M_\varkappa)\})$.

T413,T186$_m$ >——

TEOREM 415

$s.Id_{ab}(M_\varkappa) = Mod_{ab}(M_\varkappa)$.

T415 >——<

TEOREM 416

$p.Id_{ba}(M_\varkappa) = Mod_{ba}(M_\varkappa)$.

T413,T415 >——

TEOREM 417

$(\forall u\in\Omega)(s.Id_{ab}(M_\varkappa) \subseteq s.Id_u(M_\varkappa))$.

T417 >——<

TEOREM 418

$(\forall u\in\Omega)(p.Id_{ba}(M_\varkappa) \subseteq p.Id_u(M_\varkappa))$.

D88,T186$_m$ >——

TEOREM 419

$s.Id_{ba}(M_\varkappa) = M_\varkappa$.

T419 >——<

TEOREM 420

$p.Id_{ab}(M_\varkappa) = M_\varkappa$.

1.12. m-L O G I K A

1.12.1. m_ζ-LOGIKA

DEFINICIJE 90-96

90 $\mathbf{T}_o = ba.$

91 $\perp_o = ab.$

92 $(\forall x \in M_\varkappa)(\neg_m x = Kx).$

93 $(\forall x,y \in M_\varkappa)(x \ \&_m \ y \ = \ x \cup y).$

94 $(\forall x,y \in M_\varkappa)(x \ \vee_m \ y \ = \ x \cap y).$

95 $(\forall x,y \in M_\varkappa)(x \rightarrow_m y \ = Kx \cap y).$

96 $(\forall x,y \in M_\varkappa)(x \longleftrightarrow_m y \ = (Kx \cap y) \cup (Ky \cap x)).$

DEFINICIJA 97
$(\forall \varkappa \in N^+)(L_\varkappa = Mod_{ba}(M_\varkappa) \cup Mod_{ab}(M_\varkappa)).$

KONVENCIJA 2
$(\forall \varkappa \in N^+)(XL_\varkappa Y) \rightharpoondown (XL_\varkappa Y).$

D80,D90-D97,$T79_m$,$T92_m$,$T95_m$,$T102_m$,$T104_m$,$T124_m$,$T125_m$,$T137_m$,$T138_m$,
T400 $>\!\!-\!\!-$

TEOREMI 421-424

421 $(\forall x \in L_\varkappa)(q(x) = \mathbf{T}_o \ \ \vee \ \ q(x) = \perp_o).$

422 $(\forall x \in L_\varkappa)(Dx \in L_\varkappa \ \ \& \ \ Ix \in L_\varkappa \ \ \& \ \ \neg_m x \in L_\varkappa).$

423 $(\forall x,y \in L_\varkappa)((\ x \ \ \&_m \ y) \in L_\varkappa \ \ \& \ \ (x \ \vee_m \ y) \in L_\varkappa).$

424 $(\forall x,y \in L_\varkappa)((\ x \rightarrow_m y) \in L_\varkappa \ \ \& \ \ (x \longleftrightarrow_m y) \in L_\varkappa).$

DEFINICIJA 98

$(\forall \varkappa \in N^{+})(\mathscr{L}_{\varkappa} = (L_{\varkappa},\ D,\ I,\ \neg_m,\ \&_m,\ v_m,\ \rightarrow_m,\ \leftrightarrow_m\ )).$

1.12.2. IZOMORFIZAM ALGEBRE SUDOVA I m_{ζ}-LOGIKE PRVOG REDA

$D31_m$, D90-D97, $T8_m$, $T124_m$, $T125_m$, T392, T396, T397, T421 $\succ$——

TEOREMI 425-432

425 $L_1 = \{\top_o, \bot_o\}$.

426 $(\forall x \in L_1)((\neg_m x = \top_o \leftrightarrow x = \bot_o)\ \&$
$(\neg_m x = \bot_o \leftrightarrow x = \top_o))$.

427 $(\forall x,y \in L_1)(x\ \&_m\ y = \top_o \leftrightarrow x = \top_o\ \&\ y = \top_o)$.

428 $(\forall x,y \in L_1)(x\ v_m\ y = \bot_o \leftrightarrow x = \bot_o\ \&\ y = \bot_o)$.

429 $(\forall x,y \in L_1)(x \rightarrow_m y = \bot_o \leftrightarrow x = \top_o\ \&\ y = \bot_o)$.

430 $(\forall x,y \in L_1)(x \leftrightarrow_m y = \top_o \leftrightarrow x = y)$.

431 $(\forall x \in L_1)(\neg_m x = Dx)$.

432 $(\forall x \in L_1)(\ Ix\ = x\ )$.

D98, T425-T432, (V.Devidé, Matematička logika, 1964, str.37, Def.2.1.)
$\succ$——

TEOREM 433

$\mathscr{L}_1 \in S_{\text{algebra sudova}}$.

1.12.3. SISTEM M KAO GENERALIZACIJA LOGIKE

D90,D91,D32 $\succ\!\!-$

TEOREM 434

$IT_o = T_o \;\&\; I\bot_o = \bot_o$.

D92,T113,T82 $\succ\!\!-$

TEOREM 435

$(\forall x\in M)(\neg_m x = DIx)$.

D92,D31,T8 $\succ\!\!-$

TEOREM 436

$\neg_m T_o = \bot_o \;\&\; \neg_m\bot_o = T_o$.

D92,T98 $\succ\!\!-$

TEOREM 437

$(\forall x\in M)(\neg_m\neg_m x = x)$.

TEOREM 438

$(\forall x\in M)(x \,\&_m\, x = Q^{\lambda_s(x)}x)$.

D o k a z

(1)	$x\in M$	Sup.
(2)	$x \cup x = Q^{2\lambda_s(x)}(q_s(x) \cup q_s(x))$	1,T225,T137
(3)	$= Q^{2\lambda_s(x)}q_s(x)$	2,T209,T211
(4)	$= Q^{\lambda_s(x)}x$	3,T225

1,4,D93 $\succ\!\!-$ T438.

T438 $\succ\!\!-\!\!\prec$

TEOREM 439

$(\forall x\in M)(x \,\mathrm{v}_m\, x = Q^{\lambda_p(x)}x)$.

D93,D94,T190,T178 $\succ\!\!-$

TEOREM 440

$(\forall x,y\in M)(x \,\&_m\, (y \,\mathrm{v}_m\, x) = (x \,\&_m\, y) \,\mathrm{v}_m\, x)$.

T440 $\succ\!\!-\!\!\prec$

TEOREM 441

$(\forall x,y\in M)(x \,\mathrm{v}_m\, (y \,\&_m\, x) = (x \,\mathrm{v}_m\, y) \,\&_m\, x)$.

T145,D93,D90 $\succ\!\!-\!\!-$

TEOREM 442

$(\forall x\in M)(x \mathbin{\&_m} \mathsf{T}_o = x).$

T442 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 443

$(\forall x\in M)(x \vee_m \bot_o = x).$

T438 $\succ\!\!-\!\!-$

TEOREM 444

$(\forall x\in M)(x \mathbin{\&_m} \bot_o = Q^{\lambda_s(x)}\bot_o).$

T444 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 445

$(\forall x\in M)(x \vee_m \mathsf{T}_o = Q^{\lambda_p(x)}\mathsf{T}_o).$

T154,D91;D92 $\succ\!\!-\!\!-$

TEOREM 446

$(\forall x\in M)(x \mathbin{\&_m} (\neg_m x) = \bot_o).$

T446 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 447

$(\forall x\in M)(x \vee_m (\neg_m x) = \mathsf{T}_o).$

TEOREM 448

$(\forall x,y\in M)(x \mathbin{\&_m} y = \mathsf{T}_o \longleftrightarrow q(x) = \mathsf{T}_o \mathbin{\&} y = Ix).$

D o k a z

(1) $x,y\in M \mathbin{\&} x \cup y = ba$	Sup.
(2) $q(x \cup y) = ba$	1,T37
(3) $q(x) = q(y) = ba$	2,T139,T124
(4) $(\exists m,n\in N)(x = Q^m q(x) \mathbin{\&} y = Q^n q(x))$	1,T79
(5) $Q^{m+n}ba = ba$	4,3,1,T137
(6) $Q^m\, ba = Q^{-n}ba$	5,T69
(7) $y = Ix$	6,4,3,T109,T112
(8) $(\forall x,y\in M)(x \cup y = ba \rightarrow q(x) = ba \mathbin{\&} y = Ix).$	1,3,7
(9) $(\forall x,y\in M)(q(x) = ba \mathbin{\&} y = Ix \rightarrow x \cup y = ba).$	T109,T112,T137

8,9,D90,D93 $\succ\!\!-\!\!-$ T448.

T448 $\succ\!\!\!-\!\!\!\prec$

TEOREM 449

$(\forall x,y\in M)(x \vee_m y = \bot_o \quad \longleftrightarrow \quad q(x) = \bot_o \;\&\; y = Ix).$

T448,T106 $\succ\!\!-\!\!-$

TEOREM 450

$(\forall x,y\in M)(x \,\&_m\, Iy = \top_o \quad \longleftrightarrow \quad q(x) = \top_o \;\&\; y = x).$

T450 $\succ\!\!\!-\!\!\!\prec$

TEOREM 451

$(\forall x,y\in M)(x \vee_m Iy = \bot_o \quad \longleftrightarrow \quad q(x) = \bot_o \;\&\; y = x).$

TEOREM 452

$(\forall x,y\in M)(x \rightarrow_m y = \bot_o \quad \longleftrightarrow \quad q(x) = \top_o \;\&\; y = Dx).$

D o k a z

(1) $x,y\in M \;\&\; Kx \frown y = ab$	Sup.
(2) $(\exists m,n\in N)(Kx = Q^{-m}\,\bar{q}(x) \;\&\; y = Q^{n}q(y))$	1,T79,T102,T105
(3) $Q^{-m+n}(\bar{q}(x) \frown q(y)) = ab$	2,1,T137
(4) $q(x) = ba \;\&\; q(y) = ab$	3,T78,T125
(5) $Q^{n}ab = Q^{m}ab$	4,3,T69
(6) $y = DQ^{m}ba = Dx$	5,4,2,T95
(7) $(\forall x,y\in M)(Kx \frown y{=}ab \;\rightarrow\; q(x){=}ba \;\&\; y{=}Dx).$	1,4,6
(8) $x,y\in M \;\&\; q(x) = ba \;\&\; y = Dx$	Sup.
(9) $Kx \frown y = KDy \frown y$	8,T83
(10) $Kx \frown y = y \frown Iy$	9,D32
(11) $(\forall x,y\in M)(q(x){=}ba \;\&\; y{=}Dx \;\rightarrow\; Kx \frown y{=}ab).$	8,10,T451,D95

7,11,D90,D91,D95 $\succ\!\!-\!\!-$ T452.

T452 $\succ\!\!\!-\!\!\!\prec$

TEOREM 453

$(\forall x,y\in M)(\neg_m x \;\&_m\; y = \top_o \quad \longleftrightarrow \quad q(x) = \bot_o \;\&\; y = Dx).$

TEOREM 454

$(\forall x,y\in M)(\,Dx \;\vee_m\; y = \bot_o \quad \longleftrightarrow \quad q(x) = \top_o \;\&\; y = \neg_m x).$

D o k a z

(1) $x,y\in M \;\&\; Dx \frown y = ab$	Sup.
(2) $Q^{\lambda_p(Dx)+\lambda_p(y)}\,q_p(Dx) \frown q_p(y) = ab$	1,T238,T138
(3) $\lambda_p(Dx) + \lambda_p(y) = 0 \;\&\; q_p(Dx) = q_p(y) = ab$	2,T125,T222
(4) $\lambda_s(x) = -\lambda_p(y) \;\&\; q_s(x) = ba \;\&\; q_p(y) = ab$	3,T266,T264

D o k a z

(1) $x, y \in M \;\&\; (Kx \cap y) \cup (x \cap Ky) = ba$ Sup.

1,T448 $\succ$—

(2) $q(Kx \cap y) = ba \;\&\; Kx \cap y = I(x \cap Ky)$

2,T109,T217 $\succ$—

(3) $q_p(Kx \cap y) = q_p(x \cap Ky) = BA$

3,T238,T102 $\succ$—

(4) $q_p(Dx) \cap q_p(y) = q_p(x) \cap q_p(Dy) = BA$

1,4,T125 $\succ$—

(5) $(\forall x, y \in M)((Kx \cap y) \cup (x \cap Ky) = ba \rightarrow q(x) = q(y))$.

(6) $x, y \in M \;\&\; q(x) = q(y) \;\&\; (Kx \cap y) \cup (x \cap Ky) = z$. Sup.

6,T226,T137,T238 $\succ$—

(7) $z = Q^{\lambda_p(Kx)+\lambda_p(y)+\lambda_p(x)+\lambda_p(Ky)}(q_p(Kx \cap y) \cup q_p(x \cap Ky))$

7,T272,T140,6,T102,T220 $\succ$—

(8) $z = Q^{-2}(q_p(q_p(\bar{x}) \cap q_p(x)) \cup q_p(q_p(x) \cap q_p(\bar{x})))$

8,T125 $\succ$—

(9) $z = Q^{-2}(q_p(BA) \cup q_p(BA))$

9,T217 $\succ$—

(10) $z = Q^{-2}BABA$

6,10 $\succ$—

(11) $(\forall x, y \in M)(q(x) = q(y) \rightarrow (Kx \cap y) \cup (x \cap Ky) = ba)$.

5,11,D90,D96 $\succ$— T456.

(5) $x = Q^{-\lambda_p(y)}ba$	4,T225
(6) $Kx = Q^{\lambda_p(y)}Kba$	5,T105
(7) $Kx = y$	5,4,T226
(8) $(\forall x,y \in M)(Dx \cap y = ab \rightarrow q(x)=ba \ \& \ y=Kx)$.	1,4,7
(9) $x,y \in M \ \& \ q(x) = ba \ \& \ y = Kx$	Sup.
(10) $Dx \cap y = DKy \cap y$	9,T99
(11) $\qquad = Iy \cap y$	10,T113
(12) $(\forall x,y \in M)(q(x) = ba \ \& \ y=Kx \rightarrow Dx \cap y=ab)$.	9,11,T451

8,12,D90-D93 $\succ\!\!-$ T454.

T454 $\succ\!\!-\!\!\prec$

TEOREM 455

$(\forall x,y \in M)(Dx \ \&_m \ y = \mathsf{T}_o \leftrightarrow q(x) = \bot_o \ \& \ y = \neg_m x)$.

TEOREM 456

$(\forall x,y \in M)(x \leftrightarrow_m y = \mathsf{T}_o \leftrightarrow q(x) = q(y))$.

T456 $\succ\!\!-\!\!\!-\!\!\prec$

TEOREM 457

$(\forall x,y\in M)((\neg_m x \,\&_m\, y)\ \ v_m\ \ (x \,\&_m\, \neg_m y) = \bot_o \ \longleftrightarrow\ q(x) = q(y))$.

TEOREM 458

$(\forall x,y\in M)((\,Dx\ \ v_m\ y)\ \ \&_m\ \ (x\ v_m\ Dy\ \ ) = \top_o \ \longleftrightarrow\ \ x\ = \ Iy\,)$.

D o k a z

(1)	$x,y\in M \ \ \&\ \ (Dx \frown y) \smile (x \frown Dy) = ba$	Sup.
(2)	$q(Dx \frown y) = ba \ \ \&\ \ Dx \frown y = I(x \frown Dy)$	1,T448
(3)	$q_p(Dx) \frown q_p(y){=}BA \ \ \&\ \ q_p(x) \frown q_p(Dy){=}BA$	2,T238,T109
(4)	$q(x) = q(y)$	3,T125
(5)	$Q^{\lambda_p(Dx)+\lambda_p(y)}{}_{BA} = Q^{-\lambda_p(x)-\lambda_p(Dy)}{}_{IBA}$	4,2,T238,T112
(6)	$Q^{\lambda_s(x)+\lambda_p(y)+1}{}_{ba} = Q^{-\lambda_p(x)-\lambda_s(y)-1}{}_{ba}$	5,T266
(7)	$\lambda_s(x)+\lambda_p(x)+1 = -(\lambda_s(y)+\lambda_p(y)+1)$	6,T222
(8)	$\lambda(x) = -\lambda(y)$	7,T283
(9)	$(\forall x,y\in M)((Dx \frown y) \smile (x \frown Dy){=}ba \rightarrow x{=}Iy)$.	1,4,8,T292,T109
(10)	$x,y\in M \ \ \&\ \ x = Iy$	Sup.
(11)	$(Dx \frown y) \smile (x \frown Dy){=}(DIy \frown y) \smile (x \frown DIx)$	10
(12)	$={(Ky \frown y)} \smile (x \frown Kx)$	11,T118
(13)	$=ba \smile ba$	12,T155
(14)	$(\forall x,y\in M)(x{=}Iy \rightarrow (Dx \frown y) \smile (x \frown Dy){=}ba)$.	10,13,T124

9,14,D90,D93,D94 $\succ\!\!-\!\!\!-$ T458.

T458 $\succ\!\!-\!\!\!-\!\!\prec$

TEOREM 459

$(\forall x,y\in M)((Dx \,\&_m\, y)\ \ v_m\ \ (x \,\&_m\, Dy) = \bot_o \ \longleftrightarrow\ x = Iy)$.

T157,D93,D94 $\succ\!\!-\!\!\!-$

TEOREM 460

$(\forall x,y\in M)(D(x \,\&_m\, y) = Dx\ v_m\ Dy)$.

T460 $\succ\!\!-\!\!\!-\!\!\prec$

TEOREM 461

$(\forall x,y\in M)(D(x\ v_m\ y) = Dx \,\&_m\, Dy)$.

1.13. m_ζ-S T R U K T U R A

1.13.1. m-ZEROID KAO INDUKTIVNA KLASA GENERIRANA IZ m-NULA OPERACIJAMA ZBRAJANJA

DEFINICIJA 99

$M_\zeta = \{x\colon x \in M_{nepoz} \ \& \ (q(x) = ba \ \vee \ q(x) = ab)\}.$

DEFINICIJA 100

$\mathcal{M}_\zeta = (M_\zeta, ab, ba, \smile, \frown).$

D99,T311 $\succ\!\!-\!\!-$

TEOREM 462

$x \in M_\zeta \ \longleftrightarrow \ \lambda(x) \leq 0 \ \& \ \neg\, l(x) = d(x).$

T462,T147,T148 (tot.indukc.) $\succ\!\!-\!\!-$

TEOREM 463

$(\forall G \in S)((\Omega_o \subseteq G \ \& \ G \subseteq M_\zeta \ \&$
$(x, y \in G \ \rightarrow \ (x \smile y) \in G \ \& \ (x \frown y) \in G)) \ \rightarrow \ G = M_\zeta).$

TEOREM 464

$(\forall x, y \in M_\zeta)((q(x) = ba \ \& \ q(y) = ab) \ \longrightarrow$
$x \smile y = D(x \smile Dy) \ \& \ x \frown y = D(x \smile y)).$

D o k a z

(1)	$x, y \in M_\zeta \ \& \ q(x) = ba \ \& \ q(y) = ab$	Sup.
(2)	$(\exists m, n \in N)(x = Q^m ba \ \& \ y = Q^n ab)$	1,T79
(3)	$D(x \smile Dy) = DQ^{m+n} ba$	2,T137,T145
(4)	$= Q^{m+n} ab$	3,T95
(5)	$= Q^m ba \smile Q^n ab$	4,T137,T145
(6)	$(\forall x, y \in M_\zeta)(q(x)=ba \ \& \ q(y)=ab \rightarrow x \smile y = D(x \smile Dy)).$	1,2,5
(7)	$x, y \in M_\zeta \ \& \ x = Q^m ba \ \& \ y = Q^n ab$	Sup.
(8)	$D(x \smile y) = DQ^{m+n}(ba \smile ab)$	7,T137
(9)	$= Q^{m+n}(ab \frown ba)$	8,T95,T157
(10)	$= Q^m ba \frown Q^n ab$	9,T137
(11)	$(\forall x, y \in M_\zeta)(q(x)=ba \ \& \ q(y)=ab \rightarrow x \frown y = D(x \smile y)).$	7,10

6,11 $\succ\!\!-\!\!-$ T464.

T464 $\succ\!\!\!-\!\!\!\prec$

TEOREM 465

$$(\forall x,y\in M_\zeta)((q(x) = ab \ \& \ q(y) = ba) \rightarrow x \cap y = D(x \cap Dy) \ \& \ x \cup y = D(x \cap y)) \ .$$

T464,T465 $\succ\!\!-$

TEOREM 466

$$(\forall x,y\in M_\zeta)(\neg q(x) = q(y) \rightarrow x \cup y = Dx \cup Dy \ \& \ x \cap y = Dx \cap Dy \ .$$

TEOREM 467

$$(\forall x,y\in M_\zeta)(q(x) = q(y) = ba \rightarrow (x \cap y) = Q'(x \cup y)).$$

D o k a z

(1)	$x,y\in M \ \& \ x = Q^m ba \ \& \ y = Q^n ba$	Sup.
(2)	$x \cap y = Q^{m+n-1}\, ba$	1,T138,T125
(3)	$= Q'(Q^m ba \cup Q^n ba)$	2,T137,T145

1,T78,3 $\succ\!\!-$ T467.

T467 $\succ\!\!\!-\!\!\!\prec$

TEOREM 468

$$(\forall x,y\in M_\zeta)(q(x) = q(y) = ab \rightarrow (x \cup y) = Q'(x \cap y)).$$

T464,T467 $\succ\!\!-$

TEOREM 469

$$(\forall x\in M_\zeta)(q(x) = ba \rightarrow x \cup ab = Dx \ \& \ x \cap ba = Q'x).$$

T469 $\succ\!\!\!-\!\!\!\prec$

TEOREM 470

$$(\forall x\in M_\zeta)(q(x) = ab \rightarrow x \cap ba = Dx \ \& \ x \cap ab = Q'x).$$

1.13.2. IZOMORFIZAM m_ζ-STRUKTURE I STRUKTURE DOBIVENE DUALIZIRANJEM ADITIVNE POLUGRUPE PRIRODNIH BROJEVA

DEFINICIJA 101

(i) $\mathscr{E} = X \ \& \ \mathscr{S} = Y \longleftrightarrow X = \{0,1,2,3,\ldots\} \ \& \ (X,0,Y)\in S_{\text{Peano s.}}$

(ii) $\bar{\mathscr{E}} = \bar{X} \ \& \ \bar{\mathscr{S}} = \bar{Y} \longleftrightarrow \bar{X} = \{\bar{0},\bar{1},\bar{2},\bar{3},\ldots\} \ \& \ (\bar{X},\bar{0},\bar{Y})\in S_{\text{Peano s.}}$

DEFINICIJA 102

$$\begin{array}{lll}
\mathscr{N}_\zeta = (N_\zeta, 0, \bar{0}, \Sigma, \Delta, +, \hat{+}) \longleftrightarrow (N_\zeta = \mathscr{E} \cup \bar{\mathscr{E}} \ \& \ \Sigma = \mathscr{S} \cup \bar{\mathscr{S}} & & \& \\
\Delta\in S_{\text{funkc}}(N_\zeta, N_\zeta) \ \& \ \Delta 0 = \bar{0} \ \& \ \Delta\bar{0} = 0 \ \& \ (\forall x\in N_\zeta)(\Delta\Sigma x = \Sigma\Delta x) & & \& \\
+\in S_{\text{funkc}}(N_\zeta \times N_\zeta, N_\zeta) & \& \ \hat{+}\in S_{\text{funkc}}(N_\zeta \times N_\zeta, N_\zeta) & \& \\
(\forall x\in N_\zeta)(\quad x + 0 = x & \& \quad x \hat{+} \bar{0} = x \quad) & \& \\
(\forall x,y\in N_\zeta)(\quad x + \Sigma y = \Sigma(x+y) & \& \quad x \hat{+} \Sigma y = \Sigma(x \hat{+} y) \quad) & \& \\
(\forall x,y\in N_\zeta)(\quad x + y = y + x & \& \quad x \hat{+} y = y \hat{+} x \quad) & \& \\
(\forall x,y,z\in N_\zeta)(x + (y+z) = (x+y) + z & \& \quad x \hat{+} (y \hat{+} z) = (x \hat{+} y) \hat{+} z \quad) & \& \\
\qquad (\quad \bar{0} + \bar{0} = \Sigma\bar{0} & \& \quad 0 \hat{+} 0 = \Sigma 0 \ )). &
\end{array}$$

PRIMJERI

267 $7 + 0 = 7$. 269 $\bar{3} \hat{+} \bar{0} = \bar{3}$. 271 $\bar{0} + \bar{0} = \bar{1}$.

268 $\bar{3} + 0 = \bar{3}$. 270 $5 \hat{+} \bar{0} = 5$. 272 $0 \hat{+} 0 = 1$.

D101,D102 $\succ\!\!\!-\!\!-$ T471 - T478.

TEOREM 471

$(\forall x\in N_\zeta)(x\in\mathscr{E} \longleftrightarrow \Delta x\in\bar{\mathscr{E}})$.

PRIMJERI

273 $\Delta 1 = \Delta\Sigma 0 = \Sigma\Delta 0 = \Sigma\bar{0} = \bar{1}$. 275 $\Delta 35 = \overline{35}$.

274 $\Delta\bar{1} = \Delta\Sigma\bar{0} = \Sigma\Delta\bar{0} = \Sigma 0 = 1$. 276 $\Delta\overline{73} = 73$.

TEOREM 472

$(\forall x\in N_\zeta)(\Delta\Delta x = x)$.

PRIMJERI

277 $\Delta\Delta 0 = \Delta\bar{0} = 0$. 278 $\Delta\Delta\overline{69} = 69$.

TEOREM 473

$(\forall x, y \in \mathcal{E})(x + y = \Delta(\Delta x \overset{\frown}{+} \Delta y)).$

PRIMJER 279

$1 + 2 = \Sigma\Sigma\Sigma 0 = \Sigma\Sigma\Sigma\Delta(\bar{0} \overset{\frown}{+} \bar{0}) = \Delta(\Sigma\bar{0} \overset{\frown}{+} \Sigma\Sigma\bar{0}) = \Delta(\bar{1} \overset{\frown}{+} \bar{2}).$

TEOREM 474

$(\forall x, y \in \mathcal{E})(x + \Delta y = \Delta(x + y)).$

PRIMJERI

280 $2 + \bar{0} = \Sigma\Sigma(0 + \bar{0}) = \Sigma\Sigma\bar{0} = \bar{2} = \Delta 2 = \Delta(2 + 0).$

281 $2 + \bar{3} = 2 + \Delta 3 = \Delta(2 + 3) = \bar{5}.$ 282 $\bar{4} + 2 = 2 + \bar{4} = \bar{6}.$

TEOREM 475

$(\forall x, y \in \mathcal{E})(\Delta x + \Delta y = \Sigma\Delta(x + y)).$

PRIMJERI

283 $\bar{0} + \bar{2} = \Sigma\Sigma(\bar{0} + \bar{0}) = \Sigma\Sigma\Sigma\bar{0} = \bar{3} = \Sigma\Delta(0 + 2).$

284 $\bar{3} + \bar{2} = \Sigma\Delta(3 + 2) = \Sigma\Delta 5 = \Sigma\bar{5} = \bar{6}.$

TEOREM 476

$(\forall x, y \in \mathcal{E})(x \overset{\frown}{+} y = \Sigma(x + y)).$

PRIMJERI

285 $0 \overset{\frown}{+} 2 = \Sigma\Sigma(0 \overset{\frown}{+} 0) = \Sigma\Sigma\Sigma 0 = 3 = \Sigma(0 + 2).$

286 $3 \overset{\frown}{+} 2 = \Sigma(3 + 2) = 6.$

TEOREM 477

$(\forall x, y \in \mathcal{E})(x \overset{\frown}{+} \Delta y = x + y).$

PRIMJERI

287 $2 \overset{\frown}{+} \bar{1} = \Sigma(2 \overset{\frown}{+} \bar{0}) = \Sigma 2 = 3 = 2 + 1.$

288 $\bar{3} \overset{\frown}{+} 2 = 2 \overset{\frown}{+} \bar{3} = 2 + 3 = 5.$

TEOREM 478

$(\forall x,y\in\mathscr{E})(\Delta x \overset{\frown}{+} \Delta y = \Delta(x + y)).$

PRIMJERI

289 $\bar{2} \overset{\frown}{+} \bar{1} = \Sigma(\bar{2} \overset{\frown}{+} \bar{0}) = \Sigma\bar{2} = \bar{3} = \Delta(2 + 1).$

290 $\bar{3} \overset{\frown}{+} \bar{4} = \Delta(3 + 4) = \Delta 7 = \bar{7}.$

T473,T478 $\succ\!\!-\!\!-$

TEOREM 479

$(\forall x,y\in N_\zeta)(\Delta(x + y) = \Delta x \overset{\frown}{+} \Delta y \;\;\&\;\; \Delta(x \overset{\frown}{+} y) = \Delta x + \Delta y).$

T474,T476 $\succ\!\!-\!\!-$

TEOREM 480

$(\forall x\in\mathscr{E})(x + \bar{0} = \Delta x \;\;\&\;\; x \overset{\frown}{+} 0 = \Sigma x).$

T480,T479 $\succ\!\!-\!\!-$

TEOREM 481

$(\forall x\in\bar{\mathscr{E}})(x \overset{\frown}{+} 0 = \Delta x \;\;\&\;\; x + \bar{0} = \Sigma x).$

T474,T477 $\succ\!\!-\!\!-$

TEOREM 482

$(\forall x\in N_\zeta)(x + 1 = \Sigma x \;\;\&\;\; x \overset{\frown}{+} \bar{1} = \Sigma x).$

T462-T482,D101,D102,T72,T137,T138,T84,T157,T158 $\succ\!\!-\!\!-$

TEOREM 483

$\mathscr{N}_\zeta = (N_\zeta,\ 0,\ \bar{0},\ \Sigma,\ \Delta,\ +,\ \overset{\frown}{+}\ ) \longrightarrow$

$(\exists! F\in S_{bijekc}(N_\zeta,M_\zeta))(F(0) = ba \;\;\&\;\; F(\bar{0}) = ab \;\;\&$

$(\forall x,y\in N_\zeta)(F(x + y) = F(x) \smile F(y) \;\;\&\;\; F(x \overset{\frown}{+} y) = F(x) \frown F(y)) \;\;\&$

$(\forall x\in N_\zeta)(F(\Sigma x) = QF(x) \;\;\&\;\; F(\Delta x) = DF(x))).$

Drugi dio

m_t-STRUKTURA

2.1. U T E M E L J E N J E m_t-S T R U K T U R E

DEFINICIJA 103

$(\{A,B\}\in S \ \ \& \ \ \kappa\{A,B\} = 2) \longrightarrow$

$\Psi_t\{A,B\} = \{(G,f_s,f_p)\colon G\in S \ \ \& \ \ \{f_s,f_p\} \subseteq S_{funkc}(G \times G,G) \qquad \&$

$(\forall H\in S)((\{A,B\} \subseteq H \ \ \& \ \ H \subseteq G \ \ \&$

$(x,y\in H \rightarrow (f_s(x,y)\in H \ \ \& \ \ f_p(x,y)\in H)) \longrightarrow H = G) \qquad \&$

$(x,y\in G \longrightarrow (f_s(x,y) = f_s(y,x) \ \ \& \ \ f_p(x,y) = f_p(y,x)) \qquad \&$

$(x,y,z\in G \longrightarrow (f_s(f_s(x,y),z) = f_s(x,f_s(y,z)) \ \ \&$

$f_p(f_p(x,y),z) = f_p(x,f_p(y,z)))) \qquad \&$

$(x\in\{A,B\} \longrightarrow f_s(x,x) = x \ \ \& \ \ f_p(x,x) = x) \qquad \&$

$(x\in G \longrightarrow f_s(x,f_p(B,A)) = f_p(x,f_s(A,B))) \qquad \&$

$(x,y\in G \ \ \& \ \ f_s(x,f_p(B,A)) = f_p(y,f_s(A,B))) \longrightarrow x = y) \qquad \&$

$(x\in G \longrightarrow \neg f_s(x,f_p(B,A))\in\{A,\ B,\ f_s(A,B),\ f_p(B,A)\})\}.$

A K S I O M 3

$\kappa(\Psi_t\{A,B\}) = 1.$

DEFINICIJA 104

$(M_t = G \ \ \& \ \ \cup = f_s \ \ \& \ \ \cap = f_p) \longleftrightarrow (G,f_s,f_p)\in \Psi_t\{A,B\}.$

DEFINICIJA 105

$\Omega_t = \{A,B\}$.

DEFINICIJA 106

$\Omega_t^+ = \{A,\ B,\ A \cup B,\ B \cap A\}$.

D103-D106,A3 $\succ\!\!-\!\!-$ T484-T494.

TEOREMI 484-485.

484 $\cup \in S_{funkc}((M_t \times M_t),\ M_t)$.

485 $\cap \in S_{funkc}((M_t \times M_t),\ M_t)$.

TEOREMI 486-494.

486 $\neg A = B$.

487 $\Omega_t \subseteq M_t$.

488 $(\forall G \in S)((\Omega_t \subseteq G \subseteq M_t \ \&\ (x,y \in G \longrightarrow (x \cup y) \in G \ \&\ (x \cap y) \in G) \longrightarrow G = M_t)$.

489 $(\forall x,y \in M_t)(x \cup y = y \cup x \ \&\ x \cap y = y \cap x)$.

490 $(\forall x,y,z \in M_t)(x \cup (y \cup z) = (x \cup y) \cup z \ \&\ x \cap (y \cap z) = (x \cap y) \cap z)$.

491 $(\forall u \in \Omega_t)(u \cup u = u \ \&\ u \cap u = u)$.

492 $(\forall x \in M_t)(x \cup (B \cap A) = x \cap (A \cup B))$.

493 $(\forall x,y \in M_t)(x \cup (B \cap A) = y \cap (A \cup B) \longrightarrow x = y)$.

494 $(\forall x \in M_t)\ \neg((x \cup (B \cap A)) \in \Omega_t^+)$.

DEFINICIJA 107

$\mathscr{M}_t = (M_t,\ \Omega_t,\ \cup\ ,\ \cap\ )$.

TEOREM 495

$(M_{poz}, \cup, \cap) \in \psi_t\{A,B\}$.

D o k a z

T309,T280,T18 $\succ\!\!-$

(1) $A \in M_{poz}$ & $B \in M_{poz}$.

T309,T281,T282,T237,T238 $\succ\!\!-$

(2) $x,y \in M_{poz} \rightarrow (x \cup y) \in M_{poz}$ & $(x \cap y) \in M_{poz}$

T278,T309 $\succ\!\!-$

(3) $(\forall x \in M_{poz})(\exists n \in N^+)(x = P^n A \lor y = P^n B)$.

(4) $G \in S$ & $(\Omega_t \subseteq G \subseteq M_{poz})$ & $(x,y \in G \rightarrow (x \cup y) \in G$ & $(x \cap y) \in G)$. — Def.

(5) $x \in G \rightarrow (A \cup x) \in G$ & $(B \cap x) \in G$ — 4

(6) $x \in G \rightarrow Px \in G$ — 5,T133,T134

(7) $M_{poz} \subseteq G$ — 6,3,4

4,7 $\succ\!\!-$

(8) $(\forall G \in S)((\Omega_t \subseteq G \subseteq M_{poz})$ & $(x,y \in G \rightarrow (x \cup y) \in G$ & $(x \cap y) \in G)) \rightarrow G = M_{poz}$.

T14,T2 $\succ\!\!-$

(9) $\neg A = B$.

T18,T124,T125 $\succ\!\!-$

(10) $(\forall u \in \Omega_t)(u \cup u = u$ & $u \cap u = u)$.

(11) $x \in M_{poz}$ & $(x \cup BA) \in \{A,B,BA,AB\}$ — Sup.

(12) $(\exists x \in M_{poz})(x = Q'A \lor x = Q'B \lor x = Q'BA \lor x = Q'AB)$ — 11,T152,T69

(13) $\lambda(x) = -1 \lor \lambda(x) = 0$ — 12,T288

(14) $x \in M_{poz} \rightarrow \neg((x \cup (B \cap A)) \in \{A,B,A \cup B,B \cap A\})$. — 11,13,T309

D103,1,2,8,9,10,14,T141-T144,T152,T153,T72 $\succ\!\!-$ T495.

2.2. O P E R A T O R Q^n

DEFINICIJA 108

$(\forall x \in M_t)(Qx = x \cup (B \cap A))$.

D108,T492 $\succ\!\!-$

TEOREM 496

$(\forall x \in M_t)(Qx = x \cap (A \cup B))$.

D108,T490 $\succ\!\!-$

TEOREM 497

$(\forall x,y\in M_t)(x \cup Qy = Q(x \cup y))$.

T496,T490 $\succ\!\!-$

TEOREM 498

$(\forall x,y\in M_t)(x \cap Qy = Q(x \cap y))$.

T490,T491,D106,D108 $\succ\!\!-$

TEOREM 499

$u,w\in\Omega_t^+ \;\rightarrow\; ((u \cup w)\in\Omega_t^+ \;\vee\; u \cup w = Qu \;\vee\; u \cup w = Qw)$.

T499 $\succ\!\!-\!\!\prec$

TEOREM 500

$u,w\in\Omega_t^+ \;\rightarrow\; ((u \cap w)\in\Omega_t^+ \;\vee\; u \cap w = Qu \;\vee\; u \cap w = Qw)$.

TEOREM 501

$Q\in S_{bijekc}(M_t,\, M_t\setminus\Omega_t^+)$.

D o k a z

(1) $Q\in S_{funkc}(M_t,\, M_t\setminus\Omega_t^+)$.	D108,D106,T484,T494
(2) $(\forall x,y\in M_t)(Qx = Qy \;\rightarrow\; x = y)$.	D108,T492,T493
(3) $G = \{x\colon x\in\Omega_t^+ \;\vee\; (y\in M_t \;\&\; x = Qy)\}$	Sup.
(4) $\Omega_t \subseteq G$	3,D105,D106
(5) $x,y\in G \;\rightarrow\; (x \cup y)\in G \;\&\; (x \cap y)\in G$	3,T497-T500
(6) $G = M_t$	3,4,5,T488
(7) $M_t = \Omega_t^+ \cup \mathrm{Adom}(Q)$	6,3
(8) $M_t\setminus\Omega_t^+ = \mathrm{Adom}(Q)$	7,1

1,2,8 $\succ\!\!-$ T501.

T501 $\succ\!\!-$

TEOREM 502

$(\forall x\in M_t)(\exists! F_x\in S_{funkc}(N^+,M_t))(F_x(0) = \mathbb{X} \;\&$

$(\forall n\in N^+)(F_x(n+1) = QF_x(n)))$.

DEFINICIJA 109

$(\forall x\in M_t)(\forall n\in N^+)((Q^n(x) = F_x(n)) \;\longleftrightarrow\;$ (F_x je jedinstveno određena funkcija iz teorema 502)).

D109,T502 $\succ\!\!-$

TEOREM 503

$(\forall x\in M_t)(Q^o x = x \ \ \& \ \ Q^1 x = Qx)$.

D109,T501-T503 (tot.ind.) $\succ\!\!-$ T504-T506.

TEOREM 504

$(\forall n\in N^+)(Q^n \in S_{injekc}(M_t,M_t))$.

TEOREM 505

$(\forall x,y\in M_t)(\forall n\in N^+)(x \cup Q^n y = Q^n(x \cup y) \ \ \& \ \ x \cap Q^n y = Q^n(x \cap y))$.

TEOREM 506

$(\forall x\in M_t)(\forall m,n\in N^+)(Q^m Q^n x = Q^{m+n} x)$.

TEOREM 507

$(\forall u\in \Omega_t^+)(\forall n\in N_{poz}) \neg (Q^n u\in \Omega_t^+)$.

D o k a z

(1) $u\in\Omega_t \ \ \& \ \ (\exists n\in N_{poz})(Q^n u\in\Omega_t^+)$	Sup.
(2) $QQ^{n-1}u\in\Omega_t^+$	1,T506,T503

1,2,T501 $\succ\!\!-$ T507.

TEOREM 508

$(\forall x\in M_t)(\exists! u\in\Omega_t^+)(\exists! n\in N^+)(x = Q^n u)$.

D o k a z

(1) $G = \{x\colon x\in M_t \ \& \ (\exists u\in\Omega_t^+)(\exists n\in N^+)(x = Q^n u)\}$	Def.
(2) $Q^o A = A \ \ \& \ \ Q^o B = B$	T503
(3) $\Omega_t \subseteq G$	1,2,D105
(4) $(\exists x,y\in G)(\exists u,w\in\Omega_t^+)(\exists m,n\in N^+)(x=Q^m u \ \ \& \ \ y=Q^n w)$	Sup.
(5) $x \cup y = Q^{m+n}(u \cup w) \ \ \& \ \ x \cap y = Q^{m+n}(u \cap w)$	4,T505,T506

5,T499,T500,T506 $\succ\!\!-$

$$(6)(\exists z,\dot{z}\in\Omega_t)((x \cup y = Q^{m+n}z \ \ \vee \ \ x \cup y = Q^{m+n+1}z) \ \ \& \ \ (x \cap y = Q^{m+n}\dot{z} \ \ \vee \ \ x \cap y = Q^{m+n+1}\dot{z}))$$

(7) $x,y\in G \ \rightarrow \ (x \cup y)\in G \ \ \& \ \ (x \cap y)\in G$	1,4,6
(8) $(\forall x\in M_t)(\exists u\in\Omega_t^+)(\exists n\in N^+)(x = Q^n u)$.	1,3,7,T488
(9) $(\exists x\in M_t)(\exists u,w\in\Omega_t^+)(\exists m,n\in N^+)(x = Q^m u \ \ \& \ \ x = Q^n w)$	Sup.
(10) $(\exists k\in N^+)(m = n + k \ \ \vee \ \ m + k = n)$	9
(11) $Q^{n+k} u = Q^n w \ \ \vee \ \ Q^m u = Q^{m+k} w$	9,10
(12) $Q^k u = w \ \ \vee \ \ u = Q^k w$	11,T504
(13) $k = 0 \ \ \& \ \ u = w$	9,10,12,T507

8,9,10,13 $\succ\!\!-$ T508.

D115,T514,D113,D114,D105 $\succ\!\!-$

TEOREM 515

$l_t \in S_{surjekc}(M_t, \Omega_t)$.

D116,T513,D111,D112,D105 $\succ\!\!-$

TEOREM 516

$d_t \in S_{surjekc}(M_t, \Omega_t)$.

D113-D115,T511 $\succ\!\!-$

TEOREM 517

$$(\forall x \in M_t)((l_t(x) = A \;\leftrightarrow\; A \cup x = x \;\leftrightarrow\; \neg B \cap x = x) \;\&\; (l_t(x) = B \;\leftrightarrow\; B \cap x = x \;\leftrightarrow\; \neg A \cup x = x)).$$

D116,D111,D112,T512 $\succ\!\!-$

TEOREM 518

$$(\forall x \in M_t)((d_t(x) = A \;\leftrightarrow\; x \cap A = x \;\leftrightarrow\; \neg x \cup B = x) \;\&\; (d_t(x) = B \;\leftrightarrow\; x \cup B = x \;\leftrightarrow\; \neg x \cap A = x)).$$

T517,T518,T491 $\succ\!\!-$

TEOREM 519

$l_t(A) = A \;\&\; d_t(A) = A.$

T519 $\succ\!\!-\!\!\prec$

TEOREM 520

$l_t(B) = B \;\&\; d_t(B) = B.$

T517,T518,T490,T491 $\succ\!\!-$

TEOREM 521

$l_t(A \cup B) = A \;\&\; d_t(A \cup B) = B.$

T521 $\succ\!\!-\!\!\prec$

TEOREM 522

$l_t(B \cap A) = B \;\&\; d_t(B \cap A) = A.$

TEOREM 523

$(\forall x \in M_t)(\forall n \in N^+)(l_t(Q^n x) = l_t(x)).$

D o k a z

(1) $x \in M_t \;\&\; n \in N^+ \;\&\; l_t(Q^n x) = A$	Sup.
(2) $A \cup Q^n x = Q^n x$	1,T517
(3) $Q^n(A \cup x) = Q^n x$	2,T505

(4) $A \cup x = x$ 3,T504

(5)$(\forall x \in M_t)(\forall n \in N^+)(1_t(Q^n x) = A \longleftrightarrow 1_t(x) = A)$. 1,4,T517

5 $\succ\!\!\!-\!\!\!-\!\!\prec$

(6)$(\forall x \in M_t)(\forall n \in N^+)(1_t(Q^n x) = B \longleftrightarrow 1_t(x) = B)$.

5,6,T515 $\succ\!\!-\!\!-$ T523.

T518,T505,T504,T516 $\succ\!\!-\!\!-$

TEOREM 524

$(\forall x \in M_t)(\forall n \in N^+)(d_t(Q^n x) = d_t(x))$.

DEFINICIJA 117

$(\forall x \in M_t)(\forall u \in \Omega_t)(\bar{1}_t(x) = u \longleftrightarrow \neg u = 1_t(x))$.

DEFINICIJA 118

$(\forall x \in M_t)(\forall u \in \Omega_t)(\bar{d}_t(x) = u \longleftrightarrow \neg u = d_t(x))$.

PRIMJERI

298 $\bar{1}_t(A) = B$. 299 $\bar{d}_t(B) = A$. 300 $\bar{1}_t(B \cup A) = B$.

301 $\bar{d}_t(A \cup B) = A$.

2.4. OPERATORI P_1^n I P_d^n

DEFINICIJA 119

$$(\forall x \in M_t)(P_1(x) = Px = y \longleftrightarrow (1_t(x) = A \ \& \ y = B \cap x) \lor (1_t(x) = B \ \& \ y = A \cup x))\ .$$

DEFINICIJA 120

$$(\forall x \in M_t)(P_d(x) = xP = y \longleftrightarrow (d_t(x) = A \ \& \ y = x \cup B) \lor (d_t(x) = B \ \& \ y = x \cap A))\ .$$

PRIMJERI

302 $PA = B \cap A$. 303 $PB = A \cup B$. 304 $P(B \cap A) = A \cup (B \cap A)$.

305 $AP = A \cup B$. 306 $BP = B \cap A$. 307 $(A \cup B)P = (A \cup B) \cap A$.

308 $P((B \cap A) \cup (B \cap A)) = A \cup (B \cap A) \cup (B \cap A)$.

309 $(B \cup (B \cap A))P = ((B \cap A) \cup B) \cap A$.

TEOREM 525

$(\forall x \in M_t)(l_t(Px) = \bar{l}_t(x) \ \& \ d_t(Px) = d_t(x))$.

D o k a z

(1) $x \in M_t \ \& \ l_t(x) = A \ \& \ d_t(x) = A$	Sup.
(2) $A \cup x = x \ \& \ x \cap A = x \ \& \ Px = B \cap x$	1,T517,D119
(3) $B \cap Px = Px \ \& \ Px \cap A = Px$	2,T490,T491
(4) $l_t(Px) = B \ \& \ d_t(Px) = A$	3,T517,T518

1,4,D117 >——

(5)$(\forall x \in M_t)(l_t(x) = A \ \& \ d_t(x) = A \rightarrow l_t(Px) = \bar{l}_t(x) \ \& \ d_t(Px) = d_t(x))$.

5 >——<

(6)$(\forall x \in M_t)(l_t(x) = B \ \& \ d_t(x) = B \rightarrow l_t(Px) = \bar{l}_t(x) \ \& \ d_t(Px) = d_t(x))$.

T517,T518,D119,T490,T491,D117 >——

(7)$(\forall x \in M_t)(l_t(x) = A \ \& \ d_t(x) = B \rightarrow l_t(Px) = \bar{l}_t(x) \ \& \ d_t(Px) = d_t(x))$.

7 >——<

(8)$(\forall x \in M_t)(l_t(x) = B \ \& \ d_t(x) = A \rightarrow l_t(Px) = \bar{l}_t(x) \ \& \ d_t(Px) = d_t(x))$.

5,6,7,8,T515,T516,D105 >—— T525.

D120,D118,D105,T490,T491,T515-T518 >——

TEOREM 526

$(\forall x \in M_t)(l_t(xP) = l_t(x) \ \& \ d_t(xP) = \bar{d}_t(x))$.

TEOREM 527

$P_1 \in S_{bijekc}(M_t, M_t \setminus \Omega_t)$.

D o k a z

D119,T484,T485 >——

(1) $P_1 \in S_{funkc}(M_t, M_t)$.

(2) $x \in M_t \ \& \ Px = A$	Sup.
(3) $l_t(x) = B \ \& \ d_t(x) = A$	2,T525,D117
(4) $A \cup x = A$	3,2,D119
(5)$(\exists n \in N^+)(A \cup Q^n(B \cap A) = A)$	4,3,2,T508,T522
(6) $Q^n A \cup (B \cap A) = A$	5,T505
(7)$(\forall x \in M_t) \neg (Px = A)$.	6,T494,2

7 >——<

(8)$(\forall x \in M_t) \neg (Px = B)$.

(9) $Adom(P_1) \subseteq (M_t \setminus \Omega_t)$. 1,7,8,D105

D105,T508 >——

(10) $y \in M_t \setminus \Omega_t \longrightarrow$
$(\exists m \in N_{poz})(y = Q^m A \ \ v \ \ y = Q^m B) \ \ v$
$(\exists n \in N^+)(y = Q^n(B \cap A) \ \ v \ \ y = Q^n(A \cup B))$.

(11) $y = A \cup Q^{m-1}(B \cap A) \quad v$
$y = B \cap Q^{m-1}(A \cup B) \quad v$
$y = B \cap Q^n A \ \ v \ \ y = A \cup Q^n B$ 10,D108,T505,T506,T496

(12) $y = PQ^{m-1}(B \cap A) \quad v$
$y = PQ^{m-1}(A \cup B) \quad v$
$y = PQ^n A \ \ v \ \ y = PQ^n B$ 11,T519-T524,D119

(13) $y \in M_t \setminus \Omega_t \longrightarrow (\exists x \in M_t)(y = Px)$. 10,12,T504

(14) $Adom(P_1) = M_t \setminus \Omega_t$. 9,13

(15) $x, y \in M_t \ \ \& \ \ Px = Py$ Sup.

(16) $l_t(x) = l_t(y) \ \ \& \ \ d_t(x) = d_t(y)$ 15,T515,T516,T525

(17) $(\exists u \in \Omega_t^+)(\exists m, n \in N^+)(PQ^m u = PQ^n u)$ 15,16,T519-T522,T508

(18) $Q^m(A \cup u) = Q^n(A \cup u) \quad v$
$Q^m(B \cap u) = Q^n(B \cap u)$ 17,D119,T505

(19) $m = n$ 18,T504,T507

(20) $(\forall x, y \in M_t)(Px = Py \rightarrow x = y)$ 15,17,19

1,14,20 >—— T527.

T484,T485,T494,T496,T504-T508,T515-T525,D105,D117,D119 >——

TEOREM 528

$P_d \in S_{bijekc}(M_t, M_t \setminus \Omega_t)$.

TEOREM 529

$(\forall x \in M_t)(PPx = Qx)$.

D o k a z

(1) $x \in M_t \ \ \& \ \ l_t(x) = A \ \ \& \ \ d_t(x) = A$ Sup.

(2) $(\exists n \in N^+)(x = Q^n A)$ 1,T519,T508

(3) $PPx = Q^n(A \cup (B \cap A))$ 2,T523,D119,T505

(4) $PPx = Q^n QA$ 3,D108

(5) $(\forall x \in M_t)((l_t(x) = A \ \& \ d_t(x) = A) \rightarrow PPx = Qx)$. 1,2,4,T506

5 $\rightarrowtail$

(6) $(\forall x \in M_t)((l_t(x)=B \ \& \ d_t(x)=B) \rightarrow PPx = Qx)$.

(7) $x \in M_t \quad \& \quad l_t(x) = A \quad \& \quad d_t(x) = B$ — Sup.

(8) $(\exists n \in N^+)(x = Q^n(A \cup B))$ — 7,T521,T508

(9) $PPx = Q^n(A \cup (B \cap (A \cup B)))$ — 8,T523,D119,T505

(10) $PPx = Q^nQ(A \cup B)$ — 9,T496,T505

(11) $(\forall x \in M_t)((l_t(x)=A \ \& \ d_t(x)=B) \rightarrow PPx=Qx)$. — 7,8,10,T506

11 $\rightarrowtail$

(12) $(\forall x \in M_t)((l_t(x) = B \ \& \ d_t(x) = A \rightarrow PPx = Qx)$.

5,6,11,12,T515,D105 $\rightarrowtail$ T529.

T519,T508,T524,D120,T505,D108,T506,T521,T522,T496(analog T529) $\rightarrowtail$

TEOREM 530

$(\forall x \in M_t)(xPP = Qx)$.

PRIMJERI

310 $PPA = A \cup (B \cap A) = QA$.

311 $PP(A \cup B) = A \cup (B \cap (A \cup B)) = A \cup QB = Q(A \cup B)$.

312 $(B \cap A)PP = ((B \cap A) \cup B) \cap A = QB \cap A = Q(B \cap A)$.

DEFINICIJA 121

$(\forall x \in M_t)(\forall n \in N^+)(P^{2n}x = Q^nx \quad \& \quad P^{2n+1}x = PQ^nx)$.

DEFINICIJA 122

$(\forall x \in M_t)(\forall n \in N^+)(xP^{2n} = xQ^n \quad \& \quad xP^{2n+1} = (Q^nx)P)$.

D121,D122,T503 $\rightarrowtail$ T531-T533.

TEOREM 531

$(\forall x \in M_t)(P^ox = xP^o = x)$.

TEOREM 532

$(\forall x \in M_t)(P^1x = Px \quad \& \quad xP^1 = xP)$.

TEOREM 533

$(\forall x \in M_t)(\forall n \in N^+)(P^{n+1}x = PP^nx \quad \& \quad xP^{n+1} = xP^nP)$.

D121,D122,T504,T527,T528 $\rightarrowtail$ T534-T536.

TEOREM 534

$(\forall n \in N^+)(P_1^n \in S_{injekc}(M_t,M_t))$.

TEOREM 535

$(\forall n\in N^+)(P_d^n \in S_{injekc}(M_t,M_t))$.

TEOREM 536

$(\forall x\in M_t)(\forall m,n\in N^+)(P^mP^nx = P^{m+n}x \quad \& \quad xP^mP^n = xP^{m+n})$.

TEOREM 537

$(\forall x\in M_t)(\forall m,n\in N^+)(P^m(xP^n) = (P^mx)P^n)$.

D o k a z

(1) $x\in M_t \quad \& \quad l_t(x) = A \quad \& \quad d_t(x) = A$ Sup.

(2) $P(xP) = B \frown (x \smile B)$ 1,D119,D120,T525,T526

(3) $(\exists n\in N^+)(P(xP) = B \frown (Q^nA \smile B))$ 2,1,T519,T508

(4) $= Q^n(B \frown (A \smile B))$ 3,T505

(5) $= Q^n(B \smile (B \frown A))$ 4,T492

(6) $= B \smile (B \frown x)$ 3,5,T505

(7) $= (Px)P$ 1,6,D119,D120,T525,T526

(8) $(\forall x\in M_t)((l_t(x) = A \quad \& \quad d_t(x) = A) \rightarrow$
$P(xP) = (Px)P)$. 1,7

8 $\succ\!\!\!-\!\!\!-\!\!\prec$

(9) $(\forall x\in M_t)((l_t(x) = B \quad \& \quad d_t(x) = B) \rightarrow$
$P(xP) = (Px)P)$.

(10) $x\in M_t \quad \& \quad l_t(x) = A \quad \& \quad d_t(x) = B$ Sup.

(11) $P(xP) = B \frown (x \frown A)$ 10,D119,D120,T525,T526

(12) $P(xP) = (B \frown x) \frown A$ 11,T490

(13) $(\forall x\in M_t)((l_t(x) = A \quad \& \quad d_t(x) = B) \rightarrow$
$P(xP) = (Px)P)$. 10,12,T525,T526

13 $\succ\!\!\!-\!\!\!-\!\!\prec$

(14) $(\forall x\in M_t)((l_t(x) = B \quad \& \quad d_t(x) = A) \rightarrow$
$P(xP) = (Px)P)$.

8,9,13,14,T515,T516,D105 $\succ\!\!\!-\!\!\!-$

(15) $(\forall x\in M_t)(P(xP) = (Px)P)$.

15,D121,D122 $\succ\!\!\!-\!\!\!-$ T537.

TEOREM 538

$(\forall x\in M_t)(\exists! n\in N)(x = P^nd_t(x) \quad \& \quad x = l_t(x)P^n)$.

D o k a z

T508,D106 $\succ\!\!\!-\!\!\!-$

(1) $(\forall x\in M_t)(\exists! m\in N^+)(x = Q^mA \quad v \quad x = Q^mB \quad v$
$x = Q^m(A \smile B) \quad v \quad x = Q^m(B \frown A)$

1,D119-D122 $\succ\!\!-$

$(2)(x = P^{2m}A \ \& \ x = AP^{2m}) \vee (x = P^{2m}B \ \& \ x = BP^{2m}) \vee$
$x = P^{2m+1}B \ \& \ x = AP^{2m+1}) \vee (x = P^{2n+1}A \ \& \ x = BP^{2m+1}).$

1,2,T519-T524 $\succ\!\!-$ T538.

DEFINICIJA 123

$(\forall x \in M_t)(\forall n \in N^+)(\lambda_t(x) = n \longleftrightarrow x = P^n d_t(x)).$

DEFINICIJA 124

$(\forall x \in M_t)(\lambda(x) = \lambda_t(x) + 1).$

D123,T538 $\succ\!\!-$

TEOREM 539

$(\forall x \in M_t)(x = P^{\lambda_t(x)} d_t(x) = 1_t(x) P^{\lambda_t(x)}).$

D123,T519,T520,T531 $\succ\!\!-$

TEOREM 540

$\lambda_t(A) = \lambda_t(B) = 0.$

D123,T521,T522,D119,D120,T532 $\succ\!\!-\!\!-$

TEOREM 541

$\lambda_t(A \cup B) = \lambda_t(B \cap A) = 1.$

TEOREM 542

$(p.Id_A(M_t), A, P_1) \in S_{Peano\ s}.$

D o k a z

D111,T487,T491 $\succ\!\!-\!\!-$

(1) $A \in p.Id_A(M_t)$

D111,D116,T518,T525 $\succ\!\!-\!\!-$

(2) $x \in p.Id_A(M_t) \rightarrow Px \in p.Id_A(M_t)$

T527,D105 $\succ\!\!-\!\!-$

(3) $x \in p.Id_A(M_t) \rightarrow \neg Px = A.$

T527 $\succ\!\!-\!\!-$

(4) $(\forall x,y \in p.Id_A(M_t))(Px = Py \rightarrow x = y).$

D111,T518,T538 $\succ\!\!-\!\!-$

(5) $(\forall x \in p.Id_A(M_t))(\exists n \in N^+)(x = P^n A).$

5,T531-T533 $\succ\!\!-\!\!-$

(6) $(\forall G \subseteq p.Id_A(M_t)(A \in G \ \&\ (x \in G \rightarrow Px \in G)) \rightarrow G = p.Id_A(M_t).$

1,2,3,4,6 $\succ\!\!-\!\!-$ T542.

D112-D116,T487,T491,T517,T518,T525-T528,T538,T531-T533 $\succ\!\!-$

TEOREMI 543-545

543 $(s.Id_B(M_t),\ B,\ P_l)\in S_{Peano\ s.}$

544 $(s.Id_A(M_t),\ A,\ P_d)\in S_{Peano\ s.}$

545 $(p.Id_B(M_t),\ B,\ P_d)\in S_{Peano\ s.}$

2.5. N A D O V E Z I V A N J E m_t-P O L I N O M A

DEFINICIJA 125

$$(\forall x,y\in M_t)(f_k(x,y) = xy = z \longleftrightarrow ((d_t(x) = l_t(y)\ \&\ z = P^{\lambda_t(x)}y) \lor (d_t(x) = \bar{l}_t(y)\ \&\ z = P^{\lambda_t(x)+1}y)).$$

TEOREM 546

$(\forall x\in M_t)(Ax = A \cup x\ \ \&\ \ Bx = B \cap x).$

D o k a z

(1) $x\in M_t$	Sup.
(2) $l_t(x) = A \lor l_t(x) = B$	1,T515,D105
(3) $Ax = P^o x \lor Ax = Px$	2,D125,T540
(4) $(\forall x\in M_t)(Ax = A \cup x)$.	1,2,3,T517,D119

4 $\succ\!\!-\!\!\prec$

(5) $(\forall x\in M_t)(Bx = B \cap x)$.

4,5 $\succ\!\!-$ T546.

TEOREM 547

$(\forall x\in M_t)(xA = x \cap A\ \ \&\ \ xB = x \cup B).$

D o k a z

(1) $x\in M_t$	Sup.
(2) $xA = P^{\lambda_t(x)}A \lor xA = P^{\lambda_t(x)+1}A$	1,D125
(3) $xA = (P^{\lambda_t(x)}A) \cap A \lor xA = (P^{\lambda_t(x)+1}A) \cap A$	2,T525,T519,T518
(4) $(\forall x\in M_t)(xA = x \cap A)$.	1,2,3,D125

4 $\succ\!\!-\!\!\prec$

(5) $(\forall x\in M_t)(xB = x \cup B)$.

4,5 $\succ\!\!-$ T547.

TEOREM 548

$(\forall x,y\in M_t)((Px)y = P(xy))$.

D o k a z

D125,D117,D123,T536 $\succ\!\!-$

(1) $(\forall x,y\in M_t)((Px)y = P^{\lambda_t(x)+1}y \ \vee \ (Px)y = P^{\lambda_t(x)+2}y)$.

1,T536 $\succ\!\!-$

(2) $(\forall x,y\in M_t)((Px)y = P(P^{\lambda_t(x)}y) \ \vee \ (Px)y = P(P^{\lambda_t(x)+1}y))$.

1,2,D125,D117 $\succ\!\!-$ T548.

D125,D123,T548,T533 $\succ\!\!-$

TEOREM 549

$(\forall x,y\in M_t)(\forall n\in N^+)((P^n x)y = P^n(xy))$.

TEOREM 550

$(\forall x,y\in M_t)(\forall n\in N^+)(x(yP^n)(= (xy)P^n)$.

D o k a z

(1)	$x,y\in M_t \ \& \ n\in N^+$	Sup.
(2)	$x(yP^n) = P^{\lambda_t(x)}(yP^n) \ \vee \ x(yP^n) = P^{\lambda_t(x)+1}(yP^n)$	1,D125
(3)	$x(yP^n) =(P^{\lambda_t(x)}y)P^n \ \vee \ x(yP^n) =(P^{\lambda_t(x)+1}y)P^n$	2,T537

1,3,D125 $\succ\!\!-$ T550.

T546,T491 $\succ\!\!-$

TEOREM 551

$AA = A \ \& \ BB = B$.

T546 $\succ\!\!-$

TEOREM 552

$AB = A \cup B \ \& \ BA = B \cap A$.

T552,D119,D120,T519,T520 $\succ\!\!-$

TEOREM 553

$AB = PB = AP \ \& \ BA = PA = BP$.

T515,D105,T546,T517 $\succ\!\!-$

TEOREM 554

$(\forall x\in M_t)(1_t(x)x = x)$.

TEOREM 555

$(\forall x\in M_t)(\bar{1}_t(x)x = Px).$

D o k a z

(1)	$x\in M_t$	Sup.
(2)	$1_t(x) = A \quad v \quad 1_t(x) = B$	1,T515
(3)	$\bar{1}_t(x)x = Bx \quad v \quad \bar{1}_t(x)x = Ax$	2,D117
(4)	$\bar{1}_t(x)x = B \cap x \quad v \quad \bar{1}_t(x)x = A \cup x$	3,T546

1,2,4,D119 >—— T555.

TEOREM 556

$(\forall x\in M_t)(xd_t(x) = x).$

D o k a z

(1)	$x\in M_t$	Sup.
(2)	$xd_t(x) = (P^{\lambda_t(x)}d_t(x))d_t(x)$	1,T539
(3)	$= P^{\lambda_t(x)}(d_t(x)d_t(x))$	2,T548
(4)	$= P^{\lambda_t(x)}d_t(x)$	3,T516,T551

1,4,T539 >—— T556.

T516,D118,T547,D120 >——

TEOREM 557

$(\forall x\in M_t)(x\bar{d}_t(x) = xP).$

TEOREM 558

$$(\forall x,y\in M_t)((d_t(x) = 1_t(y) \;\&\; xy = xP^{\lambda_t(y)}) \quad v$$
$$(\bar{d}_t(x) = 1_t(y) \;\&\; xy = xP^{\lambda_t(y)+1})).$$

D o k a z

(1)	$x,y\in M_t$	Sup.
(2)	$(d_t(x) = 1_t(y) \;\&\; xy = x(d_t(x)P^{\lambda_t(y)})) \quad v$ $(\bar{d}_t(x) = 1_t(y) \;\&\; xy = x(\bar{d}_t(x)P^{\lambda_t(y)}))$.	1,T515,T516,T539

1,2,T550,T556,T557,T533 >—— T558.

D125,T558,T525,T526,D121,D122,T523,T524 >——

TEOREM 559

$(\forall x,y\in M_t)(1_t(xy) = 1_t(x) \;\&\; d_t(xy) = d_t(y)).$

TEOREM 560

$(\forall x,y,z\in M_t)((xy)z = x(yz))$.

D o k a z

(1) $x,y,z\in M$ Sup.

(2) $(xy)z = (P^{\lambda_t(x)} d_t(x)y)z$ 1,T539

(3) $(xy)z = P^{\lambda_t(x)}(l_t(y)y)z \quad v$

$(xy)z = P^{\lambda_t(x)}(\bar{l}_t(y)y)z$ 2,T515,T516,D117,T549

(4) $(xy)z = P^{\lambda_t(x)} yz \quad v$

$(xy)z = P^{\lambda_t(x)+1} yz$ 3,T554,T555,T536

1,2,4,D125 $\succ\!\!-\!\!-$ T560.

TEOREM 561

$(\forall x\in M_t)(q_t^+(x) = l_t(x)d_t(x))$.

D o k a z

T519-T522,T551,T552 $\succ\!\!-\!\!-$

(1) $(\forall u\in\Omega_t^+)(u = l_t(u)d_t(u))$.

(2) $x\in M_t$ Sup.

(3) $l_t(x) = l_t(Q^{\lambda_t^+(x)} q_t^+(x))$ &

$d_t(x) = d_t(Q^{\lambda_t^+(x)} q_t^+(x))$ 2,T510

(4) $l_t(x) = l_t(q_t^+(x))$ & $d_t(x) = d_t(q_t^+(x))$ 3,T523,T524

2,4,T509,D106,1,T559 $\succ\!\!-\!\!-$ T561.

2.6. D U A L N O S T m_t-P O L I N O M A

DEFINICIJA 126

$(\forall x \in M_t)(Dx = P^{\lambda_t(x)}\bar{d}_t(x))$.

D126,T539,D118,D105 >——

TEOREM 562

$D \in S_{bijekc}(M_t, M_t)$.

D126,T540,T519,T520,D118 >——

TEOREM 563

$DA = B \;\;\&\;\; DB = A$.

D126,T541,T521,T522,D118 >——

TEOREM 564

$D(A \cup B) = B \cap A \;\;\&\;\; D(B \cap A) = A \cup B$.

D126,T562-T564,T510,T509 >——

TEOREM 565

$(\forall x,y \in M_t)(D(x \cup y) = Dx \cap Dy \;\;\&\;\; D(x \cap y) = Dx \cup Dy)$.

2.7. K A N O N S K E   F O R M E m_t-P O L I N O M A

DEFINICIJA 030

Ako je x m_t-polinom, onda kanonska forma od x je y, ako i samo ako je y = x i za svaki m_t-polinom z, ako je z = y, onda broj simbola za operacije zbrajanja koje dolaze u x nije veći od broja simbola za operacije zbrajanja koje dolaze u z.

TEOREM 08

Ako je x m_t-polinom, onda kanonska forma od x je y, ako i samo ako je y = x i broj simbola za operacije zbrajanja koje dolaze u y jednak je t-duljini od x, tj. jednak je $\lambda_t(x)$.

PRIMJERI 313-317

Kanonske forme m_t-polinoma iz ideala s.$Id_A(M_t)$, kojima t-duljina nije veća od četiri

	x	λ_t	skup kanonskih formi od x
313	A	0	$\{A\}$.
314	AB	1	$\{A \cup B\}$.
315	ABA	2	$\{A \cup (B \cap A), (A \cup B) \cap A\}$.
316	ABAB	3	$\{((A \cup B) \cap A) \cup B,\ A \cup (B \cap A) \cup B,$ $(A \cup B) \cap (A \cup B),\ A \cup (B \cap (A \cup B))\}$.
317	ABABA	4	$\{(((A \cup B) \cap A) \cup B) \cap A,\ (A \cup B) \cap (A \cup (B \cap A)),$ $((A \cup B) \cap A) \cup (B \cap A),\ A \cup (B \cap (A \cup B) \cap A),$ $(A \cup (B \cap A) \cup B) \cap A,\ A \cup (B \cap (A \cup (B \cap A))),$ $(A \cup B) \cap (A \cup B) \cap A,\ (A \cup (B \cap (A \cup B))) \cap A,$ $A \cup (B \cap A) \cup (B \cap A),\ (A \cap (B \cup (A \cap B))) \cup A\}$.

PRIMJER 318

$x \in \{A \cup A,\ B \cup B,\ B \cup A \cup B,\ B \cup ((A \cup B) \cap (A \cup B))\} \longrightarrow$ x nije kanonska forma m_t-polinoma.

DEFINICIJA 031

Ako je x m_t-polinom, onda F_s-forma od x je y, ako i samo ako y je primitivni m_t-polinom ili y je kanonska forma serijskog zbroja primitivnih m_t-polinoma.

PRIMJER 319

F_s-forme m_t-polinoma kojima t-duljina nije veća od četiri su:

$\{A,B\}$, $\{A \cup B, B \cap A\}$, $\{A \cup (B \cap A), (B \cap A) \cup B\}$,

$\{A \cup (B \cap A) \cup B, (B \cap A) \cup (B \cap A)\}$,

$\{A \cup (B \cap A) \cup (B \cap A), (B \cap A) \cup (B \cap A) \cup B\}$.

D031,T510,D108 >——

TEOREM 09

Ako je x m_t-polinom, onda se x dade na jedinstven način prikazati u F_s-formi, odnosno u formi serijskog zbroja iz $q_t^+(x)$ i n-pribrojnika $(B \cap A)$, pri čemu je $n = \lambda_t^+(x)$.

DEFINICIJA 032

Ako je x m_t-polinom, onda F_p-forma od x je y, ako i samo ako y je primitivni m_t-polinom ili y je kanonska forma paralelnog zbroja primitivnih m_t-polinoma.

PRIMJER 320

F_p-forme m_t-polinoma kojima t-duljina nije veća od četiri su:

$\{B,A\}$, $\{B \cap A, A \cup B\}$, $\{B \cap (A \cup B), (A \cup B) \cap A\}$,
$\{B \cap (A \cup B) \cap A, (A \cup B) \cap (A \cup B)\}$,
$\{B \cap (A \cup B) \cap (A \cup B), (A \cup B) \cap (A \cup B) \cap A\}$.

D032,T510,T496 >——

TEOREM 010

Ako je x m_t-polinom, onda se x dade na jedinstven način prikazati u F_p-formi, odnosno u formi paralelnog zbroja iz $q_t^+(x)$ i n-pribrojnika $(A \cup B)$, pri čemu je $n = \lambda_t^+(x)$.

DEFINICIJA 033

C_1-forma m_t-polinoma je svaki generator ili svaki izraz

$P_n(\ldots(P_3(P_2(P_1 u)_1)_2)_3\ldots)_n$, u kojem je svako "P" supstituirano

odgovarajućom operacijom zbrajanja saglasno definiciji 119
i $u = A$ ili $u = B$.

PRIMJER 321

C_1-forme m_t-polinoma kojima t-duljina nije veća od četiri su:

$\{A,B\}$, $\{B \cap A, A \cup B\}$, $\{A \cup (B \cap A), B \cap (A \cup B)\}$,
$\{B \cap (A \cup (B \cap A)), A \cup (B \cap (A \cup B))\}$,
$\{A \cup (B \cap (A \cup (B \cap A))), B \cap (A \cup (B \cap (A \cup B)))\}$.

D033,T539,D119 >——

TEOREM 011

Ako je x m_t-polinom, onda se x dade na jedinstven način prikazati u C_1-formi, odnosno u formi iz D033 i to tako da je $u = d_t(x)$ i broj simbola "P" jednak je t-duljini od x, tj. $\lambda_t(x)$.

DEFINICIJA 034

C_d-forma m_t-polinoma je svaki generator ili svaki izraz

$(_n \ldots (_3 (_2 (_1 uP_1) P_2) P_3) \ldots)_n P_n$, u kojem je svako "P" supstituirano odgovarajućom operacijom zbrajanja saglasno definiciji 120 i $u = A$ ili $u = B$.

PRIMJER 322

C_d-forme m_t-polinoma kojima t-duljina nije veća od četiri su:

$\{A,B\}$, $\{A \cup B, B \cap A\}$, $\{(A \cup B) \cap A, (B \cap A) \cup B\}$,
$\{((A \cup B) \cap A) \cup B, ((B \cap A) \cup B) \cap A\}$,
$\{(((A \cup B) \cap A) \cup B) \cap A, (((B \cap A) \cup B) \cap A) \cup B\}$.

D032,T539,D110 >---

TEOREM 012

Ako je x m_t-polinom, onda se x dade na jedinstven način prikazati u C_d-formi, odnosno u formi iz D034 i to tako da je $u = l_t(x)$ i broj simbola "P" jednak je t-duljini od x, tj. $\lambda_t(x)$.

Treći dio

ALGEBRA DVOGENERATORSKIH IMITANCIJA

3.1. UTEMELJENJE t-STRUKTURE

DEFINICIJA 127

$(\forall e,f\in Re)(e < f \rightarrow \Omega_{e,f} = \{x:\ k\in Re_{poz}\ \&\ s\in\Xi\ \&$
$(x = (\Lambda s)(ks^{e})\ \vee\ x = (\Lambda s)(ks^{f}))\}).$

PRIMJERI

323 $\{(\Lambda s)(2.3s^{-3}),\ (\Lambda s)(\pi s^{2}),\ (\Lambda s)(0.5s^{-3})\} \subseteq \Omega_{-3,2}.$

324 $\{(\Lambda s)(7),\ (\Lambda s)(\frac{2}{3}),\ (\Lambda s)(3),\ (\Lambda s)(0.1s^{\pi})\} \subseteq \Omega_{0,\pi}.$

DEFINICIJA 128

$s\in\Xi \longrightarrow (\Lambda s)(F_1) + (\Lambda s)(F_2) = (\Lambda s)(F_1 + F_2).$

DEFINICIJA 129

$s\in\Xi \longrightarrow (\Lambda s)(F_1) \hat{+} (\Lambda s)(F_2) = (\Lambda s)(\frac{1}{\frac{1}{F_1} + \frac{1}{F_2}}).$

PRIMJERI

325 $(\Lambda s)(\frac{s}{1 + s}) + (\Lambda s)(3) = (\Lambda s)(\frac{3 + 4s}{1 + s}).$

326 $(\Lambda s)(\frac{s}{1 + s}) \hat{+} (\Lambda s)(3) = (\Lambda s)(\frac{3s}{3 + 4s}).$

AKSIOM 4

$(\forall e,f\in Re)(e < f \rightarrow (\exists! G\in S)(\Omega_{e,f} \subseteq G\quad \&$

$(x,y\in G \rightarrow (x + y)\in G\ \&\ (x \hat{+} y)\in G)\quad \&$

$(\forall W\in S)((\Omega_{e,f} \subseteq W\ \&\ W \subseteq G\quad \&$

$(x,y\in W \rightarrow (x + y)\in W\ \&\ (x \hat{+} y)\in W)) \rightarrow W = G).$

DEFINICIJA 130

$(\forall e,f\in Re)(e < f \rightarrow (T_{e,f}$ je jedinstveno odreden skup G iz aksioma 4).

DEFINICIJA 131

$T = \{x\colon e,f\in Re \ \& \ e < f \ \& \ x\in T_{e,f}\setminus \Omega_{e,f}\}$.

KONVENCIJA 3

$(\forall e,f\in Re)(e < f \rightarrow (X)(Y_{e,f})(Z)) \frown (X)(Y_{ef})(Z)$.

KONVENCIJA 4

$(\forall X\in T_{ef})(X = (\Lambda s)(F) \frown X = F)$.

D130,A4,D127-D129,K3,K4 $\rightarrowtail$

TEOREMI 566-571

566 $\Omega_{ef} \subseteq T_{ef}$.

567 $x,y\in T_{ef} \rightarrow ((x + y)\in T_{ef} \ \& \ (x \overset{\frown}{+} y)\in T_{ef})$.

568 $(\forall W\in S)((\Omega_{ef} \subseteq W \ \& \ W \subseteq T_{ef} \ \&$
$(x,y\in W \rightarrow ((x + y)\in W \ \& \ (x \overset{\frown}{+} y)\in W)) \rightarrow W = T_{ef})$.

569 $(\forall x,y\in T_{ef})(x \overset{\frown}{+} y = \dfrac{xy}{x + y})$.

570 $(\forall x,y\in T_{ef})(x \overset{\frown}{+} y = y \overset{\frown}{+} x)$.

571 $(\forall x,y,z\in T_{ef})(x \overset{\frown}{+} (y \overset{\frown}{+} z) = (x \overset{\frown}{+} y) \overset{\frown}{+} z)$.

DEFINICIJA 132

$(\forall e,f\in Re)(e < f \rightarrow \mathscr{T}_{e,f} = (T_{e,f}, \Omega_{e,f}, +, \overset{\frown}{+}))$.

DEFINICIJE 133-134

133 $(\forall e,f\in Re)(e < f \rightarrow A_{e,f} = \{x\colon k\in Re_{poz} \ \& \ s\in\Xi \ \& \ x=(\Lambda s)(ks^{e})\})$.

134 $(\forall e,f\in Re)(e < f \rightarrow B_{e,f} = \{x\colon k\in Re_{poz} \ \& \ s\in\Xi \ \& \ x=(\Lambda s)(ks^{f})\})$.

PRIMJERI

327 $(f\in Re \ \& \ 2 < f \ \& \ s\in\Xi) \longrightarrow$
$\{(\Lambda s)(3s^2), (\Lambda s)(\pi s^2), (\Lambda s)(\frac{3}{4} s^2)\} \subseteq A_{2,f}$.

328 $(e\in Re \ \& \ e < -1 \ \& \ s\in\Xi) \rightarrow$
$\{(\Lambda s)(2s^{-1}), (\Lambda s)(\pi s^{-1}), (\Lambda s)(\frac{2}{3} s^{-1})\} \subseteq B_{e,-1}$.

TEOREM 572

$(\forall x,y\in A_{ef})((x + y)\in A_{ef} \ \& \ (x \hat{+} y)\in A_{ef})$.

D o k a z

(1) $e,f\in Re \ \& \ e < f \ \& \ x,y\in A_{e,f}$ Sup.

(2) $(\exists k_1,k_2\in Re_{poz})(x=(\Lambda s)(k_1 s^e) \ \& \ y=(\Lambda s)(k_2 s^e))$ 1,D133

(3) $x+y = (k_1+k_2)s^e \ \& \ x\hat{+}y = \frac{k_1k_2}{k_1+k_2}\, s^e$ 2,D128,K4,T569

1,2,3,D133,K3 $\succ\!\!-\!\!-$ T572.

T572 $\succ\!\!-\!\!-\!\!\prec$

TEOREM 573

$(\forall x,y\in B_{ef})((x + y)\in B_{ef} \ \& \ (x \hat{+} y)\in B_{ef})$.

D133,D134,D127 $\succ\!\!-\!\!-$

TEOREM 574

$A_{ef} \cup B_{ef} = \Omega_{ef}$.

TEOREM 575

$A_{ef} \cap B_{ef} = 0$.

D o k a z

(1) $e,f\in Re \ \& \ e < f \ \& \ x\in A_{ef} \ \& \ x\in B_{ef}$ Sup.

(2) $(\exists k_1,k_2\in Re_{poz})(x = k_1 s^e \ \& \ x = k_2 s^f)$ 1,D133,D134,K4

(3) $k_1 s^e = k_2 s^f$ 2

(4) $k_1 = k_2 \ \& \ e = f$ 3

1,4 $\succ\!\!-\!\!-$ T575.

DEFINICIJA 135

$(BA)_{ef} = \{z\colon x\in A_{ef} \ \& \ y\in B_{ef} \ \& \ z = x \hat{+} y\}$.

DEFINICIJA 136

$(AB)_{ef} = \{z\colon x\in A_{ef} \ \& \ y\in B_{ef} \ \& \ z = x + y\}$.

TEOREM 576

$$x\in(BA)_{ef} \longleftrightarrow (\exists k_1,k_2\in Re_{poz})(x = s^f \frac{k_1}{k_2 + s^{f-e}}).$$

D o k a z

(1) $x\in(BA)_{ef}$ Sup.

(2) $(\exists k_1,k_2\in Re_{poz})(x = k_1 s^e \hat{+} k_2 s^f)$ 1,D133-D135

$$(3)\ x = \frac{k_1 s^f}{\frac{k_1}{k_2} + s^{f-e}} \qquad 2,T569$$

1,2,3, $\succ\!\!-\!\!-$ T576.

D124,D121,D122 $\succ\!\!-\!\!-$

TEOREM 577

$$x \in (AB)_{ef} \longleftrightarrow (\exists k_1, k_2 \in Re_{poz})(x = s^e \frac{k_2 + s^{f-e}}{k_1}).$$

PRIMJERI

329 $s^2 \frac{3}{1.5 + s} \in (BA)_{1,2}.$ 330 $s \frac{1.5 + s}{0.5} \in (AB)_{1,2}.$

3.2. FORME I PARAMETRI FORMI DVOGENERATORSKIH IMITANCIJA

3.2.1. s-FORMA I p-FORMA IMITANCIJE

DEFINICIJA 137

$$(i \in N^+ \ \& \ A_i \in Re \ \& \ c \in Re \ \& \ s \in \Xi \ \& \ x_i = (\Lambda s)(A_i s^{ci})) \longrightarrow$$
$$(\bigvee_{i=o}^{o} x_i = x_o \ \& \ (\forall n \in N^+)(\bigvee_{i=o}^{n+1} x_i = (\bigvee_{i=o}^{n} x_i) + x_{n+1})).$$

DEFINICIJA 138

$$(i \in N^+ \ \& \ A_i \in Re \ \& \ c \in Re \ \& \ s \in \Xi \ \& \ x_i = (\Lambda s)(A_i s^{ci})) \longrightarrow$$
$$(\bigwedge_{i=o}^{o} x_i = x_o \ \& \ (\forall n \in N^+)(\bigwedge_{i=o}^{n+1} x_i = (\bigwedge_{i=o}^{n} x_i) \overset{\frown}{+} x_{n+1})).$$

PRIMJERI

331 $\bigwedge_{i=o}^{o} A_i s^{3i} = A_o.$ 332 $\bigvee_{i=o}^{3} B_i s^{3i} = B_o + B_1 s^3 + B_2 s^6 + B_3 s^9.$

333 $\bigwedge_{i=o}^{1} A_i s^{\pi i} = A_o \overset{\frown}{+} A_1 s^{\pi}.$

334 $\bigwedge_{i=o}^{2} C_i s^{1.5i} = C_o \overset{\frown}{+} C_1 s^{1.5} \overset{\frown}{+} C_2 s^3.$

TEOREM 578

$$(\forall e,f,u\in Re)(\forall m,n\in N^{+})\left(s^{u}\frac{\bigvee_{i=o}^{m}A_{i}s^{(f-e)i}}{\bigvee_{i=o}^{n}B_{i}s^{(f-e)i}} = s^{v}\frac{\bigwedge_{i=o}^{n}\frac{s^{(f-e)i}}{B_{n-i}}}{\bigwedge_{i=o}^{m}\frac{s^{(f-e)i}}{A_{m-i}}} \quad \& \right.$$

$$\left. v = u + (m - n)(f - e)\right).$$

D o k a z

(1) $e,f,u\in Re \quad \& \quad \varkappa = f - e \quad \& \quad m,n\in N^{+} \;\&\; x = s^{u}\dfrac{\bigvee_{i=o}^{m}A_{i}s^{\varkappa i}}{\bigvee_{i=o}^{n}B_{i}s^{\varkappa i}}$ Sup.

1,D137 $\succ\!\!-\!\!-$

(2) $x = s^{u+(m-n)\varkappa}\dfrac{\bigvee_{i=o}^{m}A_{i}s^{\varkappa(i-m)}}{\bigvee_{i=o}^{n}B_{i}s^{\varkappa(i-n)}}$

2 $\succ\!\!-\!\!-$

(3) $x = s^{v}\dfrac{\bigvee_{i=o}^{m}\frac{A_{i}}{s^{\varkappa(m-i)}}}{\bigvee_{i=o}^{n}\frac{B_{i}}{s^{\varkappa(n-i)}}} \quad \& \quad v = u + (m - n)\varkappa$

3,T569,D138 $\succ\!\!-\!\!-$

(4) $x = s^{v}\dfrac{\dfrac{1}{\bigwedge_{i=o}^{m}\frac{s^{\varkappa(m-i)}}{A_{i}}}}{\dfrac{1}{\bigwedge_{i=o}^{n}\frac{s^{\varkappa(n-i)}}{B_{i}}}} = s^{v}\dfrac{\bigwedge_{i=o}^{n}\frac{s^{\varkappa(n-i)}}{B_{i}}}{\bigwedge_{i=o}^{m}\frac{s^{\varkappa(m-i)}}{A_{i}}}$

1,3,4 $\succ\!\!-\!\!-$ T578.

PRIMJER 335

$$s^{u}\,\frac{A_{o}+A_{1}s^{\varkappa}+A_{2}s^{2\varkappa}+A_{3}s^{3\varkappa}}{B_{o}+B_{1}s^{\varkappa}} = s^{u+2\varkappa}\,\frac{\frac{1}{B_{1}}\hat{+}\frac{1}{B_{o}}s^{\varkappa}}{\frac{1}{A_{3}}\hat{+}\frac{1}{A_{2}}s^{\varkappa}\hat{+}\frac{1}{A_{1}}s^{2\varkappa}\hat{+}\frac{1}{A_{o}}s^{3\varkappa}}$$

T576,T578 $\succ\!\!-$

TEOREM 579

$$x \in (BA)_{ef} \longleftrightarrow (\exists k_1, k_2 \in Re_{poz})(x = s^{e}\frac{k_2 \hat{+} s^{f-e}}{k_1}).$$

T577,T578 $\succ\!\!-$

TEOREM 580

$$x \in (AB)_{ef} \longleftrightarrow (\exists k_1, k_2 \in Re_{poz})(x = s^{f}\frac{k_1}{k_2 \hat{+} s^{f-e}}).$$

TEOREM 581

$$(\forall e,f \in Re)(\forall m_x,n_x,m_y,n_y \in N^+)((e < f \ \& \ x = s^{u_x}\frac{\bigvee_{i=o}^{m_x} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n_x} B_i s^{(f-e)i}} \ \&$$

$$y = s^{u_y}\frac{\bigvee_{i=o}^{m_y} C_i s^{(f-e)i}}{\bigvee_{i=o}^{n_y} D_i s^{(f-e)i}} \ \& \ u_x,u_y \in \{e,f\} \ \& \ (\forall i)(A_i,B_i,C_i,D_i \in Re_{poz}))$$

$$\longrightarrow \quad (x + y) = s^{u}\frac{\bigvee_{i=o}^{m} E_i s^{(f-e)i}}{\bigvee_{i=o}^{n} F_i s^{(f-e)i}} \quad \& \quad m,n \in N^+ \quad \&$$

$$(\forall i)(E_i,F_i \in Re_{poz}) \quad \& \quad u = \inf(u_x,u_y) \quad \&$$

$$u + (m - n)(f - e) = \sup(u_x+(m_x-n_x)(f-e),\ u_y+(m_y-n_y)(f-e))).$$

D o k a z

(1) $e,f \in Re \ \& \ e < f \ \& \ m_x,n_x,m_y,n_y \in N^+ \quad \&$

$$x = s^{u_x}\frac{\bigvee_{i=o}^{m_x} A_i s^{\pi i}}{\bigvee_{i=o}^{n_x} B_i s^{\pi i}} \quad \& \quad y = s^{u_y}\frac{\bigvee_{i=o}^{m_y} C_i s^{\pi i}}{\bigvee_{i=o}^{n_y} D_i s^{\pi i}} \quad \&$$

$u_x,u_y \in \{e,f\} \quad \& \quad \pi = f - e \quad \& \quad (\forall i)(A_i,B_i,C_i,D_i \in Re_{poz}) \quad$ Sup.

1 $\succ$——

$$(2)\ x + y = s^{u}\frac{s^{u_x-u}\left(\bigvee_{i=o}^{m_x} A_i s^{\varkappa i}\right)\left(\bigvee_{i=o}^{n_y} D_i s^{\varkappa i}\right) + s^{u_y-u}\left(\bigvee_{i=o}^{n_x} B_i s^{\varkappa i}\right)\left(\bigvee_{i=o}^{m_y} C_i s^{\varkappa i}\right)}{\left(\bigvee_{i=o}^{n_x} B_i s^{\varkappa i}\right)\left(\bigvee_{i=o}^{n_y} D_i s^{\varkappa i}\right)}$$

& $u = \inf(u_x,u_y)$

2,1 $\succ$——

(3) $(u_x = u_y \;\&\; u_x - u = 0 \;\&\; u_y - u = 0) \quad v$

$(u_x \neq u_y \;\&\; ((u_x - u = \varkappa \;\&\; u_y - u = 0) \; v \; (u_x - u = 0 \;\&$

$u_y - u = \varkappa)))$.

3,2,1 $\succ$——

(4) $x + y = s^{u}\frac{\bigvee_{i=o}^{m} E_i s^{\varkappa i}}{\bigvee_{i=o}^{n} F_i s^{\varkappa i}}$ & $(\forall i)(E_i,F_i \in Re_{poz})$ & $u = \inf(u_x,u_y)$ &

$n = n_x+n_y$ & $\varkappa m = \sup(u_x-u+\varkappa(m_x+n_y),\ u_y-u+\varkappa(m_y+n_x))$.

(5) "$a+\sup(b,c) = \sup(a+b,a+c)$ & $a+\inf(b,c) = \inf(a+b,a+c)$".
(G.Birkhoff, Lattice-ordered groups, Annals of Mathematics, Vol 43,No 2,April 1942).

4,5 $\succ$——

(6) $u + m\varkappa = \sup(u_x + \varkappa(m_x + n_y),\ u_y + \varkappa(m_y + n_x))$

6,5 $\succ$——

(7) $u + m\varkappa - \varkappa(n_x + n_y) = \sup(u_x + \varkappa(m_x - n_x),\ u_y + \varkappa(m_y - n_y))$

7,4,1 $\succ$——

(8) $u + (m - n)(f - e) = \sup(u_x+(m_x-n_x)(f-e),\ u_y+(m_y-n_y)(f-e))$.

1,4,8 $\succ$—— T581.

TEOREM 582

$$(\forall e,f\in Re)(\forall m_x,n_x,m_y,n_y\in N^+)((e < f \quad \& \quad x = s^{u_x \frac{\bigvee_{i=o}^{m_x} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n_x} B_i s^{(f-e)i}}} \quad \&$$

$$y = s^{u_y \frac{\bigvee_{i=o}^{m_y} C_i s^{(f-e)i}}{\bigvee_{i=o}^{n_y} D_i s^{(f-e)i}}} \quad \& \quad u_x,u_y\in\{e,f\} \;\&\; (\forall i)(A_i,B_i,C_i,D_i\in Re_{poz}))$$

$$\longrightarrow \quad (x \hat{+} y = s^{u \frac{\bigvee_{i=o}^{m} E_i s^{(f-e)i}}{\bigvee_{i=o}^{n} F_i s^{(f-e)i}}} \quad \& \quad m,n\in N^+ \quad \&$$

$$(\forall i)(E_i,F_i\in Re_{poz}) \quad \& \quad u = \sup(u_x,u_y) \quad \&$$

$$u + (m - n)(f - e) = \inf(u_x+(m_x-n_x)(f-e),\ u_y+(m_y-n_y)(f-e))).$$

D o k a z

(1) $e,f\in Re \quad \& \quad e < f \quad \& \quad m_x,n_x,m_y,n_y\in N^+ \quad \&$

$$x = s^{u_x \frac{\bigvee_{i=o}^{m_x} A_i s^{\varkappa i}}{\bigvee_{i=o}^{n_x} B_i s^{\varkappa i}}} \quad \& \quad y = s^{u_y \frac{\bigvee_{i=o}^{m_y} C_i s^{\varkappa i}}{\bigvee_{i=o}^{n_y} D_i s^{\varkappa i}}} \quad \&$$

$u_x,u_y\in\{e,f\} \quad \& \quad \varkappa = f-e \quad \& \quad (\forall i\in N^+)(A_i,B_i,C_i,D_i\in Re_{poz})$ Sup.

1,D129 $\succ\!\!-$

$$(2)\ x \hat{+} y = s^{-w} \frac{s^{u_x+u_y}\left(\bigvee_{i=o}^{m_x} A_i s^{\varkappa i}\right)\left(\bigvee_{i=o}^{m_y} C_i s^{\varkappa i}\right)}{s^{u_x-w}\left(\bigvee_{i=o}^{m_x} A_i s^{\varkappa i}\right)\left(\bigvee_{i=o}^{n_y} D_i s^{\varkappa i}\right) + s^{u_y-w}\left(\bigvee_{i=o}^{n_x} B_i s^{\varkappa i}\right)\left(\bigvee_{i=o}^{m_y} C_i s^{\varkappa i}\right)}$$

$\& \quad w = \inf(u_x,u_y).$

2,1 $\succ\!\!-$

(3) $(u_x = u_y \quad \& \quad u_x - w = 0 \quad \& \quad u_y - w = 0) \quad v$

$(u_x \neq u_y \quad \& \quad ((u_x - w = \varkappa \quad \& \quad u_y - w = 0) \quad v$

$(u_x - w = 0 \quad \& \quad u_y - w = \varkappa))).$

2,1,3 $\succ\!\!-$

(4) $x \hat{+} y = s^u \dfrac{\bigvee_{i=o}^{m} E_i s^{\varkappa i}}{\bigvee_{i=o}^{n} F_i s^{\varkappa i}}$ & $(\forall i)(E_i, F_i \in Re_{poz})$ &

$u = u_x + u_y - w = u_x + u_y - \inf(u_x, u_y)$ & $m = m_x + m_y$ &

$\varkappa n = \sup(u_x - w + (m_x + n_y)\varkappa,\ u_y - w + (m_y + n_x)\varkappa)$.

4 $\succ\!\!-$

(5) $u+(m-n)\varkappa = u_x+u_y-w+(m_x+m_y)\varkappa-\sup(u_x-w+(m_x+n_y)\varkappa, u_y-w+(m_y+n_x)\varkappa)$.

(6)"$a + b = \inf(a,b) + \sup(a,b)$". (G.Birkhoff, Lattice-ordered Groups, Annals of Mathematics, Vol 43, No 2, April 1942).

5,6 $\succ\!\!-$

(7) $u+(m-n)\varkappa = w-(n_x+n_y)\varkappa + \inf(u_x-w+(m_x+n_y)\varkappa,\ u_y-w+(m_y+n_x)\varkappa)$.

7,(5 u demonstr. T581) $\succ\!\!-$

(8) $u + (m - n)\varkappa = \inf(u_x + (m_x - n_x)\varkappa,\ u_y + (m_y - n_y)\varkappa)$.

4,6 $\succ\!\!-$

(9) $u = \sup(u_x, u_y)$.

1,4,8,9 $\succ\!\!-$ T582.

TEOREM 583

$(\forall x \in T_{ef})(\exists! u \in \{e,f\})(\exists! v \in \{e,f\})(\exists m,n \in N^+)$

$(x = s^u \dfrac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} = s^v \dfrac{\bigwedge_{i=o}^{n} C_i s^{(f-e)i}}{\bigwedge_{i=o}^{m} D_i s^{(f-e)i}}$ &

$(\forall i)(A_i, B_i, C_i, D_i \in Re_{poz})$ & $v - u = (m - n)(f - e)$ &

$(\forall i)(C_i B_{n-i} = 1$ & $D_i A_{m-i} = 1))$.

D o k a z

(1) $G = \{x\colon x \in T_{ef}$ & $(\exists! u \in \{e,f\})(\exists m,n \in N^+)(x = s^u \dfrac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}}$ &

$(\forall i)(A_i, B_i \in Re_{poz})$ & $(u + (m - n)(f - e)) \in \{e,f\})\}$. Def.

1,D127,T566 >——

(2) $\Omega_{ef} \subseteq G$.

1,T581,T582 >——

(3) $x,y \in G \rightarrow (x + y) \in G \;\&\; (x \overset{\frown}{+} y) \in G$.

1,2,3,T568 >——

$$(4)(\forall x \in T_{ef})(\exists! u \in \{e,f\})(\exists m,n \in N^{+})\left(x = s^{u} \frac{\bigvee\limits_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee\limits_{i=o}^{n} B_i s^{(f-e)i}} \quad \& \right.$$

$$\left. (\forall i)(A_i, B_i \in Re_{poz}) \;\&\; (u + (m - n)(f - e)) \in \{e,f\}\right).$$

4,T578 >—— T583.

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$$(1)\ x = s^{-0.5} \overset{\frown}{+} (2s^{0.7} + (1.5s^{0.7} \overset{\frown}{+} 4s^{0.5})) \qquad \text{Sup.}$$

1,D127,T567,T569 >——

$$(2)\ x \in T_{-0.5,0.7} \;\&\; f - e = 1.2 \;\&$$

$$x = s^{0.7} \frac{14 + 3s^{1.2}}{4 + 15.5s^{1.2} + 3s^{2.4}} = s^{-0.5} \frac{\frac{1}{3} \overset{\frown}{+} \frac{1}{15.5} s^{1.2} \overset{\frown}{+} \frac{1}{4} s^{2.4}}{\frac{1}{3} \overset{\frown}{+} \frac{1}{14} s^{1.2}}$$

$$\& \quad -0.5 - 0.7 = (1 - 2)(0.7 - (-0.5)).$$

3.2.2. ZAVRŠECI IMITANCIJE

DEFINICIJA 139

$$(\forall x \in T_{ef})\left(x = s^{u} \frac{\bigvee\limits_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee\limits_{i=o}^{n} B_i s^{(f-e)i}} \longrightarrow w_1(x) = u \;\& \; w_d(x) = u + (m - n)(f - e)\right).$$

D139,T583 >——

TEOREM 584

$$(\forall x \in T_{ef})\left(x = s^{u} \frac{\bigwedge\limits_{i=o}^{m} A_i s^{(f-e)i}}{\bigwedge\limits_{i=o}^{n} B_i s^{(f-e)i}} \longrightarrow w_1(x) = u + (m - n)(f - e) \;\& \; w_d(x) = u\right).$$

D139,D133 >——

TEOREM 585

$x \in A_{ef} \rightarrow w_l(x) = w_d(x) = e.$

D139,D134 >——

TEOREM 586

$x \in B_{ef} \rightarrow w_l(x) = w_d(x) = f.$

D139,T583,T585 >——

TEOREM 587

$w_l \in S_{surjekc}(T_{ef}, \{e,f\}).$

D139,T583,T586 >——

TEOREM 588

$w_d \in S_{surjekc}(T_{ef}, \{e,f\}).$

D139,T581,T582 >—— T589-T592.

TEOREM 589

$(\forall x,y \in T_{ef})(w_l(x + y) = \inf(w_l(x), w_l(y))).$

TEOREM 590

$(\forall x,y \in T_{ef})(w_l(x \overset{\frown}{+} y) = \sup(w_l(x), w_l(y))).$

TEOREM 591

$(\forall x,y \in T_{ef})(w_d(x + y) = \sup(w_d(x), w_d(y))).$

TEOREM 592

$(\forall x,y \in T_{ef})(w_d(x \overset{\frown}{+} y) = \inf(w_d(x), w_d(y))).$

D139,T576 >——

TEOREM 593

$x \in (BA)_{ef} \rightarrow w_l(x) = f \ \& \ w_d(x) = e.$

D139,T577 >——

TEOREM 594

$x \in (AB)_{ef} \rightarrow w_l(x) = e \ \& \ w_d(x) = f.$

DEFINICIJA 140

$(\forall x \in T_{ef})((l(x) = a \longleftrightarrow w_l(x) = e) \ \& \ (l(x) = b \longleftrightarrow w_l(x) = f)).$

DEFINICIJA 141

$(\forall x \in T_{ef})((d(x) = a \longleftrightarrow w_d(x) = e) \ \& \ (d(x) = b \longleftrightarrow w_d(x) = f)).$

D140,T587 $\succ\!\!-\!\!-$
TEOREM 595
$l \in S_{surjekc}(T_{ef},\{a,b\})$.

D141,T588 $\succ\!\!-\!\!-$
TEOREM 596
$d \in S_{surjekc}(T_{ef},\{a,b\})$.

D140,D141,T585 $\succ\!\!-\!\!-$
TEOREM 597
$x \in A_{ef} \;\rightarrow\; l(x) = d(x) = a.$

D140,D141,T586 $\succ\!\!-\!\!-$
TEOREM 598
$x \in B_{ef} \;\rightarrow\; l(x) = d(x) = b.$

D140,D141,T593 $\succ\!\!-\!\!-$
TEOREM 599
$x \in (BA)_{ef} \;\rightarrow\; l(x) = b \;\&\; d(x) = a.$

D140,D141,T594 $\succ\!\!-\!\!-$
TEOREM 600
$x \in (AB)_{ef} \;\rightarrow\; l(x) = a \;\&\; d(x) = b.$

DEFINICIJA 142
$(\forall x \in T_{ef})(q(x) = l(x)d(x)).$

D142,T595,T596,T3 $\succ\!\!-\!\!-$
TEOREM 601
$q \in S_{surjekc}(T_{ef},\Omega)$.

TEOREM 602

$$(\forall x \in (T_{ef} \setminus \Omega_{ef}))\Big(\; x = s^u \frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} \quad \longrightarrow$$

$$((A_oB_1-A_1B_o) < 0 \;\&\; l(x) = a) \;\vee\; ((A_oB_1-A_1B_o) > 0 \;\&\; l(x) = b) \;\&$$
$$((A_mB_{n-1}-A_{m-1}B_n) > 0 \;\&\; d(x) = a) \;\vee\; ((A_mB_{n-1}-A_{m-1}B_n) < 0 \;\&\; d(x) = b)\Big).$$

D o k a z

$$(1)\ G = \{x\colon x \in T_{ef}\ \ \&\ \ x \in \Omega_{ef}\ \ \mathrm{v}\ \ (x = s^{u\frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}}} \longrightarrow$$

$$((A_oB_1 - A_1B_o) < 0\ \&\ l(x)=a)\ \mathrm{v}\ ((A_oB_1 - A_1B_o) < 0\ \&\ l(x)=b))\}.\quad \text{Def.}$$

1 >——

$$(2)\ \Omega_{ef} \subseteq G$$

$$(3)\ x,y \in G\ \ \&\ \ \neg(x+y)\in\Omega_{ef}\ \ \&\ \ \pi = f - e\ \ \&$$

$$x = s^{u_x\frac{\bigvee_{i=o}^{m_x} A_i s^{\pi i}}{\bigvee_{i=o}^{n_x} B_i s^{\pi i}}}\quad \&\quad y = s^{u_y\frac{\bigvee_{i=o}^{m_y} C_i s^{\pi i}}{\bigvee_{i=o}^{n_y} D_i s^{\pi i}}}\ \ \&\ \ u_x = u_y = u.\ \ \text{Sup.}$$

3,D127,T581,T583 >——

(4)

$$x + y = s^{u\frac{\bigvee_{i=o}^{m} E_i s^{\pi i}}{\bigvee_{i=o}^{n} F_i s^{\pi i}}}\quad \&\quad (m > 0\ \mathrm{v}\ n > 0)\ \&\ (\forall i)(E_i, F_i \in Re_{poz})$$

$$E_o = A_oD_o + B_oC_o\quad \&\quad E_1 = A_oD_1 + A_1D_o + B_1C_o + B_oC_1\quad \&$$
$$F_o = B_oD_o\quad \&\quad F_1 = B_oD_1 + B_1D_o.$$

4 >——

$$(5)\ E_oF_1 - E_1F_o = D_o^2(A_oB_1 - A_1B_o) + B_o^2(C_oD_1 - C_1D_o)$$

3,1 >——

$$(6)\ ((l(x) = a\ \ \&\ \ A_oB_1 < A_1B_o\ \ \&\ \ C_oD_1 < C_1D_o)\ \ \mathrm{v}$$
$$(l(x) = b\ \ \&\ \ A_1B_o < A_oB_1\ \ \&\ \ C_1D_o < C_oD_1))\ .$$

5,6 >——

$$(7)\ (l(x) = a\ \ \&\ \ E_oF_1 - E_1F_o < 0)\ \mathrm{v}\ (l(x) = b\ \ \&\ \ E_1F_o < E_oF_1)$$

1,3,4,7,D139-D141 >——

$$(8)\ (x,y\in G\ \ \&\ \ l(x) = l(y))\ \ \rightarrow\ \ (x+y)\in G.$$

(9) $x, y \in G \;\;\&\;\; \neg (x + y) \in \Omega_{ef} \;\;\&\;\; \pi = (f - e) \;\;\&$

$$x = s^{u} \frac{\bigvee_{i=o}^{m_x} A_i s^{\pi i}}{\bigvee_{i=o}^{n_x} B_i s^{\pi i}} \;\;\&\;\; y = s^{v} \frac{\bigvee_{i=o}^{m_y} C_i s^{\pi i}}{\bigvee_{i=o}^{n_y} D_i s^{\pi i}} \;\;\&$$

$l(x) = a \;\;\&\;\; l(y) = b$ Sup.

9,D127,D139-D141,T581,T583 $\succ\!\!-$

(10)

$$x + y = s^{u} \frac{\bigvee_{i=o}^{m} E_i s^{\pi i}}{\bigvee_{i=o}^{n} F_i s^{\pi i}} \;\;\&\;\; (m > 0 \;\; v \;\; n > 0) \;\;\&$$

$(\forall i)(E_i, F_i \in Re_{poz}) \;\;\&$

$E_o = A_o D_o \;\;\&\;\; E_1 = A_o D_1 + A_1 D_o + B_o C_o \;\;\&$

$F_o = B_o D_o \;\;\&\;\; F_1 = B_o D_1 + B_1 D_o.$

10 $\succ\!\!-$

(11) $E_o F_1 - E_1 F_o = D_o^2 (A_o B_1 - A_1 B_o) - B_o^2 C_o D_o$

9,1 $\succ\!\!-$

(12) $A_o B_1 < A_1 B_o$

10,11,12,D139-D141 $\succ\!\!-$

(13) $l(x + y) = a \;\;\&\;\; E_o F_1 < E_1 F_o$

1,9,10,13 $\succ\!\!-$

(14) $(x, y \in G \;\;\&\;\; \neg (l(x) = l(y)) \rightarrow (x + y) \in G.$

8,14 $\succ\!\!-$

(15) $(\forall x, y \in G)((x + y) \in G).$

1,D127,T581,T583,D139-D141 (analog 15) $\succ\!\!-$

(16) $(\forall x, y \in G)((x \overset{\frown}{+} y) \in G).$

1,2,15,16,T568 $\succ\!\!-$

(17) $$(\forall x \in (T_{ef} \setminus \Omega_{ef}))\left(x = s^{u} \frac{\bigvee_{i=o}^{m} A_i s^{\pi i}}{\bigvee_{i=o}^{n} B_i s^{\pi i}} \longrightarrow\right.$$

$$\left.((A_o B_1 - A_1 B_o < 0 \;\;\&\;\; l(x) = a) \;\; v \;\; (A_o B_1 - A_1 B_o > 0 \;\;\&\;\; l(x) = b))\right).$$

D127,T578,D139-D141,T568 (analog 17) $\succ\!\!-$

$$(18)(\forall x\in T_{ef}\setminus \Omega_{ef}))(x = s^{u}\frac{\bigvee_{i=o}^{m} A_i s^{\varkappa i}}{\bigvee_{i=o}^{n} B_i s^{\varkappa i}} \longrightarrow$$

$((A_mB_{n-1}-A_{m-1}B_n > 0 \ \& \ d(x)=a) \lor (A_mB_{n-1}-A_{m-1}B_n < 0 \ \&$
$d(x) = b)).$

17,18 $\succ\!\!-$ T602.

T602,T583 $\succ\!\!-$

TEOREM 603

$$(\forall x\in(T_{ef}\setminus \Omega_{ef}))(x = s^{u}\frac{\bigwedge_{i=o}^{m} A_i s^{(f-e)i}}{\bigwedge_{i=o}^{n} B_i s^{(f-e)i}} \longrightarrow$$

$((A_oB_1-A_1B_o) < 0 \ \& \ d(x) = b) \quad \lor ((A_oB_1-A_1B_o > 0 \ \& \ d(x) = a)) \ \&$
$((A_mB_{n-1}-A_{m-1}B_n) > 0 \ \& \ l(x)=b) \lor ((A_mB_{n-1}-A_{m-1}B_n) < 0 \ \& \ l(x)=a)).$

3.2.3. VALUACIJA I DEFEKT IMITANCIJE

DEFINICIJA 143

$(\forall x\in T_{ef})(w(x) = w_d(x) - w_l(x)).$

D143,T587,T588 $\succ\!\!-$

TEOREM 604

$w\in S_{surjekc}(T_{ef},\{e - f,\ 0,\ f - e\}).$

TEOREM 605

$(\forall x,y\in T_{ef})(w(x + y) + w(x \hat{+} y) = w(x) + w(y)).$

D o k a z

(1) $x,y\in T_{ef}$ & $w(x + y) + w(x \hat{+} y) = z$	Sup.
(2) $z = w_d(x+y) - w_l(x+y) + w_d(x\hat{+}y) - w_l(x\hat{+}y)$	1,D143
(3) $z = \sup(w_d(x),w_d(y)) - \inf(w_l(x),w_l(y)) +$ $+ \inf(w_d(x),w_d(y)) - \sup(w_l(x),w_l(y))$	2,T589-T592

3,(6 u demonstr. T582) $\succ\!\!-$

(4) $z = w_d(x) + w_d(y) - w_l(x) - w_l(y)$

1,4,D143 $\succ\!\!-$ T605.

DEFINICIJA 144

$(\forall x \in T_{ef})(h(x) = \frac{w(x)}{f-e}).$

D144,D131,T604 $\succ\!\!-$

TEOREM 606

$h \in S_{surjekc}(T,\{-1,0,+1\}).$

T605,D144 $\succ\!\!-$

TEOREM 607

$(\forall x,y \in T_{ef})(h(x+y) + h(x \overset{\frown}{+} y) = h(x) + h(y)).$

D139-D141,D143,D144,T606,T607 $\succ\!\!-$ T608-T610.

TEOREM 608

$$(\forall x \in T_{ef})\left(x = s^{u} \frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} \longrightarrow h(x) = m - n\right).$$

TEOREM 609

$$(\forall x \in T_{ef})\left(x = s^{u} \frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} \longrightarrow ((m=n \;\&\; l(x)=d(x)) \vee (m=n+1 \;\&\; l(x)=a \;\&\; d(x)=b) \vee (m=n-1 \;\&\; l(x)=b \;\&\; d(x)=a))\right).$$

TEOREM 610

$$(\forall x \in T_{ef})\left(x = s^{u} \frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} \longrightarrow \lim_{s \to 0}(x) = \frac{A_o}{B_o} s^{w_l(x)} \;\&\; \lim_{s \to \infty}(x) = \frac{A_m}{B_n} s^{w_d(x)}\right).$$

D143,D144,T574,T585,T586,T593,T594 $\succ\!\!-$

TEOREMI 611-613

611 $x \in \Omega_{ef} \longrightarrow h(x) = 0.$

612 $x \in (BA)_{ef} \longrightarrow h(x) = -1.$

613 $x \in (AB)_{ef} \longrightarrow h(x) = +1.$

3.2.4. KANONSKA s-FORMA IMITANCIJE

T583 >——

TEOREM 614

$$(\forall x \in T_{ef})((x = s^u \frac{\bigvee_{i=0}^{m} A_i s^{(f-e)i}}{\bigvee_{i=0}^{n} B_i s^{(f-e)i}} = s^u \frac{\bigvee_{i=0}^{p} C_i s^{(f-e)i}}{\bigvee_{i=0}^{q} D_i s^{(f-e)i}} \quad \&$$

$$\neg \mathrm{El}\left(\sum_{i=0}^{m} A_i s^i, \sum_{i=0}^{n} B_i s^i\right) = 0 \quad \& \quad \neg \mathrm{El}\left(\sum_{i=0}^{p} C_i s^i, \sum_{i=0}^{q} D_i s^i\right) = 0 \quad \&$$

$$((d(x) = a \;\&\; B_n = D_q = 1) \quad v \quad (d(x) = b \;\&\; A_m = C_p = 1))) \longrightarrow$$

$$m = p \;\;\&\;\; n = q \;\;\&\;\; (\forall i)(A_i = C_i \;\;\&\;\; B_i = D_i)).$$

DEFINICIJA 145

$$(\forall x \in T_{ef})(\mathrm{Slog}(x) = (A_i^m, B_i^n) \longleftrightarrow (x = s^u \frac{\bigvee_{i=0}^{m} A_i s^{(f-e)i}}{\bigvee_{i=0}^{n} B_i s^{(f-e)i}} \quad \&$$

$$\neg \mathrm{El}\left(\sum_{i=0}^{m} A_i s^i, \sum_{i=0}^{n} B_i s^i\right) = 0 \quad \&$$

$$((d(x) = a \;\&\; B_n = 1) \quad v \quad (d(x) = b \;\&\; A_m = 1)) \quad \&$$

$$(A_i^m, B_i^n) = (A_o, A_1, \ldots A_m, B_o, B_1, \ldots B_n))).$$

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$$(1)\quad x = s^7 \frac{14 + 3s^{12}}{4 + 15.5s^{12} + 3s^{24}} \quad \& \quad x \in T_{-5,7} \qquad \text{Sup.}$$

1,D139,D141 >——

$$(2)\quad d(x) = a \quad \&$$

$$\mathrm{El}((14 + 3s^i),(4 + 15.5s^i + 3s^{2i})) = \begin{vmatrix} 3 & 14 & 0 \\ 0 & 3 & 14 \\ 3 & 15.5 & 4 \end{vmatrix} = -27$$

1,2,D145 $\succ\!\!-$

(3) $Slog(x) = (\frac{14}{3}, 1, \frac{4}{3}, \frac{15.5}{3}, 1)$.

D145,T614 $\succ\!\!-$

TEOREM 615

$(\forall x,y \in T_{ef})((Slog(x) = Slog(y) \ \& \ (l(x) = l(y) \lor d(x) = d(y)))$
$\longrightarrow \ x = y)$.

PRIMJER 338

(1) $x \in T_{ef} \ \& \ Slog(x) = (3,1,4,2,1)$ Sup.

1,T609 $\succ\!\!-$

(2) $(A_o=3 \ \& \ A_1=1 \ \& \ A_2=4 \ \& \ B_o=2 \ \& \ B_1=1 \ \& \ l(x)=a \ \& \ d(x)=b) \ \lor$
$(A_o=3 \ \& \ A_1=1 \ \& \ B_o=4 \ \& \ B_1=2 \ \& \ B_2=1 \ \& \ l(x)=b \ \& \ d(x)=a)$

2,1,D145 $\succ\!\!-$

(3) $d(x) = a$

3,2,D145,D140,D139 $\succ\!\!-$

(4) $l(x) = b \ \& \ x = s^f \dfrac{3 + s^{f-e}}{4 + 2s^{f-e} + s^{2(f-e)}}$.

PRIMJERI

339 $(x \in T_{ef} \ \& \ Slog(x) = (1,1)) \longrightarrow x = s^e \ \lor \ x = s^f$

340 $(x \in T_{ef} \ \& \ Slog(x) = (3,1)) \longrightarrow x = s^e \frac{3}{1}$.

341 $(x \in T_{ef} \ \& \ Slog(x) = (1,3)) \longrightarrow x = s^f \frac{1}{3}$.

342 $(x \in T_{ef} \ \& \ Slog(x) = (1,1,1)) \longrightarrow x = s^e \dfrac{1+s^{f-e}}{1} \ \lor$
$x = s^f \dfrac{1}{1+s^{f-e}}$.

343 $(x \in T_{ef} \ \& \ Slog(x) = (2,1,3,4)) \longrightarrow x = s^f \dfrac{2 + s^{f-e}}{3 + 4s^{f-e}}$.

3.2.5. DULJINA IMITANCIJE

DEFINICIJA 146

$(\forall x \in T_{ef})(\lambda_t(x) = k \longleftrightarrow \mathrm{Slog}(x) = (\ A_1^m\ ,\ B_1^n\ )\ \&\ k = m + n).$

DEFINICIJA 147

$(\forall x \in T_{ef})(\lambda(x) = \lambda_t(x) + 1).$

DEFINICIJA 148

$$(\forall x \in T_{ef})(\lambda_s(x) = \frac{\lambda_t(x) - h(x)}{2} \quad \& \quad \lambda_p(x) = \frac{\lambda_t(x) + h(x)}{2}).$$

D148 >——

TEOREMI 616-617

616 $(\forall x \in T_{ef})(\lambda_s(x) + \lambda_p(x) = \lambda_t(x)).$

617 $(\forall x \in T_{ef})(h(x) = \lambda_p(x) - \lambda_s(x))$

D147,D146,D133,D134,T574,T576,T577 >——

TEOREMI 618-619

618 $(\forall x \in T_{ef})(x \in \Omega_{ef} \longleftrightarrow \lambda(x) = 1).$

619 $(\forall x \in T_{ef})((x \in (BA)_{ef}\ \vee\ x \in (AB\ _{ef}) \longleftrightarrow \lambda(x) = 2).$

D146,T618,T619 >——

TEOREMI 620-621

620 $(\forall x \in T_{ef})(x \in \Omega_{ef} \longleftrightarrow \lambda_t(x) = 0).$

621 $(\forall x \in T_{ef})((x \in (BA)_{ef}\ \vee\ x \in (AB)_{ef}) \longleftrightarrow \lambda_t(x) = 1).$

D148,T620,T611,T621,T612,T613 >——

TEOREMI 622-624

622 $(\forall x \in T_{ef})(x \in \Omega_{ef} \longleftrightarrow \lambda_s(x) = 0\ \&\ \lambda_p(x) = 0).$

623 $(\forall x \in T_{ef})(x \in (BA)_{ef} \longleftrightarrow \lambda_s(x) = 1\ \&\ \lambda_p(x) = 0).$

624 $(\forall x \in T_{ef})(x \in (AB)_{ef} \longleftrightarrow \lambda_p(x) = 1\ \&\ \lambda_s(x) = 0).$

TEOREM 625
$\lambda_t \in S_{surjekc}(T_{ef}, N^+)$.

D o k a z
D146,D147,T615 $\succ\!\!-\!\!-$
(1) $\lambda_t \in S_{funkc}(T_{ef}, N^+)$.

(2) $G = \{n\colon n \in N^+ \ \& \ (\exists x \in T_{ef})(\lambda_t(x) = n)\}$	Def.
(3) $0 \in G$	2,T620,T566
(4) $x \in T_{ef} \ \& \ \lambda_t(x) = n$	Sup.
(5)$(w_1(x) = e \ \& \ (\forall y \in B_{ef})(\lambda_t(x \mathbin{\hat{+}} y) = n+1)) \ \vee$	
$(w_1(x) = f \ \& \ (\forall y \in A_{ef})(\lambda_t(x + y) = n+1))$	4,T587,T583,T615
(6) $n \in G \longrightarrow (n+1) \in G$	4,5,T567,2
(7) $N^+ \subseteq Adom(\lambda_t)$	2,3,6

1,3,7 $\succ\!\!-\!\!-$ T625.

T625,D148,T606,T622,T611 $\succ\!\!-\!\!-$
TEOREMI 626-627
626 $\lambda_s \in S_{surjekc}(T_{ef}, N^+)$.

627 $\lambda_p \in S_{surjekc}(T_{ef}, N^+)$.

D146,D148,D145,T608 $\succ\!\!-\!\!-$
TEOREM 628
$(\forall x \in T_{ef})((\lambda_p(x) = m \ \& \ \lambda_s(x) = n) \longleftrightarrow Slog(x) = (A_1^m, B_1^n))$.

PRIMJERI
($x \in T_{1,3}$ & $Y = (\lambda_t(x), \lambda(x), \lambda_s(x), \lambda_p(x))) \longrightarrow$

344 $x = s\,\frac{2}{1}$ $\longrightarrow$ $Y = (\ 0\ ,\ 1\ ,\ 0\ ,\ 0\ )$.

345 $x = s^3\text{-}\frac{1}{2}$ $\longrightarrow$ $Y = (\ 0\ ,\ 1\ ,\ 0\ ,\ 0\ )$.

346 $x = s^3\,\frac{1}{2 + s^2}$ $\longrightarrow$ $Y = (\ 1\ ,\ 2\ ,\ 1\ ,\ 0\ )$.

347 $x = s\,\frac{2 + 3s^2}{5 + s^2}$ $\longrightarrow$ $Y = (\ 2\ ,\ 3\ ,\ 1\ ,\ 1\ )$.

348 $x = s^3\,\frac{2 + s^2}{3 + 4s^2}$ $\longrightarrow$ $Y = (\ 2\ ,\ 3\ ,\ 1\ ,\ 1\ )$.

349 $x = s\,\frac{1 + 4s^2 + s^4}{5 + 3s^2}$ $\longrightarrow$ $Y = (\ 3\ ,\ 4\ ,\ 1\ ,\ 2\ )$.

DEFINICIJA 149

$$(\forall x,y\in T_{ef})(El_s(x,y) = El\left(\sum_{i=o}^{n} B_i s^i, \sum_{i=o}^{q} D_i s^i\right) \quad \longleftrightarrow$$

$$Slog(x) = (A_i^m,\ B_i^n) \quad \& \quad Slog(y) = (C_i^p,\ D_i^q)).$$

DEFINICIJA 150

$$(\forall x,y\in T_{ef})(El_p(x,y) = El\left(\sum_{i=o}^{m} A_i s^i, \sum_{i=o}^{p} C_i s^i\right) \quad \longleftrightarrow$$

$$Slog(x) = (A_i^m,\ B_i^n) \quad \& \quad Slog(y) = (C_i^p,\ D_i^q)).$$

PRIMJER 350

$$x = s^3\frac{2 + s^2}{3 + 4s^2} \qquad \& \qquad y = s\frac{2 + 4s^2 + s^4}{5 + 3s^2} \qquad \longrightarrow$$

$$El_s(x,y) = \begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} \quad \& \quad El_p(x,y) = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 4 & 2 \end{vmatrix}.$$

TEOREM 629

$$(\forall x,y\in T_{ef})(\lambda_s(x+y) = \lambda_s(x) + \lambda_s(y) \quad \longleftrightarrow \quad El_s(x,y) \neq 0).$$

D o k a z

(1) $x,y\in T_{ef}$ & $w_1(x) = u_x$ & $w_1(y) = u_y$ &

$Slog(x) = (A_i^m,\ B_i^n)$ & $Slog(y) = (C_i^p,\ D_i^q)$ &

$$G_x = \bigvee_{i=o}^{m} A_i s^{(f-e)i} \qquad \& \qquad H_x = \bigvee_{i=o}^{n} B_i s^{(f-e)i} \qquad \&$$

$$G_y = \bigvee_{i=o}^{p} B_i s^{(f-e)i} \qquad \& \qquad H_y = \bigvee_{i=o}^{q} D_i s^{(f-e)i}.$$ Sup.

(2) $\lambda_s(x + y) = \lambda_s(x) + \lambda_s(y)$ & $El_s(x,y) = 0.$ Sup.

2,1,D149 $\succ\!\!-\!\!-$

(3) $(\exists H, H'_x, H'_y \in S_{polinom}(s))(H_x = HH'_x \ \& \ H_y = HH'_y \ \& \ \neg H \in Re)$.

3,1 $\succ\!\!-\!\!-$

(4) $$x + y = \frac{s^{u_x} G_x H'_y + s^{u_y} G_y H'_x}{H'_x H'_y}$$

4,1,T628 $\succ\!\!-\!\!-$

(5) $\lambda_s(x + y) < (\lambda_s(x) + \lambda_s(y))$

1,2,5 $\succ\!\!-\!\!-$

(6) $(\forall x, y \in T_{ef})(\lambda_s(x+y) = \lambda_s(x) + \lambda_s(y) \rightarrow El_s(x,y) \neq 0)$.

(7)"2.2.Teorem. Operator R uzimanja rezultante je distributivan prema množenju polinoma:
$R(ab,c) = R(a,c)R(b,c)$ i
$R(p.q.r,\dots,u) = R(p,u)R(q,u)R(r,u)..$"
(Kurepa, Viša algebra I, 1965, str.705).

1,T581 $\succ\!\!-\!\!-$

(8) $x + y = s^u \frac{G}{H_x H_y}$ & $G = s^{u_x - u} G_x H_y + s^{u_y - u} G_y H_x$ &
$u = \inf(u_x, u_y)$.

(9) $El(H_x H_y, G) = 0$ & $El_s(x,y) = 0$ Sup.

9,7 $\succ\!\!-\!\!-$

(10) $El(H_x G) El(H_y, G) = 0$

10 $\succ\!\!-\!\!-$

(11) $El(H_x, G) = 0 \ \vee \ El(H_y, G) = 0$

11,8 $\succ\!\!-\!\!-$

(12) $El(G_x H_y, H_x) = 0 \ \vee \ El(G_y H_x, H_y) = 0$

12,7 $\succ\!\!-\!\!-$

(13) $El(G_x, H_x) = 0 \ \vee \ El(H_y, H_x) = 0 \ \vee \ El(G_y, H_y) = 0$

13,9,1,D145,D149 $\succ\!\!-\!\!-$

(14) $El_s(x,y) \neq 0 \rightarrow El(H_x H_y, G) \neq 0$

14,8,D145,T628 $\succ\!\!-\!\!-$

(15) $(\forall x, y \in T_{ef})(El_s(x,y) \neq 0 \rightarrow \lambda_s(x + y) = \lambda_s(x) + \lambda_s(y))$.

6,15 $\succ\!\!-\!\!-$ T629.

D150,T628,D145 $\succ$——

TEOREM 630

$$(\forall x,y\in T_{ef})(\lambda_p(x \mathbin{\hat{+}} y) = \lambda_p(x) + \lambda_p(y) \longleftrightarrow El_p(x,y) \nparallel 0).$$

TEOREM 631

$$(\forall x,y\in T_{ef})(\lambda_s(x + y) = \lambda_s(x) + \lambda_s(y) \longleftrightarrow$$
$$\lambda_p(x + y) = \lambda_p(x) + \lambda_p(y) - h(x \mathbin{\hat{+}} y)).$$

D o k a z

(1) $x,y\in T_{ef}$ & $\lambda_s(x + y) = \lambda_s(x) + \lambda_s(y)$ Sup.

(2) $\lambda_p(x + y) = \lambda_s(x + y) + h(x + y)$ 1,T617

(3) $= \lambda_s(x) + \lambda_s(y) + h(x + y)$ 2,1

(4) $= \lambda_p(x) - h(x) + \lambda_p(y) - h(y) + h(x + y)$ 3,T617

(5) $(\forall x,y\in T_{ef})(\lambda_s(x + y) = \lambda_s(x) + \lambda_s(y) \rightarrow$
$\lambda_p(x + y) = \lambda_p(x) + \lambda_p(y) - h(x \mathbin{\hat{+}} y)).$ 1,4,T607

T617,T607 (analog 5) $\succ$——

(6) $(\forall x,y\in T_{ef})(\lambda_p(x + y) = \lambda_p(x) + \lambda_p(y) - h(x \mathbin{\hat{+}} y) \rightarrow$
$\lambda_s(x + y) = \lambda_s(x) + \lambda_s(y)).$

5,6 $\succ$—— T631.

T617,T607 (analog T631) $\succ$——

TEOREM 632

$$(\forall x,y\in T_{ef})(\lambda_p(x \mathbin{\hat{+}} y) = \lambda_p(x) + \lambda_p(y) \longleftrightarrow$$
$$\lambda_s(x \mathbin{\hat{+}} y) = \lambda_s(x) + \lambda_s(y) + h(x + y)).$$

3.3. INVARIJANTA IZOMORFNIH TRANSFORMACIJA t-STRUKTURE

3.3.1. KARAKTERISTIČNI POLINOM IMITANCIJE I IZOMORFIZMI t-STRUKTURE

DEFINICIJA 151

$$(\forall x\in T_{ef})(\Pi(x) = s^{\frac{u-e}{f-e}} \sum_{i=o}^{m} A_i s^{2i} + s^{\frac{f-u}{f-e}} \sum_{i=o}^{n} B_i s^{2i} \quad \longleftrightarrow$$

$$(A_i^m,\ B_i^n) = \mathrm{Slog}(x) \ \ \&\ \ u = w_1(x)).$$

PRIMJERI

$(x\in T_{ef} \ \ \&\ \ \varkappa = f - e) \longrightarrow$

351 $\Pi(s^e\frac{A_o}{1}) \quad = A_o + s.$

352 $\Pi(s^f\frac{1}{B_o}) \quad = B_o + s.$

353 $\Pi(s^e\frac{A_o+s^\varkappa}{B_o}) \quad = A_o + B_o s + s^2.$

354 $\Pi(s^f\frac{A_o}{B_o+s^\varkappa}) \quad = B_o + A_o s + s^2.$

355 $\Pi(s^e\frac{A_o+A_1 s^\varkappa}{B_o+s^\varkappa}) \quad = A_o + B_o s + A_1 s^2 + s^3.$

356 $\Pi(s^f\frac{A_o + s^\varkappa}{B_o+B_1 s^\varkappa}) \quad = B_o + A_o s + B_1 s^2 + s^3.$

357 $\Pi(s^e\frac{A_o+A_1 s^\varkappa+s^{2\varkappa}}{B_o + B_1 s^\varkappa}) \quad = A_o + B_o s + A_1 s^2 + B_1 s^3 + s^4.$

358 $\Pi(s^f\frac{A_o + A_1 s^\varkappa}{B_o+B_1 s^\varkappa+s^{2\varkappa}}) \quad = B_o + A_o s + B_1 s^2 + A_1 s^3 + s^4.$

D151,D145-D147,T609 $\succ\!\!-\!\!-$

TEOREM 633

$$(\forall x\in T_{ef})(\Pi(x) = \sum_{i=o}^{n} A_i s^i \quad \longrightarrow \quad n = \lambda(x) \ \ \&\ \ A_n = 1).$$

D151,T615 >——

TEOREM 634

$$(\forall x,y\in T_{ef})((\Pi(x) = \Pi(y) \quad \& \quad (l(x) = l(y) \quad v \quad d(x) = d(y)) \longrightarrow$$
$$x = y.$$

T568 >——

TEOREM 635

$$(\forall e,f,g,h\in Re)((e < f \quad \& \quad g < h) \longrightarrow$$
$$(\exists! F\in S_{funkc}(T_{ef},T_{gh}))((\forall k\in Re_{poz})(F(ks^{e}) = ks^{g} \quad \&$$
$$F(ks^{f}) = ks^{h}) \quad \&$$
$$(\forall x,y\in T_{ef})(F(x + y) = F(x) + F(y) \quad \&$$
$$(F(x \overset{\frown}{+} y) = F(x) \overset{\frown}{+} F(y))) .$$

DEFINICIJA 152

$(\forall e,f,g,h\in Re)(e < f \quad \& \quad g < h \longrightarrow e,f\surd^{g,h}$ je jedinstveno određen skup F iz teorema 635).

D152,T635,T568 >——

TEOREM 636

$$(\forall e,f,g,h\in Re)(e < f \quad \& \quad g < h \longrightarrow e,f\surd^{g,h}\in S_{bijekc}(T_{ef},T_{gh})).$$

KONVENCIJA 5

$$(\forall e,f,g,h\in Re)(e < f \quad \& \quad g < h \longrightarrow X) \leadsto X.$$

D152,T635 >——

TEOREMI 637-638

637 $(\forall k\in Re_{poz})(e,f\surd^{g,h}(ks^{e}) = ks^{g} \quad \& \quad e,f\surd^{g,h}(ks^{f}) = ks^{h}).$

638 $(\forall x,y\in T_{ef})(e,f\surd^{g,h}(x + y) = e,f\surd^{g,h}(x) + e,f\surd^{g,h}(y) \quad \&$

$$e,f\surd^{g,h}(x \overset{\frown}{+} y) = e,f\surd^{g,h}(x) \overset{\frown}{+} e,f\surd^{g,h}(y)).$$

T568 $\succ\!\!-$

TEOREM 639

$(\forall e,f,g,h\in Re)((e < f \;\; \& \;\; g < h) \longrightarrow$

$(\exists! F\in S_{funkc}(T_{ef},T_{gh}))((\forall k\in Re_{poz})(F(ks^{e}) = \frac{1}{k}\, s^{h} \quad \&$

$F(ks^{f}) = \frac{1}{k}\, s^{g} \quad \&$

$(\forall x,y\in T_{ef})(F(x + y) = F(x) \mathbin{\overset{\frown}{+}} F(y) \quad \&$

$(F(x \mathbin{\overset{\frown}{+}} y) = F(x) + F(y))).$

DEFINICIJA 153

$(\forall e,f,g,h\in Re)((e < f \;\; \& \;\; g < h) \longrightarrow e,f/\!\blacktriangle^{g,h}$ je jedinstveno određen skup F iz teorema 639).

D153,T639,T568 $\succ\!\!-$

TEOREM 640

$(\forall e,f,g,h\in Re)(e < f \;\; \& \;\; g < h \longrightarrow e,f/\!\blacktriangle^{g,h} \in S_{bijekc}(T_{ef},T_{gh})).$

D153,T639 $\succ\!\!-$

TEOREMI 641-642

641 $(\forall k\in Re_{poz})(e,f/\!\blacktriangle^{g,h}(ks^{e}) = \frac{1}{k}\, s^{h} \;\; \& \;\; e,f/\!\blacktriangle^{g,h}(ks^{f}) = \frac{1}{k}\, s^{g}).$

642 $(\forall x,y\in T_{ef})(e,f/\!\blacktriangle^{g,h}(x + y) = e,f/\!\blacktriangle^{g,h}(x) \mathbin{\overset{\frown}{+}} e,f/\!\blacktriangle^{g,h}(y) \quad \&$

$e,f/\!\blacktriangle^{g,h}(x \mathbin{\overset{\frown}{+}} y) = e,f/\!\blacktriangle^{g,h}(x) + e,f/\!\blacktriangle^{g,h}(y))\; .$

T568,T589,T590,T634,D151,T636-T638,T640-T642 $\succ\!\!-$

TEOREMI 643-644

643 $(\forall x\in T_{ef})(\forall y\in T_{gh})(e,f\blacktriangledown\!/^{g,h}(x) = y \longleftrightarrow (\Pi(x) = \Pi(y) \quad \&$

$l(x) = l(y))).$

644 $(\forall x\in T_{ef})(\forall y\in T_{gh})(e,f/\!\blacktriangle^{g,h}(x) = y \longleftrightarrow (\Pi(x) = \Pi(y) \quad \&$

$\neg\, l(x) = l(y))).$

TEOREM 645

$(\forall x \in T_{ef})(\Pi(x) \in S_{\text{Hurwitz p.}})$.

D o k a z

(1)"...the stability of the physical system is assured if it can be shown that the zeros of the polynomial

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_o \qquad (619)$$

lie in the left half of the z-plane. A polynomial having this property is called a Hurwitz polynomial.

If the polynomial P(z) is written in the form

$$P(z) = m(z) + n(z) \qquad (628)$$

in which m(z) represents the terms involving even powers of z (called the even part of P), and n(z) represents the terms involving odd powers of z (called the odd part of P), according to Eq.625, one has

$$\psi(z) = \frac{m(z)}{n(z)} \qquad (629)$$

One has thus gained a new formulation for the necessary and sufficient conditions that P(z) be a Hurwitz polynomial.Namely, the quotient of its even and odd parts must be a function having simple poles on the imaginary axis only, and with positive real residues in these poles.

It is collaterally useful to digress for a moment and study somewhat more carefully the properties of the function $\psi(z)$. First it should be observed that if $\psi(z)$ satisfies the conditions 627, its reciprocal $1/\psi(z)$ does so also. Hence, using Eq. 629, the function

$$\frac{1}{\psi(z)} = \frac{n(z)}{m(z)} \qquad (638)$$

must also have simple poles on the imaginary axis only and positive real residues. Both the even and odd parts m(z) and n(z) must, therefore, be polynomials, whose zeros are simple and lie on the imaginary axis".(E.A.Guillemin,The Mathematics of Circuit Analysis, 1951, str.395-400).

(2) $x \in \Omega_{\bar{1},1}$ Sup.

(3) $(\exists k \in Re_{poz})(x = s^{-1}\frac{k}{1} \ \vee \ x = s\frac{1}{k}$ 2,D127

(4) $\Pi(x) = s + k$ 3,D151,D141

(5) $(\forall x \in \Omega_{\bar{1},1})(\Pi(x) \in S_{\text{Hurwitz p.}})$ 2,4

(6) $G = \{x: x \in T_{\bar{1},1} \ \& \ \Pi(x) \in S_{\text{Hurwitz p.}}\}$ Def.

(7) $\Omega_{\bar{1},1} \subseteq G$ 6,5,T566

(8) $x,y \in G \ \& \ \text{Slog}(x) = (A_1^m, B_1^n) \ \& \ \text{Slog}(y) = (C_1^p, D_1^q)$ Sup.

$$(9)\left(x = \frac{M}{N} = s\,\frac{\bigvee_{i=o}^{m} A_i s^{2i}}{\bigvee_{i=o}^{n} B_i s^{2i}} \quad v \quad x = \frac{M}{N} = \frac{1}{s}\,\frac{\bigvee_{i=o}^{m} A_i s^{2i}}{\bigvee_{i=o}^{n} B_i s^{2i}}\right) \ \&$$

$$\left(y = \frac{P}{Q} = s\,\frac{\bigvee_{i=o}^{p} C_i s^{2i}}{\bigvee_{i=o}^{q} D_i s^{2i}} \quad v \quad y = \frac{P}{Q} = \frac{1}{s}\,\frac{\bigvee_{i=o}^{p} C_i s^{2i}}{\bigvee_{i=o}^{q} D_i s^{2i}}\right) \qquad 8,6,D145$$

(10) $(M + N) \in S_{\text{Hurwitz p.}}$ & $(P + Q) \in S_{\text{Hurwitz p.}}$ 9,8,6,D151

(11) $x + y = \frac{MQ + PN}{NQ}$ & $x \mathbin{\hat{+}} y = \frac{MP}{MQ + PN}$ 9,T569

(12) $(x + y) \in S_{\text{Hurwitz p.}}$ & $(x \mathbin{\hat{+}} y) \in S_{\text{Hurwitz p.}}$ 11,10,9,1

(13) $x,y \in G \rightarrow (x + y) \in G$ & $(x \mathbin{\hat{+}} y) \in G$ 8,12,6

(14) $(\forall x \in T_{\bar{1},1})(\Pi(x) \in S_{\text{Hurwitz p.}})$. 6,7,13,T568

14,T636,T643 >—— T645.

3.3.2. IZOMORFIZMI D, R i $\bar{R}$

DEFINICIJA 154

$(\forall x \in T_{ef})(P^o(x) = {}_{e,f}\sqrt{}^{e,f}(x))$.

D154,T637,T638,T568 >——

TEOREM 646

$(\forall x \in T_{ef})(P^o(x) = x)$.

DEFINICIJA 155

$(\forall x \in T_{ef})(D(x) = {}_{e,f}\!\not{\!\Lambda}^{e,f}(x))$.

D155,T640-T642,T568,D139 >——

TEOREMI 647-650

647 $D \in S_{\text{bijekc}}(T_{ef}, T_{ef})$.

648 $(\forall x \in T_{ef})(D(x) = s^{e+f}\,\frac{1}{x})$.

649 $(\forall x \in T_{ef})(w_1(Dx) = e+f - w_1(x)$ & $w_d(Dx) = e+f - w_d(x))$.

650 $(\forall x,y \in T_{ef})(D(x + y) = Dx \mathbin{\hat{+}} Dy$ & $D(x \mathbin{\hat{+}} y) = Dx + Dy$.

DEFINICIJA 156

$(\forall x \in T_{ef})(R(x) = {}_{e,f}\nearrow^{\bar f,\bar e}(x))$.

D156,T640–T642,T568,D139 $\succ\!\!-\!\!-$

TEOREMI 651–654

651 $R \in S_{bijekc}(T_{ef}, T_{\bar f,\bar e})$.

652 $(\forall x \in T_{ef})(R(x) = \frac{1}{x})$.

653 $(\forall x \in T_{ef})(w_1(Rx) = -w_1(x) \quad \& \quad w_d(Rx) = -w_d(x))$.

654 $(\forall x,y \in T_{ef})(R(x + y) = Rx \mathbin{\hat{+}} Ry \quad \& \quad R(x \mathbin{\hat{+}} y) = Rx + Ry)$.

DEFINICIJA 157

$(\forall x \in T_{ef})(\bar R(x) = {}_{e,f}\swarrow^{\bar f,\bar e}(x))$.

D157,T636–T638,T568,D139 $\succ\!\!-\!\!-$

TEOREMI 655–657

655 $\bar R \in S_{bijekc}(T_{ef}, T_{\bar f,\bar e})$.

656 $(\forall x \in T_{ef})(\bar R(x) = s^{-(e+f)}x)$.

657 $(\forall x \in T_{ef})(w_1(\bar R x) = w_1(x) = (e+f) \quad \& \quad w_d(\bar R x) = w_d(x) - (e+f))$.

PRIMJER 359

$$\left(x \in T_{ef} \quad \& \quad \pi = f - e \quad \& \quad x = s^e \, \frac{A_o + A_1 s^{\pi} + A_2 s^{2\pi}}{B_o + B_1 s^{\pi}}\right) \longrightarrow$$

$$D(x) = s^f \frac{B_o + B_1 s^{\pi}}{A_o + A_1 s^{\pi} + A_2 s^{2\pi}} \quad \& \quad R(x) = s^{-e} \frac{B_o + B_1 s^{\pi}}{A_o + A_1 s^{\pi} + A_2 s^{2\pi}} \quad \&$$

$$\bar R(x) = s^{-f} \, \frac{A_o + A_1 s^{\pi} + A_2 s^{2\pi}}{B_o + B_1 s^{\pi}} \; .$$

3.3.3. ČETVORNA GRUPA PERMUTACIJA DVOGENERATORSKIH IMITANCIJA

DEFINICIJE 158-161

158 $P^{o}_{T} = \{(x,y):\ x\in T\ \ \&\ \ y = x\}$.

159 $D_{T} = \{(x,y):\ x\in T\ \ \&\ \ y = D(x)\}$.

160 $R_{T} = \{(x,y):\ x\in T\ \ \&\ \ y = R(x)\}$.

161 $\bar{R}_{T} = \{(x,y):\ x\in T\ \ \&\ \ y = \bar{R}(x)\}$.

DEFINICIJA 162

$\Gamma_{T} = \{P^{o}_{T},\ D_{T},\ R_{T},\ \bar{R}_{T}\}$.

D158-D162,D131,T647,T651,T655 $\succ\!\!-\!\!-$
TEOREM 658

$(\forall F\in \Gamma_{T})(F\in S_{bijekc}(T,T))$.

DEFINICIJA 163

$(\forall F_{1},F_{2}\in \Gamma_{T})(F_{1} \bigstar F_{2} = F_{1}F_{2} = \{(x,y):\ x\in T\ \ \&\ \ y = F_{1}(F_{2}(x))\})$.

D158-D163,T648,T652,T656 $\succ\!\!-\!\!-$
TEOREMI 659-663

659 $(\forall F\in \Gamma_{T})(FF = P^{o}_{T})$.

660 $(\forall F\in \Gamma_{T})(P^{o}_{T}\,F = F\ \ \&\ \ FP^{o}_{T} = F)$.

661 $D_{T}R_{T} = R_{T}D_{T} = \bar{R}_{T}$.

662 $D_{T}\bar{R}_{T} = \bar{R}_{T}D_{T} = R_{T}$.

663 $R_{T}\bar{R}_{T} = \bar{R}_{T}R_{T} = D_{T}$.

D162,D163,T659-T663 $\succ\!\!-\!\!-$
TEOREM 664

$(\Gamma_{T}\ ,\ \bigstar)\in S_{Klein\ gr.}$.

3.4. RASTAV IMITANCIJE U SERIJSKI I PARALELNI ZBROJ IREDUCIBILNIH IMITANCIJA

TEOREM 665

$$(s \in \Xi \;\&\; x = (\Lambda s)(F)) \longrightarrow$$

$$(x \in T_{ef} \longleftrightarrow (A_0, A_1, .. A_m, B_0, B_1, .. B_n \in Re_{poz} \quad \&$$

$$\left(x = (\Lambda s)\left(s^e \frac{\sum_{i=o}^{m} A_i s^{(f-e)i}}{\sum_{i=o}^{n} B_i s^{(f-e)i}}\right) \;\vee\; x = (\Lambda s)\left(s^f \frac{\sum_{i=o}^{n} B_i s^{(f-e)i}}{\sum_{i=o}^{m} A_i s^{(f-e)i}}\right)\right) \quad \&$$

$$\left(\sum_{i=o}^{m} A_i s^{2i} + s \sum_{i=o}^{n} B_i s^{2i}\right) \in S_{\text{Hurwitz p.}})).$$

D o k a z

(1) $A_o, ... A_m, B_o, ... B_n \in Re_{poz}$ &

$$\left(\sum_{i=o}^{m} A_i s^{2i} + s \sum_{i=o}^{n} B_i s^{2i}\right) \in S_{\text{Hurwitz p.}} \quad \&$$

$$\left(F(s) = \frac{\sum_{i=o}^{m} A_i s^{2i}}{s \sum_{i=o}^{n} B_i s^{2i}} \quad \vee \quad F(s) = s \frac{\sum_{i=o}^{n} B_i s^{2i}}{\sum_{i=o}^{m} A_i s^{2i}}\right) \qquad \text{Sup.}$$

1,(1 u demonstr. T645),D127,T576 >——

(2) $F(s) = \sum_{i=o}^{n} x_i \;\&\; (\forall i)(x_i \in \Omega_{\bar{1},1} \;\vee\; x_i \in (BA)_{\bar{1},1})$

2,D127,D135,T566,T567 >——

(3) $F(x) \in T_{\bar{1},1}$

1,3,T636,T645,T583,D137 >—— T665.

DEFINICIJA 164

$$\Omega_{s(e,f)} = \{x:\; x \in T_{ef} \;\&\; (\exists y, z \in T_{ef})(x = y + z \longrightarrow$$

$$\neg(\lambda_s(y) < \lambda_s(x)) \;\&\; \neg(\lambda_s(z) < \lambda_s(x)))\}.$$

D164,T665,D133,D134,T576 $>\!\!-\!\!-$

TEOREM 666

$\Omega_{s(e,f)} = A_{ef} \cup B_{ef} \cup (BA)_{ef}.$

T666,T662,T623 $>\!\!-\!\!-$

TEOREM 667

$(\forall x \in T_{ef})(x \in \Omega_{s(e,f)} \longleftrightarrow \lambda_p(x) = 0).$

DEFINICIJA 165

$$\Omega_{p(e,f)} = \{x:\ x \in T_{ef}\ \&\ (\exists y,z \in T_{ef})(x = y \hat{+} z \longrightarrow \neg(\lambda_p(y) < \lambda_p(x))\ \&\ \neg(\lambda_p(z) < \lambda_p(x)))\}.$$

D165,T665,D133,D134,T577 $>\!\!-\!\!-$

TEOREM 668

$\Omega_{p(e,f)} = A_{ef} \cup B_{ef} \cup (AB)_{ef}.$

T668,T622,T624 $>\!\!-\!\!-$

TEOREM 669

$(\forall x \in T_{ef})(x \in \Omega_{p(e,f)} \longleftrightarrow \lambda_s(x) = 0).$

D164,D165,T666,T668,T629,T630,D148,D146,T622-T624,T611-T613 $>\!\!-\!\!-$

TEOREMI 670-671

670 $(\forall x \in T_{ef})(\lambda(x) = n \longrightarrow$ egzistira točno jedna n-torka pozitivnih realnih brojeva $(k_1, A_1, A_2, .., B_1, B_2, .., k_2)$, takvih da je

$$x = \frac{f-w_1(x)}{f-e} k_1 s^e + s^f \left(\bigvee_{i=1}^{\lambda_s(x)} \frac{A_i}{B_i + s^{f-e}} \right) + \frac{w_d(x)-e}{f-e} k_2 s^f\ ).$$

671 $(\forall x \in T_{ef})(\lambda(x) = n \longrightarrow$ egzistira točno jedna n-torka pozitivnih realnih brojeva $(k_1, A_1, A_2, .., B_1, B_2, .., k_2)$, takvih da je

$$x = \frac{w_1(x)-e}{f-e} k_1 s^f \hat{+} s^e \left(\bigwedge_{i=1}^{\lambda_p(x)} \frac{A_i + s^{f-e}}{B_i} \right) \hat{+} \frac{f-w_d(x)}{f-e} k_2 s^e\ ).$$

PRIMJERI

360 $x = s^{-0.5}\frac{16 + 30s^{1.5} + 8s^3}{8 + 6s^{1.5} + s^3} \longrightarrow (x \in T_{-0.5,1}$ &

$w_1(x) = -0.5$ & $w_d(x) = 0.5$ & $\lambda(x) = 5$ & $\lambda_s(x) = 2$ &

$x = 2s^{-0.5} + s\frac{3}{4 + s^{1.5}} + s\frac{3}{2 + s^{1.5}}$).

361 $x = s^2\frac{6 + 6s^3}{2 + 27s^3 + 5s^6} \longrightarrow (x \in T_{-1,2}$ &

$w_1(x) = 2$ & $w_d(x) = -1$ & $\lambda(x) = 4$ & $\lambda_p(x) = 1$ &

$x = 2s^2 \hat{+} s^{-1}\frac{1 + s^3}{4} \hat{+} 3s^{-1}$).

3.5. MATRICA IMITANCIJE

DEFINICIJA 166

$$(\forall x \in T_{ef})(\mu(x) = \begin{bmatrix} C_0 & C_2 & C_4 & C_6 & \cdots & & C_{2n-4} & C_{2n-2} \\ C_{-1} & C_1 & C_3 & C_5 & \cdots & & C_{2n-5} & C_{2n-3} \\ C_{-2} & C_0 & C_2 & C_4 & \cdots & & C_{2n-6} & C_{2n-4} \\ C_{-3} & C_{-1} & C_1 & C_3 & \cdots & & C_{2n-7} & C_{2n-5} \\ \cdot & \cdot & \cdot & \cdot & \cdots & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot\ \cdot & C_{n-4} & C_{n-2} & C_n \\ C_{-n+1} & \cdot & \cdot & \cdot & \cdot\ \cdot & C_{n-5} & C_{n-3} & C_{n-1} \end{bmatrix} \longleftrightarrow$$

$$(\Pi(x) = \sum_{i=0}^{n} C_i s^i \ \& \ ((i < 0 \ \vee \ i > n) \rightarrow C_i = 0))) .$$

D166,D151,(Def.13.9.1. Kurepa, Viša algebra II, str. 993) $\succ$——

TEOREM 672

$(\forall x \in T_{ef})(\mu(x)$ je matrica koja rezultira iz Hurwitzove matrice polinoma $\Pi(x)$, transpozicijom preko glavne i sporedne dijagonale) .

D139-D141,D145,D147,D151,D166,T672 $\succ$——

TEOREMI 673-676

673 $(\forall x \in T_{ef})(\mu(x)$ je kvadratna matrica reda jednakog $\lambda(x))$.

674 $(\forall x \in T_{ef})$(Svaka početna glavna subdeterminanta matrice $\mu(x)$ je pozitivna).

675 $(\forall x \in T_{ef})(\mathrm{Slog}(x) = (A_1^m, B_1^n) \longrightarrow$
((glavna dijagonala od $\mu(x)$ počinje sa $A_o \longleftrightarrow l(x) = a$) &
(glavna dijagonala od $\mu(x)$ počinje sa $B_o \longleftrightarrow l(x) = b$))).

676 $(\forall x \in T_{ef})(\mathrm{Slog}(x) = (A_1^m, B_1^n) \longrightarrow$
((glavna dijagonala od $\mu(x)$ svršava sa $A_m \longleftrightarrow d(x) = a$) &
(glavna dijagonala od $\mu(x)$ svršava sa $B_n \longleftrightarrow d(x) = b$))).

PRIMJERI

$(x \in T_{ef} \;\&\; \pi = f-e) \longrightarrow$

362 $\mu(s^e \frac{A_o}{1}) = \begin{bmatrix} A_o \end{bmatrix}$. 363 $\mu(s^f \frac{1}{B_o}) = \begin{bmatrix} B_o \end{bmatrix}$.

364 $\mu(s^e \frac{A_o + s^\pi}{B_o}) = \begin{bmatrix} A_o & 1 \\ 0 & B_o \end{bmatrix}$. 365 $\mu(s^f \frac{A_o}{B_o + s^\pi}) = \begin{bmatrix} B_o & 1 \\ 0 & A_o \end{bmatrix}$.

366 $\mu(s^e \frac{A_o + A_1 s^\pi}{B_o + s^\pi}) = \begin{bmatrix} A_o & A_1 & 0 \\ 0 & B_o & 1 \\ 0 & A_o & A_1 \end{bmatrix}$. 367 $\mu(s^f \frac{A_o + s^\pi}{B_o + B_1 s^\pi}) = \begin{bmatrix} B_o & B_1 & 0 \\ 0 & A_o & 1 \\ 0 & B_o & B_1 \end{bmatrix}$.

368 $\mu(s^e \frac{A_o + A_1 s^\pi + s^{2\pi}}{B_o + B_1 s^\pi}) = \begin{bmatrix} A_o & A_1 & 1 & 0 \\ 0 & B_o & B_1 & 0 \\ 0 & A_o & A_1 & 1 \\ 0 & 0 & B_o & B_1 \end{bmatrix}$.

369 $\mu(s^f \frac{A_o + A_1 s^\pi}{B_o + B_1 s^\pi + s^{2\pi}}) = \begin{bmatrix} B_o & B_1 & 1 & 0 \\ 0 & A_o & A_1 & 0 \\ 0 & B_o & B_1 & 1 \\ 0 & 0 & A_o & A_1 \end{bmatrix}$.

3.6. OPERATORI IZLJUŠTENJA l_t i d_t

DEFINICIJA 167

$$(\forall x\in T_{ef})((l_t(x) = k_1 s^{w_1(x)} \ \& \ d_t(x) = k_2 s^{w_2(x)}) \longleftrightarrow$$

$$x = s^u \frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} \ \& \ k_1 = \frac{A_o}{B_o} \ \& \ k_2 = \frac{A_m}{B_n}).$$

PRIMJER 370

(1) $x = s^2 \dfrac{6 + 6s^3}{2 + 27s^3 + 5s^6}$

1,Pr.361,D167 $\rightarrowtail$

(2) $l_t(x) = 3s^2 \ \& \ d_t(x) = \frac{6}{5} s^{-1}$.

D167,T585,T586,D127 $\rightarrowtail$

TEOREM 677

$(\forall x\in\Omega_{ef})(l_t(x) = d_t(x) = x)$.

D167,D127,T585,T586,T677 $\rightarrowtail$ T678,T679.

TEOREM 678

$l_t\in S_{surjekc}(T_{ef},\Omega_{ef})$.

TEOREM 679

$d_t\in S_{surjekc}(T_{ef},\Omega_{ef})$.

D167,D133,D134,T583 $\rightarrowtail$ T680-T683.

TEOREM 680

$(\forall x\in T_{ef})(l(x) = a \ \rightarrow \ (\forall u\in B_{ef})(l_t(u \hat{+} x) = u))$.

TEOREM 681

$(\forall x\in T_{ef})(l(x) = b \ \rightarrow \ (\forall u\in A_{ef})(l_t(u + x) = u))$.

TEOREM 682

$(\forall x\in T_{ef})(d(x) = a \ \rightarrow \ (\forall u\in B_{ef})(d_t(u + x) = u))$.

TEOREM 683

$(\forall x\in T_{ef})(d(x) = b \ \rightarrow \ (\forall u\in A_{ef})(d_t(x \hat{+} u) = u))$.

3.7. PRETHODNICI NEGENERATORSKIH IMITANCIJA

TEOREM 684

$$(\forall x\in T_{ef}\setminus\Omega_{ef})(\exists! y\in T_{ef})((l(x) = a \;\&\; x = l_t(x) + y) \;\vee\; (l(x) = b \;\&\; x = l_t(x) \hat{+} y)) .$$

D o k a z

(1) $x\in(T_{ef}\setminus\Omega_{ef}) \;\&\; \Pi(x) = \sum_{i=o}^{m} A_i s^{2i} + s\sum_{i=o}^{n} B_i s^{2i}$ Sup.

1,D166 $\succ\!\!-$

(2) $$\begin{bmatrix} A_o & A_1 & A_2 & A_3 & \cdot & A_{i+1} & \cdot \\ 0 & B_o & B_1 & B_2 & \cdot & B_i & \cdot \\ 0 & 0 & A_1-cB_1 & A_2-cB_2 & \cdot & A_i-cB_i & \cdot \\ 0 & 0 & B_o & B_1 & \cdot & B_{i-1} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} = X \longrightarrow$$

X je matrica koja rezultira iz matrice $\mu(x)$, kad se ovoj svaki parni redak pomnoži sa $c = \frac{A_o}{B_o}$ i doda sa suprotnim predznakom retku ispod njega

(3) Za svaku matricu Z, $\left|Z\binom{1,2,..,n}{1,2,..,n}\right|$ je determinanta početnog glavnog minora n-tog reda matrice Z. Def.

1,2,3,T573 $\succ\!\!-$

(4) $0 < r \leq \lambda(x) \longrightarrow$

$\left|X\binom{1,2,..,r+1}{1,2,..,r+1}\right| = A_o \left|Y\binom{1,2,..,r}{1,2,..,r}\right|$ & Y je matrica koja rezultira iz X, kad se ovoj ispusti prvi redak i prvi stupac.

4,3,2,T674 $\succ\!\!-\!\!-$

$$(5)\ (\sum_{i=o}^{n} B_i s^{2i} + s \sum_{i=o}^{m} (A_i - \frac{A_o}{B_o} B_i) s^{2i}) \in S_{Hurwitz\ p.}$$

5,T665 $\succ\!\!-\!\!-$

$$(6)(y_1 = s^f \frac{\underset{i=o}{\overset{m}{V}} (A_i - \frac{A_o}{B_o} B_i) s^{(f-e)i}}{\underset{i=o}{\overset{n}{V}} B_i s^{(f-e)i}} \quad \&$$

$$y_2 = s^e \frac{\underset{i=o}{\overset{n}{V}} B_i s^{(f-e)i}}{\underset{i=o}{\overset{m}{V}} (A_i - \frac{A_o}{B_o} B_i) s^{(f-e)i}}) \longrightarrow y_1 \in T_{ef} \ \& \ y_2 \in T_{ef} \ .$$

1,6,T569,D139,T587,D151 $\succ\!\!-\!\!-$

$$(7)\ (w_1(x) = e \ \& \ x = \frac{A_o}{B_o} s^e + y_1) \quad v$$
$$(w_1(x) = f \ \& \ x = \frac{A_o}{B_o} s^f \hat{+} y_2) \ .$$

1,6,7,D140,D167 $\succ\!\!-\!\!-$

$$(8)(\forall x \in T_{ef} \setminus \Omega_{ef})(\exists y \in T_{ef})((l(x) = a \ \& \ x = l_t(x) + y) \quad v$$
$$(l(x) = b \ \& \ x = l_t(x) \hat{+} y)) \ .$$

8,T678 $\succ\!\!-\!\!-$ T684.

D167,D166,T674,T665,T569,D139,T588,D141,T679 (analog T684) $\succ\!\!-\!\!-$

TEOREM 685

$$(\forall x \in T_{ef} \setminus \Omega_{ef})(\exists ! y \in T_{ef})((d(x) = a \ \& \ x = y \hat{+} d_t(x)) \quad v$$
$$(d(x) = b \ \& \ x = y + d_t(x))) \ .$$

DEFINICIJA 168

$(\forall x \in (T_{ef} \setminus \Omega_{ef}))(P_1^{-1}(x) = P_1'(x) = y \longleftrightarrow$ y je jedinstveno određena (e,f)-imitancija iz teorema 684).

DEFINICIJA 169

$(\forall x \in (T_{ef} \setminus \Omega_{ef}))(P_d^{-1}(x) = P_d'(x) = y \longleftrightarrow$ y je jedinstveno određena (e,f)-imitancija iz teorema 685).

T684,T685,D168,D169 $\succ\!\!-\!\!-$

TEOREMI 686-689

686 $(\forall x \in T_{ef} \setminus \Omega_{ef})(l(x) = a \;\rightarrow\; x = l_t(x) + P_l'(x))$.

687 $(\forall x \in T_{ef} \setminus \Omega_{ef})(l(x) = b \;\rightarrow\; x = l_t(x) \overset{\frown}{+} P_l'(x))$.

688 $(\forall x \in T_{ef} \setminus \Omega_{ef})(d(x) = a \;\rightarrow\; x = P_d'(x) \overset{\frown}{+} d_t(x))$.

689 $(\forall x \in T_{ef} \setminus \Omega_{ef})(d(x) = b \;\rightarrow\; x = P_d'(x) + d_t(x))$.

PRIMJER 371

(1) $x = s^{-0.5}\dfrac{16 + 30s^{1.5} + 8s^3}{8 + 6s^{1.5} + s^3}$ Sup.

1,Pr.360,D167,D140,D141 $\succ\!\!-\!\!-$

(2) $l_t(x) = 2s^{-0.5}$ & $d_t(x) = 8s^{-0.5}$ & $l(x) = d(x) = a$.

1,2,T686,T688

(3) $P_l'(x) = x - 2s^{-0.5} = s\dfrac{18 + 6s^{1.5}}{8 + 6s^{1.5} + s^3}$ &

$$P_d'(x) = x \overset{\frown}{-} 8s^{-0.5} = \frac{-\,8xs^{-0.5}}{x - 8s^{-0.5}} =$$

$$= s^{-0.5}\frac{16 + 30s^{1.5} + 8s^3}{6 + 2.25s^{1.5}} \quad .$$

TEOREM 690

$P_l' \in S_{surjekc}(T_{ef} \setminus \Omega_{ef},\, T_{ef})$.

D o k a z

(1) $P_l' \in S_{funkc}(T_{ef} \setminus \Omega_{ef}, T_{ef})$. D168,T684

(2) $x \in T_{ef}$ Sup.

(3) $l(x) = a \;\vee\; l(x) = b$ 2,T595

(4) $(\exists u \in B_{ef})(l_t(u \overset{\frown}{+} x) = u \;\&\; (u \overset{\frown}{+} x) \in T_{ef} \setminus \Omega_{ef}) \;\vee$
$(\exists w \in A_{ef})(l_t(w + x) = w \;\&\; (w + x) \in T_{ef} \setminus \Omega_{ef})$ 3,T680,T681

(5) $u + x = w + P_l'(u \overset{\frown}{+} x) \;\vee\; w + x = w + P_l'(w + x)$ 4,T686,T687

(6) $x \in T_{ef} \;\rightarrow\; (\exists u \in B_{ef})(x = P_l'(u \overset{\frown}{+} x)) \;\vee$
$(\exists w \in A_{ef})(x = P_l'(w + x))$

6,T567,1 $\succ\!\!-\!\!-$ T690.

T685,D169,T596,T682,T683,T688,T689,T567 (analog T690) $\succ\!\!-$

TEOREM 691

$P'_d \in S_{surjekc}(T_{ef} \setminus \Omega_{ef}, T_{ef})$.

TEOREM 692

$(\forall x \in T_{ef} \setminus \Omega_{ef})(l(P'_l(x)) \neq l(x) \;\; \& \;\; d(P'_d(x)) = d(x) \;\; \&$
$\lambda(P'_l(x)) = \lambda(x) - 1)$.

D o k a z

(1) $x \in T_{ef} \setminus \Omega_{ef} \;\; \& \;\; Slog(x) = (A_i^m, B_i^n) \;\; \& \;\; w_l(x) = e$ Sup.

1,T686,D167 $\succ\!\!-$

$$(2)\; P'_l(x) = s^e \frac{\bigvee_{i=o}^{m} A_i s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}} - \frac{A_o}{B_o} s^e$$

2 $\succ\!\!-$

$$(3)\; P'_l(x) = s^f \frac{\bigvee_{i=1}^{m} (A_i - \frac{A_o}{B_o} B_i) s^{(f-e)i}}{\bigvee_{i=o}^{n} B_i s^{(f-e)i}}$$

3,D139,D147 $\succ\!\!-$

(4) $w_l(P'_l(x)) = f \;\; \& \;\; w_d(P'_l(x)) = w_d(x) \;\; \&$
$\lambda\,(P'_l(x)) = \lambda(x) - 1$

1,4,D140,D141 $\succ\!\!-$

(5)$(x \in T_{ef} \setminus \Omega_{ef} \;\; \& \;\; l(x) = a) \;\; \rightarrow \;\; l(P'_l(x)) = b \;\; \&$
$\lambda(P'_l(x)) = \lambda(x) - 1$.

T687,D167,D139-D141,D147 $\succ\!\!-$

(6)$(x \in T_{ef} \setminus \Omega_{ef} \;\; \& \;\; l(x) = b) \;\; \rightarrow \;\; l(P'_l(x)) = a \;\; \&$
$\lambda(P'_l(x)) = \lambda(x) - 1$.

5,6,T595 $\succ\!\!-$ T692.

D139-D141,D147,D167,T688,T689,T596 (analog T692) $\succ\!\!-$

TEOREM 693

$(\forall x \in T_{ef} \setminus \Omega_{ef})(l(P'_l(x)) = l(x) \;\; \& \;\; d(P'_d(x)) \neq d(x) \;\; \&$
$\lambda(P'_l(x)) = \lambda(x) - 1)$.

DEFINICIJE 170-171

170 $(\forall x\in T_{ef})(P_l^0(x) = x \;\;\&\;\; (\forall n\in N^+)(n < \lambda(x) \;\rightarrow$
$P_l^{-(n+1)}(x) = P_l^{!}P_l^{-n}(x))).$

171 $(\forall x\in T_{ef})(P_d^0(x) = x \;\;\&\;\; (\forall n\in N^+)(n < \lambda(x) \;\rightarrow$
$P_d^{-(n+1)}(x) = P_d^{!}P_d^{-n}(x))).$

D127,D146,D170,D171,T585,T586,T620,T625,T690-T693 $\rightarrowtail$
TEOREMI 694-695

694 $(\forall x\in T_{ef})(\exists! k\in Re_{poz})(P_l^{-\lambda_t(x)}(x) = ks^{w_d(x)}).$

695 $(\forall x\in T_{ef})(\exists! k\in Re_{poz})(P_d^{-\lambda_t(x)}(x) = ks^{w_l(x)}).$

PRIMJER 372

(1) $x = s^e\,\dfrac{1 + 5s^\pi + 6s^{2\pi} + s^{3\pi}}{1 + 4s^\pi + 3s^{2\pi}}$ $\;\;\&\;\; e,f\in Re \;\;\&\;\; \pi = f - e$ Sup.

1,D139-D141,D145,D146 $\rightarrowtail$

(2) $w_l(x) = e \;\;\&\;\; w_d(x) = f \;\;\&\;\; l(x) = a \;\;\&\;\; d(x) = b \;\;\&$
$\mathrm{Slog}(x) = (1,5,6,1,1,4,3) \;\;\&\;\; \lambda_t(x) = 5$

1,2,D170,D171,T686-T689,D167 $\rightarrowtail$ (3),(4).

(3) $$P_l^{-5}(x) = P_l^{-4}\Big(s^f\,\frac{1 + 3s^\pi + s^{2\pi}}{1 + 4s^\pi + 3s^{2\pi}}\Big) = P_l^{-3}\Big(s^e\,\frac{1 + 3s^\pi + s^{2\pi}}{1 + 2s^\pi}\Big) =$$
$$= P_l^{-2}\Big(s^f\,\frac{1 + s^\pi}{1 + 2s^\pi}\Big) \qquad = P_l^{-1}\Big(s^e\Big(\frac{1 + s^\pi}{1}\Big)\Big) = s^f.$$

(4) $$P_d^{-5}(x) = P_d^{-4}\Big(s^e\,\frac{3 + 14s + 14s^2}{3 + 12s + 9s^2}\Big) = P_d^{-3}\Big(s^e\,\frac{42 + 196s + 196s^2}{15 + 42s}\Big) =$$
$$= P_d^{-2}\Big(s^e\,\frac{14 + 42s}{5 + 14s}\Big) \qquad = P_d^{-1}\Big(s^e\,\frac{42 + 126s}{1}\Big) = 42s^e.$$

3.8. RASTAV IMITANCIJE U ALTERNIRANU SUMU GENERATORA

T686-T691,T694,T695 $\succ\!\!\!-\!\!-$ T696,T697.

TEOREM 696

$(\forall x \in T_{ef})(\lambda(x) = n \longrightarrow$ egzistira točno jedna n-torka pozitivnih realnih brojeva $(G,H,I,J,K,L,\ldots)$, takvih da je

$$x = \frac{f-w_1(x)}{f-e} Gs^e + (Hs^f \overset{\frown}{+} (Is^e + (Js^f \overset{\frown}{+} \ldots \overset{\frown}{+} (Ks^e + \frac{w_d(x)-e}{f-e} Ls^f) \cdots)).$$

TEOREM 697

$(\forall x \in T_{ef})(\lambda(x) = n \longrightarrow$ egzistira točno jedna n-torka pozitivnih realnih brojeva $(G,H,I,J,K,L,\ldots)$, takvih da je

$$x = (\cdots (\frac{f-w_1(x)}{f-e} Gs^e + Hs^f) \overset{\frown}{+} Is^e) + Js^f) \overset{\frown}{+} \ldots \overset{\frown}{+} Ks^e) + \frac{w_d(x)-e}{f-e} Ls^f).$$

PRIMJER 373

$$(x = s^e \frac{1 + 5s^{\varkappa} + 6s^{2\varkappa} + s^{3\varkappa}}{1 + 4s^{\varkappa} + 3s^{2\varkappa}} \quad \& \quad e,f \in Re \quad \& \quad \varkappa = f - e) \quad \longrightarrow$$

$$x = s^e + (s^f \overset{\frown}{+} (s^e + (s^f \overset{\frown}{+} (s^e + s^f)))) \qquad \&$$

$$x = ((((42s^e + 126s^f) \overset{\frown}{+} 3s^e) + \frac{14}{3} s^f) \overset{\frown}{+} \frac{14}{9} s^e) + \frac{1}{3} s^f .$$

3.9. ALGEBRA KLASA OSTATAKA MODULO NULA DVOGENERATORKSIH IMITANCIJA KAO MODEL m_t-STRUKTURE

DEFINICIJA 172

$(\forall x,y\in T_{ef})(x\ \theta\ y \longleftrightarrow q(x) = q(y)\ \&\ \lambda_s(x) = \lambda_s(y)).$

D172,T617,T628,T608 $\succ\!\!-$

TEOREM 698

$(\forall x,y\in T_{ef})(x\ \theta\ y \longleftrightarrow q(x) = q(y)\ \&\ \lambda_p(x) = \lambda_p(y)).$

D172 $\succ\!\!-$

TEOREMI 699-701

699 $(\forall x\in T_{ef})(x\ \theta\ x).$

700 $(\forall x,y\in T_{ef})(x\ \theta\ y \rightarrow y\ \theta\ x).$

701 $(\forall x,y,z\in T_{ef})(x\ \theta\ y\ \&\ y\ \theta\ z \rightarrow x\ \theta\ z).$

TEOREM 702

$(\forall x,y,z,u\in T_{ef})((x\ \theta\ z\ \&\ y\ \theta\ u\ \&\ El_s(x,y) \nparallel 0\ \&$
$El_s(z,u) \nparallel 0 \rightarrow (x + y\ \theta\ z + u)).$

D o k a z

(1)	$x,y,z,u\in T_{ef}\ \&\ x\ \theta\ z\ \&\ y\ \theta\ u\ \&$ $El_s(x,y) \nparallel 0\ \&\ El_s(z,u) \nparallel 0)$	Sup.
(2)	$w_1(x + y) = \inf(w_1(z),w_1(u)) = w_1(z + u)\ \&$ $w_d(x + y) = \sup(w_d(z),w_d(u)) = w_d(z + u)$.	1,D172,T589,T591
(3)	$q(x + y) = q(z + u)$	2,D140-D142
(4)	$\lambda_s(x + y) = \lambda_s(x) + \lambda_s(y)$	1,T629
(5)	$= \lambda_s(z) + \lambda_s(u)$	4,1,D172
(6)	$= \lambda_s(z + u)$	6,1,T629

1,3,6,D172 $\succ\!\!-$ T702.

T702 $\succ\!\!-\!\!\prec$

TEOREM 703

$(\forall x,y,z,u\in T_{ef})((x\ \theta\ z\ \&\ y\ \theta\ u\ \&\ El_p(x,y) \nparallel 0\ \&$
$El_p(z,u) \nparallel 0) \rightarrow (x \hat{+} y\ \theta\ z \hat{+} u)).$

DEFINICIJA 173

$(\forall x \in T_{ef})(|x|_o = \{y:\ y \in T_{ef}\ \ \&\ \ y\ \theta\ x\})$.

DEFINICIJA 174

$T_{ef}(\theta) = \{z:\ x \in T_{ef}\ \ \&\ \ z = |x|_o\}$.

D173,D133,T597,T622 >——

TEOREM 704

$(\forall k \in Re_{poz})(A_{ef} = |ks^e|_o)$.

T704 >——<

TEOREM 705

$(\forall k \in Re_{poz})(B_{ef} = |ks^f|_o)$.

DEFINICIJA 175-177

175 $(\forall x \in T_{ef})(\forall F \in \{w_l, w_d, l, d, q, w, h, \lambda, \lambda_t, \lambda_p, \lambda_s\})(F|x|_o) = F(x))$.

176 $(\forall x \in T_{ef})(\forall F \in \{D, R, \bar{R}, l_t, d_t\})(F|x|_o = |F(x)|_o)$.

177 $(\forall x \in T_{ef})(\forall n \in N^+)(n < \lambda(x) \rightarrow (P_l^{-n}(|x|_o) = |P_l^{-n}(x)|_o\ \ \&$

$P_d^{-n}(|x|_o) = |P_d^{-n}(x)|_o))$.

D174,D149,T670 >——

TEOREM 706

$(\forall X, Y \in T_{ef}(\theta))(\exists x \in X)(\exists y \in Y)((El_s(x,y) \neq 0))$.

DEFINICIJA 178

$(\forall X, Y \in T_{ef}(\theta))(X \cup Y = Z \leftrightarrow x \in X\ \ \&\ \ y \in Y\ \ \&\ \ El_s(x,y) \neq 0\ \ \&$

$Z = |x + y|_o)$.

D174,D150,T671 >——

TEOREM 707

$(\forall X, Y \in T_{ef}(\theta))(\exists x \in X)(\exists y \in Y)((El_p(x,y) \neq 0))$.

DEFINCIJA 179

$(\forall X, Y \in T_{ef}(\theta))(X \cap Y = Z \leftrightarrow x \in X\ \ \&\ \ y \in Y\ \ \&\ \ El_p(x,y) \neq 0\ \ \&$

$Z = |x \hat{+} y|_o)$.

T704,T705,D174 $\succ\!\!-$

TEOREM 708

$\{A_{ef},B_{ef}\} \subseteq T_{ef}(\theta)$.

D178,D179,T702,T703,T706,T707,T567 $\succ\!\!-$

TEOREM 709

$X,Y\in T_{ef}(\theta) \rightarrow (X \cup Y)\in T_{ef}(\theta) \;\&\; (X \cap Y)\in T_{ef}(\theta)$.

TEOREM 710

$(\forall G\in S)((\{A_{ef},B_{ef}\} \subseteq G \;\&\; G \subseteq T_{ef}(\theta) \;\&$

$(X,Y\in G \rightarrow (X \cup Y)\in G \;\&\; (X \cap Y)\in G)) \longrightarrow G = T_{ef}(\theta)))$.

D o k a z

(1) $G\in S \;\&\; G \subseteq T_{ef}(\theta) \;\&\; A_{ef}\in G \;\&\; B_{ef}\in G \;\&$

$(X,Y\in G \rightarrow (X \cup Y)\in G \;\&\; (X \cap Y)\in G)$ Sup.

(2) $(\exists Z\in T_{ef}(\theta))(\neg Z\in G)$ Sup.

2,1,D177,T694,T695 $\succ\!\!-$

(3) $\neg(P^{-\lambda_t(Z)}(Z)\in G)$

3,T694,T695,T704,T705 $\succ\!\!-$

(4) $\neg A_{ef}\in G \;\vee\; \neg B_{ef}\in G$

1,2,4 $\succ\!\!-$

(5) $T_{ef}(\theta) \subseteq G$

1,5 $\succ\!\!-$ T710.

D178,D179,T570-T573,T575 $\succ\!\!-$

TEOREMI 711-714

711 $(\forall X,Y\in T_{ef}(\theta))(X \cup Y = Y \cup X \;\&\; X \cap Y = Y \cap X)$.

712 $(\forall X,Y,Z\in T_{ef}(\theta))(X \cup (Y \cup Z) = (X \cup Y) \cup Z \;\&$

$X \cap (Y \cap Z) = (X \cap Y) \cap Z)$.

713 $\neg(A_{ef} = B_{ef})$.

714 $(\forall X\in\{A_{ef},B_{ef}\})(X \cup X = X \;\&\; X \cap X = X)$.

TEOREM 715

$(\forall X\in T_{ef}(\theta))(X \cup (B_{ef} \cap A_{ef}) = X \cap (A_{ef} \cup B_{ef}))$.

D o k a z

(1) $X\in T_{ef}(\theta)$ & $X \cup (B_{ef} \cap A_{ef}) = Y$ &

$X \cap (A_{ef} \cup B_{ef}) = Z$ Sup.

1,D175-D179,T629,T630,T623,T624,T589-T592 $\succ\!\!-$

(2) $\lambda(Y) = \lambda(X) + 2$ & $l(Y) = l(X)$ & $d(Y) = d(X)$ &

$\lambda(Z) = \lambda(X) + 2$ & $l(Z) = l(X)$ & $d(Z) = d(X)$

2,D172,D173,D146-D148 $\succ\!\!-$

(3) $Y = Z$

1,3 $\succ\!\!-$ T715.

D178,D179,T589-T592,T623,T624,T629,T630 $\succ\!\!-$

TEOREM 716

$(\forall X,Y\in T_{ef}(\theta))((X \cup (B_{ef} \cap A_{ef}) = Y \cap (A_{ef} \cup B_{ef})) \rightarrow X = Y)$.

TEOREM 717

$X\in T_{ef}(\theta) \rightarrow \neg(X \cup (B_{ef} \cap A_{ef}))\in\{A_{ef},B_{ef},A_{ef} \cup B_{ef},B_{ef} \cap A_{ef}\}$.

D o k a z

(1) $X\in T_{ef}(\theta)$ &

$(X \cup (B_{ef} \cap A_{ef}))\in\{A_{ef},B_{ef},A_{ef} \cup B_{ef},B_{ef} \cap A_{ef}\})$ Sup.

1,D175-D179,T618,T619,D147,T629 $\succ\!\!-$

(2) $\lambda(X \cup (B_{ef} \cap A_{ef})) \geq 3$ &

$(Y\in\{A_{ef},B_{ef},A_{ef} \cup B_{ef},B_{ef} \cap A_{ef}\} \rightarrow \lambda(Y) \leq 2)$

1,2 $\succ\!\!-$ T717.

A3,T484-T486,T708-T717 $\succ\!\!-$

TEOREM 718

$(e,f\in Re$ & $e < f) \rightarrow (\exists!F\in S)(F\in S_{bijekc}(T_{ef}(\theta),M_t)$ &

$F(A_{ef}) = A$ & $F(B_{ef}) = B$ & $(X,Y\in T_{ef}(\theta)) \longrightarrow$

$F(X \cup Y) = F(X) \cup F(Y)$ & $F(X \cap Y) = F(X) \cap F(Y)))$.

3.10. I M E I M I T A N C I J E

DEFINICIJA 180

$(\forall x \in T_{ef})(\forall y \in M)(\vartheta(x) = y \;\longleftrightarrow\; l(x) = l(y) \;\&\; \lambda(x) = \lambda(y)).$

PRIMJERI

$(x \in T_{ef} \;\&\; \pi = f - e \;\&\; El(\sum_{i=o}^{m} A_i s^i, \sum_{i=o}^{n} B_i s^i) \nmid 0) \longrightarrow$

374 $\vartheta(s^e A_o) = A.$

375 $\vartheta(s^f \frac{1}{B_o}) = B.$

376 $\vartheta(s^e \frac{A_o + s^\pi}{B_o}) = AB.$

377 $\vartheta(s^f \frac{A_o}{B_o + s^\pi}) = BA.$

378 $\vartheta(s^e \frac{A_o + A_1 s^\pi}{B_o + s^\pi}) = ABA.$

379 $\vartheta(s^f \frac{A_o + s^\pi}{B_o + B_1 s^\pi}) = BAB.$

380 $\vartheta(s^e \frac{A_o + A_1 s^\pi + s^{2\pi}}{B_o + B_1 s^\pi}) = ABAB.$

381 $\vartheta(s^f \frac{A_o + A_1 s^\pi}{B_o + B_1 s^\pi + s^{2\pi}}) = BABA.$

D180,D172,T286,T287 $\succ\!\!-\!\!-$

TEOREM 719

$(\forall x, y \in T_{ef})(\vartheta(x) = \vartheta(y) \;\longleftrightarrow\; x \;\Theta\; y)$

D180,D147,D151,D155-D157,T643,T644,T647-T649,T651-T653, T655-T657,T90,T290 $\succ\!\!-\!\!-$

TEOREMI 720-722

720 $(\forall x \in T_{ef})(\vartheta(D(x)) = D\vartheta(x)).$

721 $(\forall x \in T_{ef})(\vartheta(R(x)) = D\vartheta(x)).$

722 $(\forall x \in T_{ef})(\vartheta(\bar{R}(x)) = \vartheta(x)).$

DEFINICIJA 181

$(\forall x \in T_{ef})(\vartheta|x|_o) = \vartheta(x)).$

D181,D175-D179,T708-T719,T495 $\succ\!\!-$

TEOREMI 723-728

723 $\theta \in S_{bijekc}(T_{ef}(\theta), M_{poz})$.

724 $\theta(A_{ef}) = A \;\;\&\;\; \theta(B_{ef}) = B$.

725 $(\forall X,Y \in T_{ef}(\theta))(\theta(X \cup Y) = \theta(X) \cup \theta(Y) \;\;\&$

$\theta(X \cap Y) = \theta(X) \cap \theta(Y))$.

726 $(\forall X \in T_{ef}(\theta))(\forall F \in \{l,d,q,h,\lambda,\lambda_t,\lambda_p,\lambda_s\})(F(\theta(X)) = F(X))$.

727 $(\forall X \in T_{ef}(\theta))(\theta(D(X)) = D(\theta(X)))$.

728 $(\forall X \in T_{ef}(\theta))(\forall n \in N^+)(n < \lambda(X) \;\rightarrow\; \theta(P_l^{-n} X) = P_l^{-n}(\theta X) \;\;\&$

$\theta(P_d^{-n} X) = P_d^{-n}(\theta X))$.

DEFINICIJA 182

$(\forall x \in M_{poz})(\forall Y \in T_{ef}(\theta))(\theta_{ef}^{-1}(x) = Y \;\longleftrightarrow\; \theta(Y) = x)$.

D182,T724 $\succ\!\!-$

TEOREM 729

$\theta_{ef}^{-1}(A) = A_{ef} \quad\&\quad \theta_{ef}^{-1}(B) = B_{ef}$.

D182,D135,D136 $\succ\!\!-$

TEOREM 730

$\theta_{ef}^{-1}(BA) = (BA)_{ef} \;\;\&\;\; \theta_{ef}^{-1}(AB) = (AB)_{ef}$.

D178,D181,D182,T719,T670,T696,T706,T723-T725 $\succ\!\!-$

TEOREM 731

$(\forall x \in M_{poz})(x = x_1 \cup x_2 \;\longrightarrow$

$(\forall y \in \theta_{ef}^{-1}(x))(\exists y_1 \in \theta_{ef}^{-1}(x_1))(\exists y_2 \in \theta_{ef}^{-1}(x_2))(y = y_1 + y_2))$.

D179,D181,D182,T719,T671,T697,T707,T723-T725 $\succ\!\!-\!\!-$

TEOREM 732

$$(\forall x\in M_{poz})(x = x_1 \frown x_2 \longrightarrow (\forall y\in \theta_{ef}^{-1}(x))(\exists y_1\in \theta_{ef}^{-1}(x_1))(\exists y_2\in \theta_{ef}^{-1}(x_2))(y = y_1 \hat{+} y_2)).$$

PRIMJER 382

(1) $(s^f \dfrac{A_o + A_1 s^{\varkappa} + A_2 s^{2\varkappa}}{B_o + B_1 s^{\varkappa} + B_2 s^{2\varkappa}} \in T_{ef}(BABAB)$ & $\varkappa = f - e$ &

BABAB = (BA $\frown$ BA) $\smile$ BAB Sup.

1,T731,T732 $\succ\!\!-\!\!-$

(2) $(\exists k_1,k_2,k_3,k_4,k_5,k_6,k_7\in Re_{poz})(s^f \dfrac{A_o + A_1 s^{\varkappa} + A_2 s^{2\varkappa}}{B_o + B_1 s^{\varkappa} + B_2 s^{2\varkappa}} =$

$$= (s^f \frac{k_1}{k_2 + s^{\varkappa}} \hat{+} s^f \frac{k_3}{k_4 + s^{\varkappa}}) + s^f \frac{k_5 + s^{\varkappa}}{k_6 + k_7 s^{\varkappa}}).$$

Četvrti dio

m-TOPOLOGIJA

4.1. m-G R A F O V I

DEFINICIJA 183

Primitivni pojmovi m-topologije su dva beskonačna,disjunktna skupa nedefiniranih objekata, koji se zovu čvorišta, odnosno grane.

Simboli za nedefinirane pojmove m-topologije su:

(i) V za skup čvorišta,

(ii) E za skup grana.

D183 >——

TEOREM 733

$V \cap E = 0.$

DEFINICIJA 184

$G = \{(X_1,X_2,X_3,X_4)\colon X_1 \subseteq V \ \& \ \neg X_1 = 0 \ \& \ X_2 \subseteq E \ \&$

$X_3 \in S_{funkc}(X_2,\{\{i,k\}\colon i,k \in X_1\}) \ \& \ X_4 \in S_{funkc}(X_2, \Omega)\}.$

PRIMJER 383

(1) $v_1,v_2,v_3 \in V \ \& \ e_1,e_2,e_3 \in E$ Sup.

1,D184 >——

(2)$(\{v_1,v_2,v_3\}\{e_1,e_2,e_3\},\{(e_1,\{v_2,v_3\}),(e_2,\{v_2,v_3\}),$

$(e_3,\{v_3,v_3\})\},\{(e_1,A),(e_2,ab),e_3,ba)\}) \in G.$

DEFINICIJA 185

$(\forall x \in G)(V_x=X_1 \ \& \ E_x=X_2 \ \& \ I_x=X_3 \ \& \ \omega_x=X_4 \longleftrightarrow x = (X_1,X_2,X_3,X_4)).$

PRIMJER 384

(1) $x \in G$ & $x = (\{v_1,v_2\},\{e_1,e_2,e_3\},\{(e_1,\{v_1,v_2\}),(e_2,\{v_1,v_2\}),$
$(e_3,\{v_1,v_1\})\},\{(e_1,A),(e_2,ab),(e_3,ba)\})$. Sup.

1,D184,D185 $\succ\!\!-$

(2) $V_x = \{v_1,v_2\}$ & $E_x = \{e_1,e_2,e_3\}$ &
$I_x = \{(e_1,\{v_1,v_2\}),(e_2,\{v_1,v_2\}),(e_3,\{v_1,v_1\})\}$ &
$\omega_x = \{(e_1,A),(e_2,ab),(e_3,ba)\}$.

D184,D185 $\succ\!\!-$

TEOREMI 734-738

734 $(\forall x \in G)(V_x \subseteq V$ & $E_x \subseteq E)$.

735 $(\forall x \in G)(I_x \in S_{funkc}(E_x,\{\{i,k\}: i,k \in V_x\}))$.

736 $(\forall x \in G)(\omega_x \in S_{funkc}(E_x,\Omega))$.

737 $(\forall x \in G)(\neg V_x = 0)$.

738 $(\forall x \in G)(E_x = 0 \rightarrow x = (V_x,0,0,0))$.

DEFINICIJE 186-189

186 $(\forall x \in G)(v(x) = \kappa(V_x))$.

187 $(\forall x \in G)(\forall u \in \Omega)(E_u(x) = \{e: e \in E_x$ & $\omega_x(e) = u\})$.

188 $(\forall x \in G)(\forall u \in \Omega)(\varepsilon_u(x) = \kappa(E_u(x))$.

189 $(\forall x \in G)(\varepsilon(x) = \varepsilon_{ba}(x) + \varepsilon_A(x) + \varepsilon_B(x))$.

D184-D189 $\succ\!\!-$

TEOREMI 739-740

739 $(\forall x \in G)(\forall u \in \Omega)(\varepsilon_u(x) = \kappa\{e: e \in E_x$ & $\omega_x(e) = u\})$.

740 $(\forall x \in G)(\varepsilon(x) = \kappa(E_x \setminus E_{ab}(x)) = \kappa(E_x) - \varepsilon_{ab}(x))$.

DEFINICIJA 190

$(\forall x \in G)(g(x) = (X_1,X_2,X_3) \longleftrightarrow X_1 = V_x$ & $X_2 = E_x \setminus E_{ab}(x)$ &
$X_3 = \{(e,\{i,k\}): e \in X_2$ & $i,k \in V_x$ & $I_x(e) = \{i,k\}\})$.

PRIMJER 385

(1) $x \in G$ & $x = (\{v_1, v_2, v_3\}, \{e_1, e_2\}, \{(e_1, \{v_1, v_2\}), (e_2, \{v_2, v_3\})\}, \{(e_1, A), (e_2, ab)\})$. Sup.

1,D190 >——

(2) $g(x) = (\{v_1, v_2, v_3\}, \{e_1\}, \{(e_1, \{v_1, v_2\})\})$.

Citat 1.

"An abstract graph, or simply a graph, may now be defined as follows: A graph consists of a nonempty set V, a (possibly empty) set E disjoint from V, and a mapping ϕ of E into V & V. The elements of V and E are called the vertices and edges of the graph, respectively, and ϕ is called the incidence mapping associated with the graph". (R.G.Busacker, T.L.Saaty, Finite Graphs and Networks, 1965, str.6).

D190,D184,D183,Cit.1 >——

TEOREM 013

Za svako $x \in G$, $g(x)$ je graf, kojem su čvorišta elementi od V, a grane elementi od E.

DEFINICIJA 035

Za svako $x \in G$, graf od x je $g(x)$.

D035,T013,D190 >——

TEOREM 014

Za svako $x \in G$, ako je y graf od x, onda broj čvorišta u y jednak je $v(x)$ i broj grana od y jednak je $\varepsilon(x)$ i funkcija incidencije u y je funkcija koja rezultira iz I_x, kad se ispuste sve "prazne grane" tj. grane karaktera "ab".

DEFINICIJA 036

Za svako $x \in G$, (geometrijska) shema od x je y (simb.$y \in Sh(x)$), ako i samo ako y je geometrijski graf (crtež) izomorfan sa grafom od x.

PRIMJER 386

(1) $x \in G$ & $x = (\{v_1, v_2, v_3, v_4\}, \{e_1, e_2, e_3, e_4\}, \{(e_1, \{v_2, v_3\}),$
$(e_2, \{v_2, v_3\})(e_3, \{v_3, v_4\}), (e_4, \{v_4, v_4\})\}, \{(e_1, B), (e_2, A),$
$(e_3, ab), (e_4, ba)\})$.

1,D190 $\succ\!\!-\!\!-$

(2) $g(x) = (\{v_1, v_2, v_3, v_4\}, \{e_1, e_2, e_4\}, \{e_1, \{v_2, v_3\}), (e_2, \{v_2, v_3\}),$
$(e_4, \{v_4, v_4\})\})$.

2,D035 $\succ\!\!-\!\!-$

(3) ( o )$\in Sh(x)$.

DEFINICIJA 191

$(\forall x \in G)(S_{m\text{-}subgr}(x) = \{y\colon y \in G \;\&\; V_y \subseteq V_x \;\&\; I_y \subseteq I_x \;\&\; \omega_y \subseteq \omega_x\})$.

PRIMJER 387

(1) $x \in G$ & $x = (\{v_1, v_2, v_3\}, \{e_1, e_2, e_3\}, \{(e_1, \{v_1, v_2\}), (e_2, \{v_1, v_2\}),$
$(e_3, \{v_2, v_3\})\}, \{(e_1, A), (e_2, B), (e_3, ab)\})$. Sup.

1,D191 $\succ\!\!-\!\!-$

(2) $(\{v_1, v_2, v_3\}, \{e_1\}, \{(e_1, \{v_1, v_2\})\}, \{(e_1, A)\}) \in S_{m\text{-}subgr}(x)$.

DEFINICIJA 192

$(\forall x, y \in G)(x \approx y \longleftrightarrow ((\exists F_e \in S_{bijekc}(E_x, E_y)) \;\&\; (\exists F_v \in S_{bijekc}(V_x, V_y)))$
$(\forall g \in E_x)(I_x(g) = \{i, k\} \;\rightarrow\; I_y(F_e(g)) = \{F_v(i), F_v(k)\} \;\&$
$\omega_x(g) = \omega_y(F_e(g))))$.

PRIMJER 388

(1) $x, y \in G$ & $x = (\{v_1, v_2\}, \{e_1, e_2\}, \{(e_1, \{v_1, v_2\}), \{(e_2, \{v_1, v_1\})\},$
$\{(e_1, A), (e_2, B)\})$ & $y = (\{v_3, v_4\}, \{e_3, e_4\},$
$\{(e_3, \{v_4, v_4\}), (e_4, \{v_3, v_4\})\}, \{(e_3, B), (e_4, A)\})$. Sup.

1,D192 $\succ\!\!-\!\!-$

(2) $x \approx y$.

4.2. NADOVEZIVANJE m-GRAFOVA

DEFINICIJA 193

$(\forall x,y\in G)(f_k(x,y) = xy = (V_x \cup V_y,\ E_x \cup E_y,\ I_x \cup I_y,\ \omega_x \cup \omega_y)).$

PRIMJER 389

(1) $x = (\{v_1,v_2,v_3\},\{e_1,e_2\},\{(e_1,\{v_1,v_2\}),(e_2,\{v_2,v_3\})\},$
$\{(e_1,ba),(e_2,A)\})$ &

$y = (\{v_2,v_3,v_4\},\{e_2,e_3\},\{(e_2,\{v_2,v_3\}),(e_3,\{v_2,v_3\}),$
$\{(e_2,A),(e_3,B)\})$ Sup.

1,D193 $\succ\!\!-\!\!-$

(2) $xy = (\{v_1,v_2,v_3,v_4\},\{e_1,e_2,e_3\},\{(e_1,\{v_1,v_2\}),(e_2,\{v_2,v_3\}),$
$(e_3,\{v_2,v_3\})\},\{(e_1,ba),(e_2,A),(e_3,B)\})$.

D193,D184 $\succ\!\!-\!\!-$

TEOREMI 741-744

741 $(\forall x\in G)(xx = x)$.

742 $(\forall x,y\in G)(xy = yx)$.

743 $(\forall x,y,z\in G)(x(yz) = (xy)z)$.

744 $(\forall x,y\in G)(xy\in G \longleftrightarrow (E_x \cap E_y = 0 \quad \vee \quad (e\in E_x \cap E_y \rightarrow$
$I_x(e) = I_y(e) \quad \& \quad \omega_x(e) = \omega_y(e)))$.

4.3. J E Z I K m-T O P O L O G I J E

DEFINICIJE 194-258

194 o = $\{x\colon i \in V \ \&\ x = (\{i\},0,0,0)\}$.

195 $\overset{i}{o}$ = $\{x\colon x \in o \ \&\ V_x = \{i\}\}$.

196 $\mathcal{O}$ = o/ = $\{x\colon i \in V \ \&\ e \in E \ \&\ u \in \Omega \ \&$
$x = (\{i\},\{e\},\{(e,\{i,i\})\},\{(e,u)\})\}$.

197 ^{i}o/ = $\{x\colon x \in$ o/ $\&\ V_x = \{i\}\}$.

198 o/^{u} = $\{x\colon x \in$ o/ $\&\ \mathrm{Adom}(\omega_x) = \{u\}\}$.

199 ^{i}o/^{u} = ^{i}o/ $\cap$ o/^{u}

200 ^{i}o/$^{(e,u)}$ = $\{x\colon x \in$ ^{i}o/^{u} $\&\ E_x = \{e\}\}$.

201 o o = $\{x\colon i,k \in V \ \&\ i \neq k \ \&\ x = (\{i,k\},0,0,0)\}$.

202 $\overset{i}{o}$ o = $\{x\colon x \in (\text{o o}) \ \&\ i \in V_x\}$.

203 $\overset{i}{o}$ $\overset{k}{o}$ = $\{x\colon x \in (\text{o o}) \ \&\ V_x = \{i,k\}\}$.

204 o——o = $\{x\colon i,k \in V \ \&\ i \neq k \ \&\ e \in E \ \&\ u \in \Omega \ \&$
$x = (\{i,k\},\{e\},\{(e,\{i,k\})\},\{(e,u)\})\}$.

205 o$\overset{u}{\text{——}}$o = $\{x\colon x \in$ o——o $\&\ \mathrm{Adom}(\omega_x) = \{u\}\}$.

206 $\overset{i}{o}$——$\overset{k}{o}$ = $\{x\colon x \in$ o——o $\&\ V_x = \{i,k\}\}$.

207 $\overset{i}{o}\overset{u}{\text{——}}\overset{k}{o}$ = o$\overset{u}{\text{——}}$o $\cap$ $\overset{i}{o}$——$\overset{k}{o}$.

208 $\overset{i}{o}\overset{(e,u)}{\text{——}}\overset{k}{o}$ = $\{x\colon x \in$ $\overset{i}{o}\overset{u}{\text{——}}\overset{k}{o}$ $\&\ E_x = \{e\}\}$.

209 $\phi_0 = \{((x,y),Z):\ x,y\in G\ \&\ ((V_x \cap V_y = 0\ \&\ E_x \cap E_y = 0\ \&\ Z=\{xy\})\ \vee\ (\neg(V_x \cap V_y = 0\ \&\ E_x \cap E_y = 0)\ \&\ Z=0))\}$.

210 $\phi_1 = \{((x,y),Z):\ x,y\in G\ \&\ ((\kappa(V_x \cap V_y) = 1\ \&\ E_x \cap E_y = 0\ \&\ Z=\{xy\})\ \vee\ (\neg(\kappa(V_x \cap V_y) = 1\ \&\ E_x \cap E_y = 0)\ \&\ Z=0))\}$.

211 $\phi_2 = \{((x,y),Z):\ x,y\in G\ \&\ ((\kappa(V_x \cap V_y) = 2\ \&\ E_x \cap E_y = 0\ \&\ Z=\{xy\})\ \vee\ (\neg(\kappa(V_x \cap V_y) = 2\ \&\ E_x \cap E_y = 0)\ \&\ Z=0))\}$.

212 $x\ \ y = Z \longleftrightarrow (x,y,Z)\in\phi_0$.

213 $\circ\!\!<^{x}_{y} = Z \longleftrightarrow (x,y,Z)\in\phi_1$.

214 $i\circ\!\!<^{x}_{y} = Z \longleftrightarrow Z = \circ\!\!<^{x}_{y}\ \&\ V_x \cap V_y = \{i\}$.

215 $\circ\!\!<^{x}_{y}\!\!>\!\circ = Z \longleftrightarrow (x,y,Z)\in\phi_2$.

216 $i\circ\!\!<^{x}_{y}\!\!>\!\circ k = Z \longleftrightarrow Z = \circ\!\!<^{x}_{y}\!\!>\!\circ\ \&\ V_x \cap V_y = \{i,k\}$.

217 $x\ \ \circ = \{z:\ z\in\phi_0(x,y)\ \&\ y\in\circ\}$.

218 $x\ \ \overset{i}{\circ} = \{z:\ z\in(x\ \ \circ)\ \&\ V \setminus V_x = \{i\}\}$.

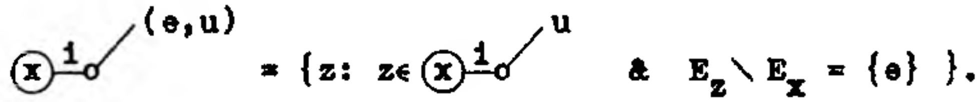
219 $x\!-\!\circ = \{z:\ z\in\phi_1(x,y)\ \&\ y\in\circ\}$.

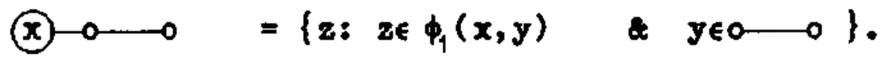
220 $x\!-\!\circ\!\!/ = \{z:\ z\in \circ\!\!<^{x}_{y}\ \&\ y\in\circ\!\!/\}$.

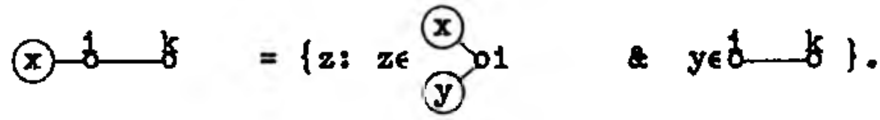
221 $x\overset{i}{-}\circ\!\!/ = \{z:\ z\in {}^{i}\circ\!\!<^{x}_{y}\ \&\ y\in {}^{i}\circ\!\!/\}$.

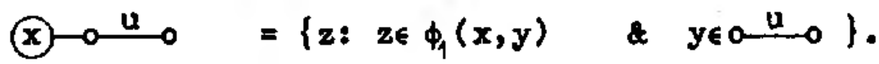
222 $x\!-\!\circ\!\!/^{u} = \{z:\ z\in \circ\!\!<^{x}_{y}\ \&\ y\in\circ\!\!/^{u}\}$.

223 $x\overset{i}{-}\circ\!\!/^{u} = x\overset{i}{-}\circ\!\!/ \cap x\!-\!\circ\!\!/^{u}$.

224 

225 

226 

227 

228 

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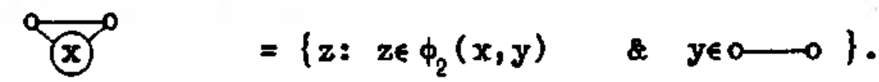
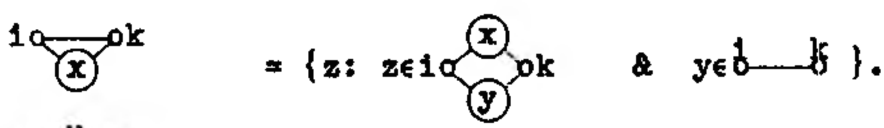
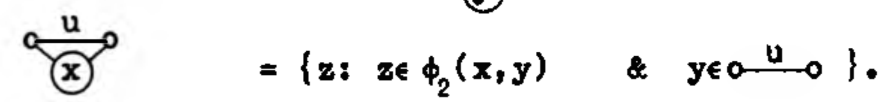
230 

231 

232 

233 

234 

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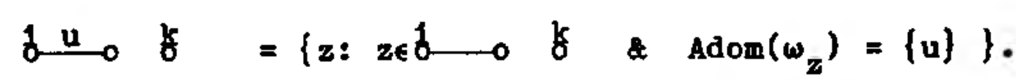
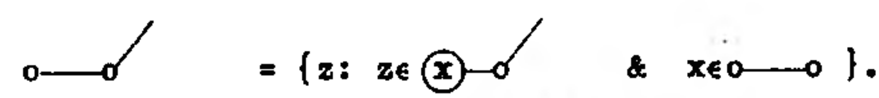
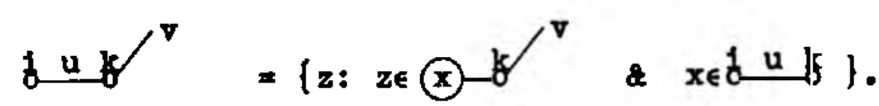
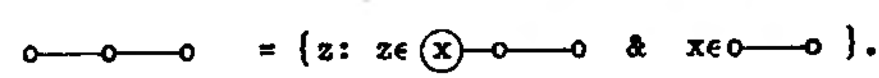
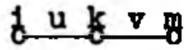
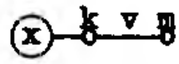
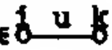
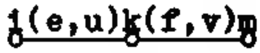
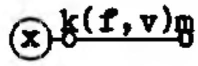
236 

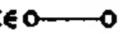
237 

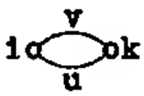
238 

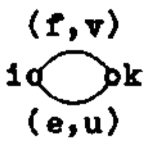
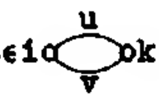
239 

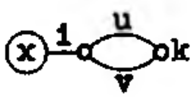
240  = {z: z∈  & x∈  }.

241  = {z: z∈  & x∈  }.

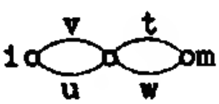
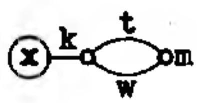
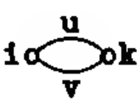
242  = {z: z∈  & x∈  }.

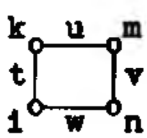
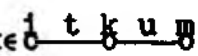
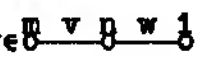
243  = {z: z∈  & $V_z=\{i,k\}$ & Adom $\omega_z=\{u,v\}$}.

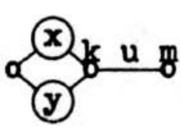
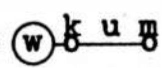
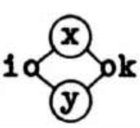
244  = {z: z∈  & $E_z=\{e,f\}$ }.

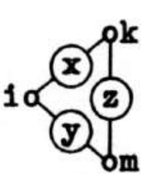
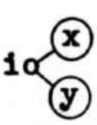
245  = {z: z∈  & y∈  }.

246  = {z: z∈ $\phi_1(x,y)$ & x,y∈  }.

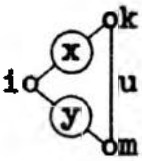
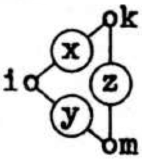
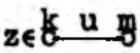
247  = {z: z∈  & x∈  }.

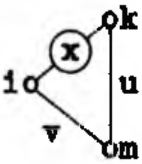
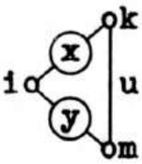
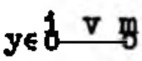
248  = {z: z∈ $\phi_2(x,y)$ & x∈  & y∈  }.

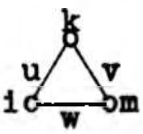
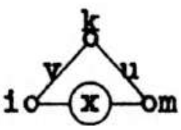
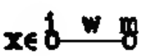
249  = {z: z∈  & w∈  }.

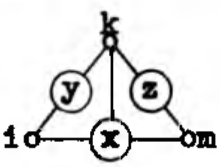
250  = {w: w∈  & ž∈  &

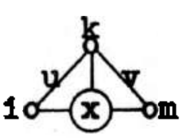
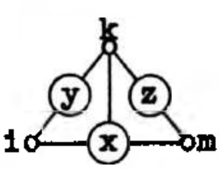
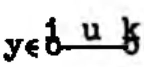
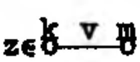
$k\in V_x$ & $m\in V_y$ & $\neg\, i\in\{k,m\}$ }.

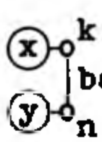
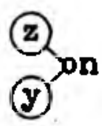
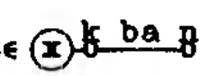
251  = {w: w∈  & z∈  }.

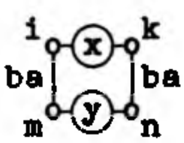
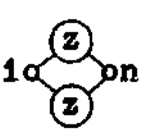
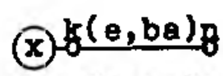
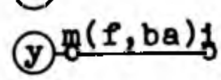
252  = {z: z∈  & y∈  }.

253  = {z: z∈  & x∈  }.

254  = {w: $w \in \phi_2(\check{z}, z)$ & $V_{\check{z}} \cap V_z = \{k,m\}$ &
$\check{z} \in \phi_2(x,y)$ & $V_x \cap V_y = \{i,k\}$ & $m \in V_x$ & $i \neq m$}.

255  = {w: w∈  & y∈  & z∈  }.

256  = {w: w∈  & z∈  }.

257  = {w: w∈  & z∈  &
ž∈  &
$e \neq f$ & $i \neq k$ & $m \neq n$ }.

258 $\mathscr{W}$ = {X: X je definiendum iz D194-D208 i D212-D257 }.

760 $x\in\circ\!\!-\!\!-\!\!\circ \;\leftrightarrow\; x\in G \;\;\&\;\; v(x) = 2 \;\;\&\;\; \kappa(E_x) = 1 \;\;\&$
$(I_x = \{(e,\{i,k\})\} \;\rightarrow\; i \neq k)$.

761 $(\forall x\in\circ\!\!-\!\!-\!\!\circ)(x\in\circ\overset{u}{\!\!-\!\!-\!\!}\circ \;\leftrightarrow\; (e\in E_x \;\rightarrow\; \omega_x(e) = u))$.

762 $x\in\circ\overset{A}{\!\!-\!\!-\!\!}\circ \;\leftrightarrow\; x\in\circ\overset{u}{\!\!-\!\!-\!\!}\circ \;\;\&\;\; u = A$.

763 $\circ\!\!-\!\!-\!\!\circ \;=\; \bigcup\{\circ\overset{A}{\!\!-\!\!-\!\!}\circ,\ \circ\overset{B}{\!\!-\!\!-\!\!}\circ,\ \circ\overset{ba}{\!\!-\!\!-\!\!}\circ,\ \circ\overset{ab}{\!\!-\!\!-\!\!}\circ\}$.

764 $\overset{i}{\circ}\!\!-\!\!-\!\!\overset{i}{\circ} \;=\; \overset{i}{\circ}\overset{u}{\!\!-\!\!-\!\!}\overset{i}{\circ} \;=\; 0$.

765 $x\in\overset{i}{\circ}\overset{u}{\!\!-\!\!-\!\!}\overset{k}{\circ} \;\rightarrow\; \neg\, i = k$.

766 $(\overset{i}{\circ}\overset{u}{\!\!-\!\!-\!\!}\overset{k}{\circ} \;=\; \overset{m}{\circ}\overset{v}{\!\!-\!\!-\!\!}\overset{n}{\circ} \;\;\&\;\; i \neq k) \;\rightarrow\; u = v \;\;\&\;\; \{i,k\} = \{m,n\}$.

767 $\overset{i}{\circ}\overset{(e,B)}{\!\!-\!\!-\!\!}\overset{k}{\circ} \;=\; \{(\{i,k\},\{e\},\{(e,\{i,k\})\},\{(e,B)\})\} \;\leftrightarrow$
$(i,k\in V \;\;\&\;\; e\in E \;\;\&\;\; \neg\, i = k)$.

768 $(X,Y\in\{\circ,\ \circ\!/\ ,\ \circ\!\!-\!\!-\!\!\circ,\ \circ\!\!-\!\!-\!\!\circ\} \;\;\&\;\; \neg\, X = Y) \;\rightarrow\; X\cap Y = 0$.

769 $\phi_0\in S_{surjekc}(G\times G,\ \{Z\colon (x,y\in G \;\;\&\;\; V_x\cap V_y = 0 \;\;\&\;\; E_x\cap E_y = 0$
$\&\;\; Z = \{xy\}) \;\;\vee\;\; Z = 0\})$.

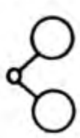
770 $\phi_1\in S_{surjekc}(G\times G,\ \{Z\colon (x\in G \;\;\&\;\; Z = \{x\}) \;\;\vee\;\; Z = 0\})$.

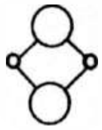
771 $\phi_2\in S_{surjekc}(G\times G,\ \{Z\colon (x\in G \;\;\&\;\; v(x) > 1 \;\;\&\;\; Z=\{x\}) \;\;\vee\;\; Z=0\})$.

772 $(\forall F\in\{\phi_0,\phi_1,\phi_2\})((x,y),Z)\in F \;\rightarrow\; ((y,x),Z)\in F)$.

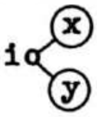
773 $(\forall F\in\{\phi_0,\phi_1,\phi_2\})((y,z,Y_1)\in F \;\;\&\;\; y_1\in Y_1 \;\;\&\;\; (x,y_1,W_1)\in F \;\;\&$
$(x,y,X_1,\in F \;\;\&\;\; x_1\in X_1 \;\;\&\;\; (x_1,z,W_2)\in F \;\rightarrow$
$W_1 = W_2)$.

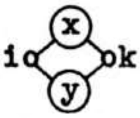
774 $\bigcirc\ \bigcirc \;=\; \phi_0$.

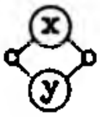
775  $=\; \phi_1$.

776  $= \phi_2$

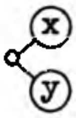
777 $z \in ($ x y $) \longleftrightarrow x, y \in G \;\&\; V_x \cap V_y = 0 \;\&\; E_x \cap E_y = 0 \;\&$
$z = (V_x \cup V_y,\ E_x \cup E_y,\ I_x \cup I_y,\ \omega_x \cup \omega_y)$.

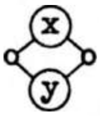
778 $z \in i$  $\longleftrightarrow x, y \in G \;\&\; V_x \cap V_y = \{i\} \;\&\; E_x \cap E_y = 0 \;\&\; z = xy$.

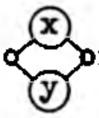
779 $z \in i$  $k \longleftrightarrow x, y \in G \;\&\; V_x \cap V_y = \{i, k\} \;\&\; i \neq k \;\&$
$E_x \cap E_y = 0 \;\&\; z = xy$.

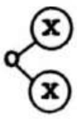
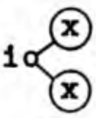
780 $(W, Z \in \{$ x y,  ,  $\} \;\&\; \neg W = Z) \rightarrow W \cap Z = 0$.

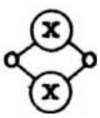
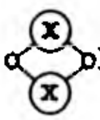
781 $z \in ($ x y $) \rightarrow v(z) = v(x) + v(y)$.

782 $z \in ($  $) \rightarrow v(z) = v(x) + v(y) - 1$.

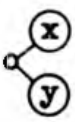
783 $z \in ($  $) \rightarrow v(z) = v(x) + v(y) - 2$.

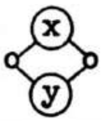
784 x x $=$ i  i $= 0$.

785  $= o \;\&\;$ i  $= \overset{i}{o}$.

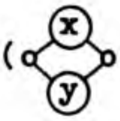
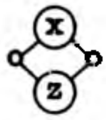
786  $= (o \;\; o) \;\&\;$ i  k $= (\overset{i}{o} \;\; \overset{k}{o})$.

787 $z \in ($ x y $) \rightarrow \neg z = x$.

788 $y \in i$  $\rightarrow x = (\{i\}, 0, 0, 0)$.

789  $= \{x\} \longleftrightarrow y\epsilon(o \quad o).$

790 $(\check{y}\epsilon(y \quad z) \;\&\; \check{w}\epsilon(x \quad \check{y}) \;\&\; \check{x}\epsilon(x \quad y)) \rightarrow$
$(x \quad \check{y}) = (\check{x} \quad z).$

791 $\neg($  $\cap$  $= 0) \rightarrow y=z.$

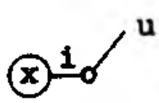
792 $\neg(x \quad y) = 0 \rightarrow ((x \quad y) = (z \quad w) \rightarrow \{x,y\}=\{z,w\}).$

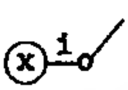
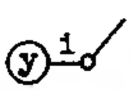
793 $x,y\epsilon o \rightarrow (x \quad y) = (o \quad o).$

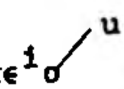
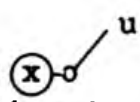
794 $(x,y\epsilon o \;\&\; \neg\, x=y) \rightarrow$  $= 0.$

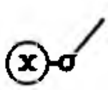
795 $y\epsilon\, x\overset{i}{o} \longleftrightarrow x\epsilon G \;\&\; i\epsilon V\setminus V_x \;\&\; y=(V_x \cup \{i\}, E_x, I_x, \omega_x).$

796 $x\epsilon\overset{i}{o} \rightarrow (x \quad o) = (\overset{i}{o} \quad o).$

797 $y\epsilon$  $\longleftrightarrow y\epsilon G \;\&\; i\epsilon V_x \;\&\; u\epsilon\Omega \;\&$
$(\exists e\epsilon E\setminus E_x)(y = x(\{i\},\{e\},\{(e,\{i,i\})\},\{(e,u)\})).$

798  $=$  $\rightarrow x=y.$

799 $x\epsilon$  $\rightarrow (y\epsilon$  $\longleftrightarrow (\exists e,f\epsilon E)(e\neq f \;\&$
$y = (\{i\},\{e,f\},\{(e,\{i,i\}),(f,\{i,i\})\},\{(e,u),(f,u)\}))).$

800 $y\epsilon$  $\rightarrow V_x=V_y \;\&\; \kappa(E_y) = \kappa(E_x) + 1.$

801 $y\epsilon$  $\longleftrightarrow x\epsilon G \;\&\; i\epsilon V_x \;\&\; e\epsilon E\setminus E_x \;\&\; u\epsilon\Omega \;\&$
$y = x(\{i\},\{e\},\{(e,\{i,i\})\},\{(e,u)\}).$

802 $z\in (x)\!-\!\circ\!\!-\!\!\!-\!\!\circ \quad \longleftrightarrow \quad z\in (x)\!-\!\circ\!-\!(y) \quad \& \quad y\in \circ\!\!-\!\!\!-\!\!\circ .$

803 $y\in (x)\!-\!\overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ} \quad \longleftrightarrow \quad x\in G \;\&\; i\in V_x \;\&\; k\in V\setminus V_x \;\&\; u\in\Omega \;\&$
$(\exists e\in E\setminus E_x)(y = x(\{i,k\},\{e\},\{(e,\{i,k\})\},\{(e,u)\}))\quad .$

804 $(x)\!-\!\overset{i}{\circ}\overset{(e,u)}{\text{———}}\overset{k}{\circ} \quad = \{x(\{i,k\},\{e\},\{(e,\{i,k\})\},\{(e,u)\})\} \quad \longleftrightarrow$
$x\in G \;\&\; i\in V_x \;\&\; k\in V\setminus V_x \;\&\; e\in E\setminus E_x \;\&\; u\in\Omega .$

805 $y\in \overset{\overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ}}{\diagdown (x) \diagup} \quad \longleftrightarrow \quad x\in G \;\&\; i,k\in V_x \;\&\; i\neq k \;\&\; u\in\Omega \;\&$
$(\exists e\in E\setminus E_x)(y = x(\{i,k\},\{e\},\{(e,\{i,k\})\},\{(e,u)\}))\ .$

806 $\overset{\overset{i}{\circ}\text{——}\overset{i}{\circ}}{\diagdown (x) \diagup} \quad = 0.$

807 $x\in(\overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ}\ \ \overset{m}{\circ}) \quad \longleftrightarrow \quad i,k,m\in V \;\&\; i\neq k \;\&\; i\neq m \;\&\; k\neq m \;\&$
$u\in\Omega \;\&\; (\exists e\in E)(x = (\{i,k,m\},\{e\},\{(e,\{i,k\})\},\{(e,u)\}).$

808 $x\in\overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ}\!\diagup^{v} \quad \longleftrightarrow \quad x\in G \;\&\; i,k\in V \;\&\; i\neq k \;\&\; u,v\in\Omega \;\&$
$(\exists e,f\in E)(x = (\{i,k\},\{e,f\},\{(e,\{i,k\}),(f\{k,k\})\},$
$\{(e,u),(f,v)\}).$

809 $x\in\overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ}\overset{v}{\text{——}}\overset{m}{\circ} \quad \longleftrightarrow \quad i,k,m\in V \;\&\; i\neq k \;\&\; i\neq m \;\&\; k\neq m \;\&$
$u,v\in\Omega \;\&\; (\exists e,f\in E)(e\neq f \;\&$
$x=(\{i,k,m\},\{e,f\},\{(e,\{i,k\}),(f,\{k,m\})\},\{(e,u),(f,v)\}\ ).$

810 $\circ\overset{u}{\text{—}}\circ\overset{v}{\text{—}}\circ \in \{\circ\overset{A}{\text{—}}\circ\overset{A}{\text{—}}\circ,\ \circ\overset{A}{\text{—}}\circ\overset{B}{\text{—}}\circ,\ \circ\overset{A}{\text{—}}\circ\overset{ba}{\text{—}}\circ,\ \circ\overset{A}{\text{—}}\circ\overset{ab}{\text{—}}\circ,$
$\circ\overset{B}{\text{—}}\circ\overset{B}{\text{—}}\circ,\ \circ\overset{B}{\text{—}}\circ\overset{ba}{\text{—}}\circ,\ \circ\overset{B}{\text{—}}\circ\overset{ab}{\text{—}}\circ,\ \circ\overset{ba}{\text{—}}\circ\overset{ba}{\text{—}}\circ,$
$\circ\overset{ba}{\text{—}}\circ\overset{ab}{\text{—}}\circ,\ \circ\overset{ab}{\text{—}}\circ\overset{ab}{\text{—}}\circ\ \}.$

811 $\overset{i}{\circ}\overset{A}{\text{—}}\overset{k}{\circ}\overset{B}{\text{—}}\overset{m}{\circ} \cap \overset{m}{\circ}\overset{A}{\text{—}}\overset{i}{\circ}\overset{B}{\text{—}}\overset{k}{\circ} \ = \ 0.$

812 $\circ\overset{u}{\text{—}}\circ\overset{\bar{u}}{\text{—}}\circ \in \{\circ\overset{A}{\text{—}}\circ\overset{B}{\text{—}}\circ,\ \circ\overset{ba}{\text{—}}\circ\overset{ab}{\text{—}}\circ\ \}.$

818 $\kappa(\bigcup\{Z:\ u\in\Omega \rightarrow Z = \circ\!\overset{u}{\text{—}}\!\circ\!\overset{\bar{u}}{\text{—}}\!\circ\}) = 2.$

819 $\kappa(\bigcup\{Z:\ u,v\in\Omega \rightarrow Z = \circ\!\overset{u}{\text{—}}\!\circ\!\overset{v}{\text{—}}\!\circ\}) = 10.$

820 $\kappa(\bigcup\{Z:\ u,v\in\Omega \rightarrow Z = \overset{i}{\circ}\!\overset{u}{\text{—}}\!\overset{k}{\circ}\!\overset{v}{\text{—}}\!\overset{m}{\circ}\}) = 4^2 = 16.$

821 $\kappa(\bigcup\{Z:\ u,v\in\Omega \rightarrow Z = i\circ\!\overset{u}{\underset{v}{=\!=}}\!\circ k\}) = 10.$

822 $\kappa(\bigcup\{Z:\ u,v\in\Omega \rightarrow Z = i\circ\!\overset{(e,u)}{\underset{(f,v)}{=\!=}}\!\circ k\}) = 4^2 = 16.$

823 $x\in \overset{i}{\circ}\!\overset{(e,A)}{\text{——}}\!\overset{k}{\circ}\!\diagup^{(f,B)} \rightarrow S_{m\text{-}subgr}(x) = \bigcup\{\overset{i}{\circ},\ \overset{k}{\circ},\ (\overset{i}{\circ}\quad \overset{k}{\circ}),$

$\overset{i}{\circ}\!\overset{(e,A)}{\text{——}}\!\overset{k}{\circ},\ \overset{k}{\circ}\!\diagup^{(f,B)},\ (\overset{i}{\circ}\quad \overset{k}{\circ}\!\diagup^{(f,B)}),\ \overset{i}{\circ}\!\overset{(e,A)}{\text{——}}\!\overset{k}{\circ}\!\diagup^{(f,B)}\}.$

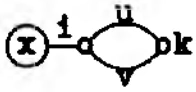
813 $x \in \overset{i}{\circ}\overset{(e,u)}{\text{———}}\circ\overset{(f,v)}{\text{———}}\overset{m}{\circ} \longleftrightarrow x \in \overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ}\overset{v}{\text{——}}\overset{m}{\circ}$ & $I_x = \{(e,\{i,k\}),$
$(f,\{k,m\})\}$.

814 $x \in i \circ\!\overset{u}{\underset{v}{\bigcirc}}\!\circ k \longleftrightarrow i,k \in V$ & $i \neq k$ & $u,v \in \Omega$ & $(\exists e,f \in E)(e \neq f$ &
$x = (\{i,k\},\{e,f\},\{(e,\{i,k\}),(f,\{i,k\})\},\{(e,u),(f,v)\}))$.

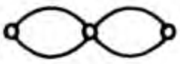
815 $(u \bigcirc u) \in \{A \bigcirc A,\ B \bigcirc B,\ ba \bigcirc ba,\ ab \bigcirc ab\}$.

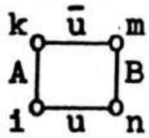
816 $(u \bigcirc A) \in \{A \bigcirc A,\ B \bigcirc A,\ ba \bigcirc A,\ ab \bigcirc A\}$.

817 $x \in (u \bigcirc \bar{u}) \longleftrightarrow x \in (A \bigcirc B) \vee x \in (ba \bigcirc ab)$.

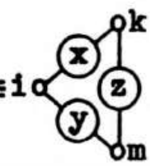
824 $z\in$ 

$\longleftrightarrow$ $x\in G$ & $i\in V_x$ & $k\in V\setminus V_x$ & $u,v\in\Omega$ & $(\exists e,f\in E\setminus E_x)(e\neq f$ & $x=(V_x\cup\{k\},E_x\cup\{e,f\},I_x\cup\{(e,\{i,k\}),(f,\{i,k\})\},\omega_x\cup\{(e,u),(f,v)\})$.

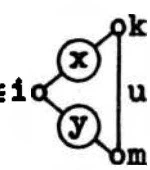
825 $x\in$  $\rightarrow$ $v(x)=3$ & $\kappa(E_x)=4$.

826 $x\in$ 

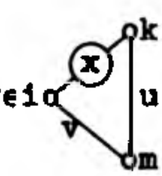
& $(e,\{k,m\})\in I_x$ & $(f,\{i,n\})\in I_x$ $\rightarrow$ $\omega_x(e) = D\omega_x(f)$.

827 $w\in$ 

$\longleftrightarrow$ $x,y,z\in G$ & $V_x\cap V_y=\{i\}$ & $V_x\cap V_z=\{k\}$ & $V_z\cap V_y=\{m\}$ & $i\neq k$ & $i\neq m$ & $k\neq m$ & $E_x\cap E_y=0$ $E_x\cap E_z=0$ & $E_y\cap E_z=0$ & $w=xyz$.

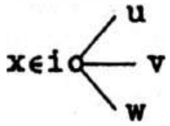
828 $w\in$ 

$\longleftrightarrow$ $x,y\in G$ & $V_x\cap V_y=\{i\}$ & $k\in V_x$ & $m\in V_y$ & $i\neq k$ & $i\neq m$ & $k\neq m$ & $u\in\Omega$ & $(\exists e\in E\setminus E_{xy})(w=xy(\{k,m\},\{e\},\{(e,\{k,m\})\},\{(e,u)\}))$.

829 $w\in$ 

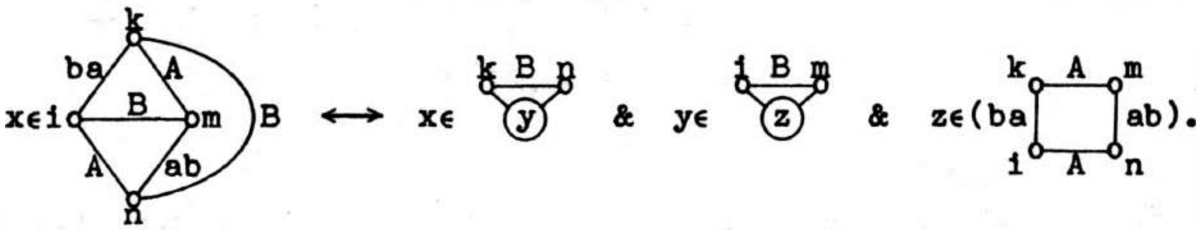
$\longleftrightarrow$ $x\in G$ & $i,k\in V_x$ & $m\in V\setminus V_x$ & $i\neq k$ & $i\neq m$ & $k\neq m$ & $u,v\in\Omega$ & $(\exists e,f\in E\setminus E_x)(w=x(\{i,k,m\},\{e,f\},\{(e,\{k,m\}),(f,\{i,m\})\},\{(e,u),(f,v)\}))$.

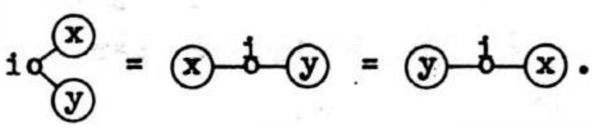
PRIMJERI

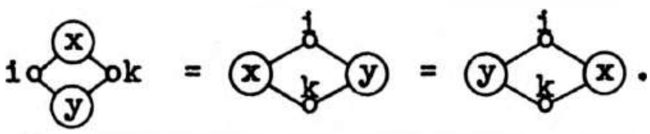
390  $\longleftrightarrow$ $i \in V$ & $u,v,w \in \Omega$ & $(\exists e,f,g \in E)(e \neq f$ & $e \neq g$ & $f \neq g$ & $x = (\{i\},\{e,f,g\},\{(e,\{i,i\}),(f,\{i,i\}),(g,\{i,i\})\},\{(e,u),(f,v),(g,w)\}))$.

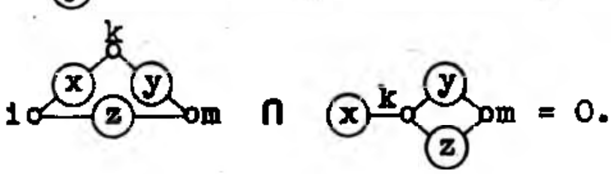
391 $x \in (\circ\ \circ\ \circ)$ $\longleftrightarrow$ $x \in (\text{(y)}\ \ \circ)$ & $y \in (\circ\ \ \circ)$.

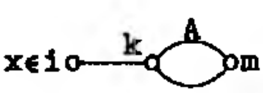
392 $z \in (\circ\overset{A}{\text{—}}\circ\ \ \circ\overset{B}{\text{—}}\circ)$ $\longleftrightarrow$ $z \in (\text{(x)}\ \ \text{(y)})$ & $x \in \circ\overset{A}{\text{—}}\circ$ & $y \in \circ\overset{B}{\text{—}}\circ$.

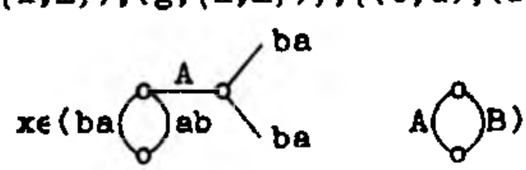
393 

394 

395 

396 

397  $\longleftrightarrow$ $x \in G$ & $(\exists e,f,g \in E)(\exists u,v \in \Omega)(e \neq f$ & $e \neq g$ & $f \neq g$ & $x = (\{i,k,m\},\{e,f,g\},\{(e,\{i,k\}),(f,\{k,m\}),(g,\{k,m\})\},\{(e,u),(f,A),(g,v)\})$.

398 (1)  Sup.

1,D186-D189,D185 $\succ\!\!-\!\!-$

(2) $v(x)=5$ & $\varepsilon_{ba}(x)=3$ & $\varepsilon_{ab}(x)=1$ & $\varepsilon_{A}(x)=2$ & $\varepsilon_{B}(x)=1$ & $\kappa(E_x)=7$ & $\varepsilon(x)=6$.

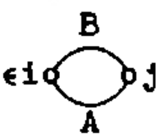
4.4. m-D V O P O L I

DEFINICIJA 259
$(\forall x \in G)(\forall i,j \in V_x)(i(x)j = (x,(i,j)))$.

KONVENCIJA 6
$i(x)j \hookrightarrow ixj$.

DEFINICIJA 260
$2G = \{z: x \in G \ \& \ i,j \in V_x \ \& \ z = ixj\}$.

PRIMJER 399

(1) $x \in$  Sup.

1,T814,D259,D260 >——
(2) $ixi,\ ixj,\ jxi,\ jxj \in 2G$.

DEFINICIJA 261
$(\forall ixj, kym \in 2G)(ixj \approx kym \longleftrightarrow ((\exists F_1 \in S_{bijekc}(E_x,E_y))$ &
$(\exists F_2 \in S_{bijekc}(V_x,V_y))((e \in E_x$ & $I_x(e)=\{n,p\}) \rightarrow I_y(F_1(e)) =$
$\{F_2(n),F_2(p)\}$ & $\omega_x(e)=\omega_y(F_1(e)))$ & $F_2(i) \in \{k,m\}$ & $F_2(j) \in \{k,m\})$.

D261,D192 >——
TEOREM 830
$(\forall ixj, kym \in 2G)(ixj \approx kym \rightarrow x \approx y)$.

PRIMJER 400
D261,T804 >——
$(x \in i \circ \overset{A}{—} \circ \overset{B}{—} \circ j$ & $y \in k \circ \overset{B}{—} \circ \overset{A}{—} \circ m) \rightarrow ixj \approx kym$.

D261,D259 >——
TEOREM 831
$(\forall x \in G)(\forall i,k \in V_x)(ixk \approx kxi)$.

DEFINICIJA 262
$(\forall Y \in S)(\forall F \in S_{relac}(2G,Y))(\forall x \in G)(F_x=\{((i,j),y): i,j \in V_x$ & $y \in Y$ &
$(ixj,y) \in F\})$.

D262 >——

TEOREM 832

$(\forall Y\in S)(\forall F\in S_{funkc}(2G,Y))(\forall x\in G)(\forall i,j\in V_x)(F_x(i,j) = F(i\times j))$.

KONVENCIJA 7

Za svako $Y\in S$, za svako $x\in G$, ako je $F_x\in S_{relac}(V_x \times V_x,Y)$, onda će se u slučaju većeg broja elemenata od V_x, relacija F_x prikazivati pomoću pravokutne sheme kao u slijedećem primjeru, tj. umjesto $F_x = \{((i,i),y_1),((i,j),y_2),((i,k),y_3),((j,i),y_4),((j,j),y_5),((j,k),y_6),((k,i),y_7),((k,j),y_8),((k,k),y_9)\}$, pisat će se:

F_x	i	j	k
i	y_1	y_2	y_3
j	y_4	y_5	y_6
k	y_7	y_8	y_9

.

4.5. TRANSFORMACIJA m-GRAFA TRANSPOZICIJOM ČVORIŠTA

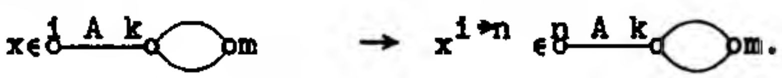
DEFINICIJA 263

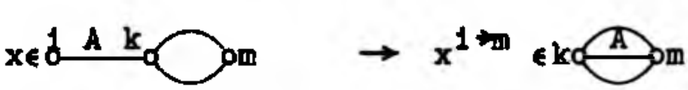
$$(\forall x\in G)(\forall i\in V_x)(\forall j\in V)(x^{i\to j} = (Y_1,Y_2,Y_3,Y_4) \longleftrightarrow$$

$$Y_1 = (V_x \setminus \{i\}) \cup \{j\} \;\&\; Y_2 = E_x \;\&$$

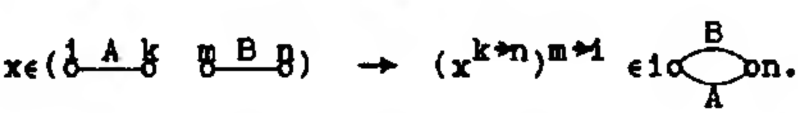
$$\begin{aligned} Y_3 = \{z:\ & (I_x(e)=\{k,m\} \;\&\; \neg\, i\in\{k,m\} \to z=(e,\{k,m\})) \;\&\\ & (I_x(e)=\{i,k\} \;\&\; \neg\, k=i \to z=(e,\{j,k\})) \;\&\\ & (I_x(e)=\{i,i\} \to z=(e,\{j,j\}))\} \;\&\; Y_4 = \omega_x \quad . \end{aligned}$$

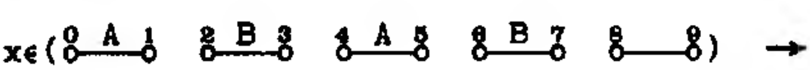
PRIMJERI

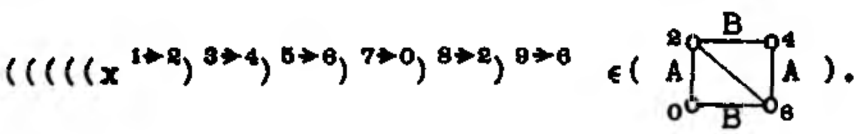
401 

402 

403 

404 

405 



D263,D185,D186 $\succ\!\!-\!\!-$

TEOREMI 833-836

833 $(\forall x\in G)(\forall i\in V_x)(\forall j\in V)(x^{i\to j} = y \;\rightarrow\; y\in G).$

834 $(\forall x\in G)(\forall i\in V_x)(x^{i\to i} = x).$

835 $(\forall x\in G)(\forall i,j\in V_x)((x^{i\to j})^{j\to i} = x^{j\to i}).$

836 $(\forall x\in G)(i,j\in V_x \;\&\; \neg\, i=j \;\rightarrow\; v(x^{i\to j}) = v(x) - 1).$

DEFINICIJA 264

$2G_{s-p} = W \;\leftrightarrow\; ((x\in \overset{i}{\circ}\!\!-\!\!-\!\!-\!\!\overset{k}{\circ} \;\rightarrow\; ixk\in W)$ &

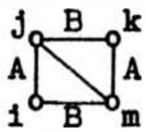
$(z\in(x \;\; y) \;\&\; ixj\in W \;\&\; kym\in W \;\rightarrow$

$i(x^{j\to k}y)m\in W \;\&\; i(x^{j\to m}y^{k\to i})m\in W)$ &

$(\forall X\subseteq W)((x\in \overset{i}{\circ}\!\!-\!\!-\!\!-\!\!\overset{k}{\circ} \;\rightarrow\; ixk\in X) \;\&\; (z\in(x \;\; y) \;\&$

$ixj\in X \;\&\; kym\in X \;\rightarrow\; i(x^{j\to k}y)m\in X \;\&\; i(x^{j\to m}y^{k\to i})m\in X) \;\rightarrow\; X=W)).$

PRIMJER 406

(1) $x\in$ 

Sup.

1,D264 $\succ\!\!-\!\!-$

(2) $jxm\in 2G_{s-p} \;\&\; \neg(ixk\in 2G_{s-p}).$

D264,D259,D260 $\succ\!\!-\!\!-$ T837-T838.

TEOREM 837

$2G_{s-p}\subset 2G.$

TEOREM 838

$(\forall x\in G)(i\in V_x \;\rightarrow\; \neg\, ixi\in 2G_{s-p}).$

4.6. LJUSKA m-GRAFA

T734-T736 $\succ\!\!-$

TEOREM 839

$(\forall x \in G)(\exists! F_x \in S_{relac}(V_x, V_x))((i,k) \in F_x \longleftrightarrow (i = k \lor (\exists e \in E_x)(l(\omega_x(e)) = b \ \& \ I_x(e) = \{i,k\}) \lor (\exists j \in V_x)((i,j) \in F_x \ \& \ (j,k) \in F_x)))$.

DEFINICIJA 265

$(\forall x \in G)$(b_x je jedinstveno određena relacija F_x iz teorema 839).

D265,T839 $\succ\!\!-$

TEOREM 840

$(\forall x \in G)(\forall i,k \in V_x)(i \ b_x \ k \longleftrightarrow i = k \lor (\exists e \in E_x)(l(\omega_x(e)) = b \ \& \ I_x(e) = \{i,k\}) \lor (\exists j \in V_x)(i \ b_x \ j \ \& \ j \ b_x \ k)))$.

T840,D190,D035 $\succ\!\!-$

TEOREM 015

$(\forall x \in G)(\forall j,k \in V_x)$($j \ b_x \ k$, ako i samo ako $j = k$ ili "j" i "k" povezani su putem u grafu od x i karakter svake grane puta počinje sa b).

T734-T736 $\succ\!\!-$

TEOREM 841

$(\forall x \in G)(\exists! F_x \in S_{relac}(V_x, V_x))((i,k) \in F_x \longleftrightarrow (i = k \lor (\exists e \in E_x)(d(\omega_x(e)) = a \ \& \ I_x(e) = \{i,k\}) \lor (\exists j \in V_x)((i,j) \in F_x \ \& \ (j,k) \in F_x)))$.

DEFINICIJA 266

$(\forall x \in G)$(a_x je jedinstveno određena relacija F_x iz teorema 841).

D266,T841 $\succ\!\!-$

TEOREM 842

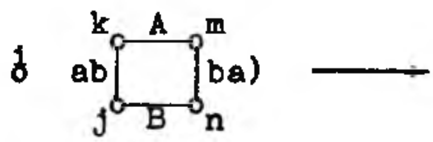
$(\forall x \in G)(\forall i,k \in V_x)((i \ a_x \ k \longleftrightarrow i = k \lor (\exists e \in E_x)(d(\omega_x(e)) = a \ \& \ I_x(e) = \{i,k\}) \lor (\exists j \in V_x)(i \ a_x \ j \ \& \ j \ a_x \ k)))$.

T842,D190,D035 $\succ\!\!-$

TEOREM 016

$(\forall x\in G)(\forall j,k\in V_x)(j\ a_x\ k$, ako i samo ako $j = k$ ili "j" i "k" povezani su putem u grafu od x i karakter svake grane puta završava sa a).

PRIMJER 407

$x\in($ 

$$b_x = \{(i,i),(j,j),(j,m),(j,n),(k,k),(m,m),(m,n),(m,j),(n,j),(n,m),(n,n)\}\ \&$$

$$a_x = \{(i,i),(j,j),(k,k),(k,m),(k,n),(m,k),(m,n),(n,k),(n,m),(n,n)\}.$$

T840,T842 $\succ\!\!-$

TEOREMI 843-845

843 $(\forall x\in G)(\forall i\in V_x)(\forall s\in\{a,b\})(i\ s_x\ i)$.

844 $(\forall x\in G)(\forall i,k\in V_x)(\forall s\in\{a,b\})(i\ s_x\ k \rightarrow k\ s_x\ i)$.

845 $(\forall x\in G)(\forall i,j,k\in V_x)(\forall s\in\{a,b\})(i\ s_x\ j\ \&\ j\ s_x\ k \rightarrow i\ s_x\ k)$.

DEFINICIJE 267-268

267 $(\forall ixk\in 2G)(l(ixk) = s \longleftrightarrow ((i\ b_x\ k\ \&\ s = b)\ \vee$
$(\neg(i\ b_x\ k)\ \&\ s = a)))$.

268 $(\forall ixk\in 2G)(d(ixk) = s \longleftrightarrow ((i\ a_x\ k\ \&\ s = a)\ \vee$
$(\neg(i\ a_x\ k)\ \&\ s = b)))$.

D267,D268 $\succ\!\!-$

TEOREM 846

$l\in S_{surjekc}(2G,\{a,b\})\ \&\ d\in S_{surjekc}(2G,\{a,b\})$.

D267,D268 $\succ\!\!-$ T847,T848.

TEOREM 847

$(\forall ixk\in 2G)((l(ixk)=b \longleftrightarrow i\ b_x\ k)\ \&\ (l(ixk)=a \longleftrightarrow \neg i\ b_x\ k))$.

TEOREM 848

$(\forall ixk\in 2G)((d(ixk)=a \longleftrightarrow i\ a_x\ k)\ \&\ (d(ixk)=b \longleftrightarrow \neg i\ a_x\ k))$.

T843-T848 $\succ\!\!-\!\!-$

TEOREMI 849-856

849 $(\forall i x i \in 2G)(l(ixi)=b \ \& \ d(ixi)=a)$.

850 $(\forall i x j \in 2G)(l(ixj)=b \rightarrow l(jxi)=b) \ \& \ (d(ixj)=a \rightarrow d(jxi)=a)$.

851 $(\forall ixj, jxk \in 2G)(l(ixj)=b \ \& \ l(jxk)=b) \rightarrow l(ixk)=b)$.

852 $(\forall ixj, jxk \in 2G)(d(ixj)=a \ \& \ d(jxk)=a) \rightarrow d(ixk)=a$.

853 $(\forall i x i \in 2G)(\neg\, l(ixi)=a \ \& \ \neg\, d(ixi)=b)$.

854 $(\forall i x j \in 2G)(l(ixj)=a \rightarrow l(jxi)=a) \ \& \ (d(ixj)=b \rightarrow d(jxi)=b)$.

855 $(\forall ixj, jxk \in 2G)((l(ixj)=a \not\leftrightarrow l(jxk)=a) \rightarrow l(ixk)=a)$.

856 $(\forall ixj, jxk \in 2G)((d(ixj)=b \not\leftrightarrow d(jxk)=b) \rightarrow d(ixk)=b)$.

DEFINICIJA 269
$(\forall i x j \in 2G)(q(ixj) = l(ixj)d(ixj))$.

D269,T846,T3 $\succ\!\!-\!\!-$
TEOREM 857
$q \in S_{surjekc}(2G, \Omega)$.

D269,D262 $\succ\!\!-\!\!-$
TEOREM 858
$(\forall x \in G)(q_x = \{((i,j),u):\ i,j \in V_x \ \& \ q(ixj) = u\})$.

PRIMJER 408

(1) $x \in$ 

Sup.

1,T858,K7 $\succ\!\!-\!\!-$

(2)

q_x	h	i	k	m	n
h	ba	ab	ab	ab	ab
i	ab	ba	ba	ba	B
k	ab	ba	ba	ba	B
m	ab	ba	ba	ba	B
n	ab	B	B	B	ba

.

T849,D269 $\succ\!\!-\!\!-$

TEOREM 859

$(\forall ixi\in 2G)(q(ixi) = ba)$.

D204,D205,T758,T777,T847,T848,D269 $\succ\!\!-\!\!-$

TEOREMI 860-862

860 $x\in(\overset{i}{\circ}\quad\overset{k}{\circ}) \;\rightarrow\; q(ixk) = ab.$

861 $x\in \overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ} \;\rightarrow\; q(ixk) = u\,.$

862 $z\in(\,x\quad y\,) \;\rightarrow\; (\forall i\in V_x)(\forall k\in V_y)(q(izk) = ab).$

DEFINICIJA 270

$(\forall ixj\in 2G)(q_s(ixj) = q_s(q(ixj)))$.

DEFINICIJA 271

$(\forall ixj\in 2G)(q_p(ixj) = q_p(q(ixj)))$.

D270,D271,T859-T862,T217 $\succ\!\!-\!\!-$

TEOREMI 863-867

863 $x\in\overset{i}{\circ} \;\rightarrow\; q_s(ixi) = ba \;\&\; q_p(ixi) = BA.$

864 $x\in\overset{i}{\circ}\overset{ba}{\text{——}}\overset{k}{\circ} \;\rightarrow\; q_s(ixk) = ba \;\&\; q_p(ixk) = BA.$

865 $(x\in(\overset{i}{\circ}\quad\overset{k}{\circ}) \;\vee\; x\in\overset{i}{\circ}\overset{ab}{\text{——}}\overset{k}{\circ}) \;\rightarrow\; q_s(ixk)=AB \;\&\; q_p(ixk)=ab.$

866 $x\in\overset{i}{\circ}\overset{A}{\text{——}}\overset{k}{\circ} \;\rightarrow\; q_s(ixk) = q_p(ixk) = A.$

867 $x\in\overset{i}{\circ}\overset{B}{\text{——}}\overset{k}{\circ} \;\rightarrow\; q_s(ixk) = q_p(ixk) = B.$

TEOREM 868

$z\in x\text{—}\overset{k}{\circ}\text{—}y \;\rightarrow\; (\forall i\in V_x)(\forall m\in V_y)(q_s(izm) = q_s(ixk) \smile q_s(kym)).$

D o k a z

(1) $x,y\in G \;\&\; V_x\cap V_y = \{k\} \;\&\; E_x\cap E_y = 0 \;\&\; z = xy.$ Sup.

1,T847,T848,T855,T856,T160,T161 $\succ\!\!-\!\!-$

(2) $(\forall i\in V_x)(\forall m\in V_y)(l(izm) = \inf(l(ixk),l(kym)) \;\&$
$d(izm) = \sup(d(ixk),d(kym)))$.

2,D269 $\succ\!\!-$

(3) $q(izm) = \inf(l(q(ixk)),l(q(kym))\sup(d(q(ixk)),d(q(kym)).$

3,T162,T164 $\succ\!\!-$

(4) $q(izm) = q(q(ixk) \cup q(kym))$

4,D270,T221 $\succ\!\!-$

(5) $q_s(izm) = q_s(q(ixk) \cup q(kym))$

5,T237,T221 $\succ\!\!-$

(6) $q_s(izm) = q_s(ixk) \cup q_s(kym)$

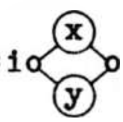
1,6 $\succ\!\!-$

(7) $(\forall x,y\in G)((V_x \cap V_y = \{k\} \;\&\; E_x \cap E_y = 0 \;\&\; xy = z) \to$
$(\forall i\in V_x)(\forall m\in V_y)(q_s(ixm) = q_s(ixk) \cup q_s(kym)).$

7,T778 $\succ\!\!-$ T868.

D269,D271,T160,T161,T163,T165,T221,T238,T779,T847,T848,T855,T856, (analog T868) $\succ\!\!-$

TEOREM 869

$z\in$ 

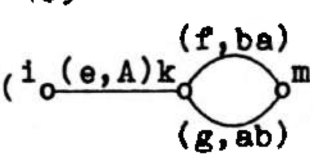
$\to q_p(izk) = q_p(ixk) \cap q_p(iyk).$

4.7. S U V I S L O S T m-G R A F A

DEFINICIJA 272

$(\forall x\in G)(x' = (Y,Z) \leftrightarrow Y = g(x) \;\&$
$Z = \{(e,u): e\in E_x \setminus E_{ab}(x) \;\&\; (e,u)\in\omega_x\}).$

PRIMJER 409

(1) $x\in($ 

$\circ^n\,)$ Sup.

1,D272,D187 $\succ\!\!-$

(2) $x' = (\{i,k,m,n\},\{e,f\},\{(e,\{i,k\}),(f\{k,m\})\},\{(e,A),(f,ba)\}).$

D272,D186,D189,D190 $\succ\!\!-$

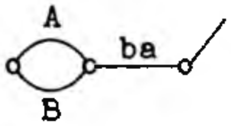
TEOREM 870

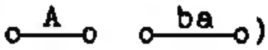
$(\forall x\in G)(x'\in G \;\&\; v(x') = v(x) \;\&\; \varepsilon(x') = \varepsilon(x) \;\&\; g(x') = g(x)).$

DEFINICIJA 273

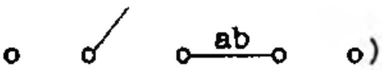
$(\forall x\in G)((\pi(x) = 1 \longleftrightarrow \neg(\exists y,z\in G)(x'\in(y \quad z)) \;\&$
$(\forall n\in N_{poz})(\pi(x) = n+1 \longleftrightarrow (\exists y,z\in G)(x'\in(y \quad z)) \;\&\; \pi(y)= n \;\&$
$\pi(z)= 1))).$

PRIMJERI

410 x∈  → $\pi(x) = 1.$

411 x∈() → $\pi(x) = 2.$

412 x∈  → $\pi(x) = 2.$

413 x∈() → $\pi(x) = 5.$

Citat 2
"If the graph G happens to be an unconnected graph, as in Fig. 1-11(a) then it is obvious that it must consist of a number of "connected pieces".We next attempt to make this intuitive concept precise.
By Problem 1-12, the existence of a path between vertices is an equivalence relation. Any such equivalence relation defines a partition of the vertices of the graph into sets such that any two vertices in a set are connected by a path in G. Alternatively, we could also construct the sets. Beginning with any vertex v_1, consider all the vertices of G which can be connected to v_1 by a path in G. Then the elements of G incident at these vertices constitute a connected subgraph G_s.Furthermore, if any other elements of G is added to this subgraph to form G_1, then G_1 is not connected. Thus G_s is a maximal connected subgraph of G. G may or may not have any more vertices that are contained in G_s.If G has other vertices(not in G_s), consider one of these vertices v_i. By a similar process, we can now construct a maximal connected subgraph containing v_i. The process can be repeated until there are no more vertices left, provided G is finite. The number of these connected subgraphs is denoted by p.
Theorem 1-3. The decomposition of a graph into maximal connected subgraphs is unique.
Theorem 1-4. p=1 for a graph G if and only if G is connected".
(Seshu-Reed, Linear graphs and electrical networks, 1961, str. 16-17).

D273,D272,D190,Cit.2 >——

TEOREM 017

$(\forall x\in G)(\pi(x)$ je broj maksimalno povezanih subgrafova (maximal connected subgraphs) od $g(x)$).

T017 $\succ\!\!-\!\!-$

TEOREM 018

$(\forall x \in G)(\varkappa(x) = 1 \longleftrightarrow g(x)$ je suvisli graf).

D273,D272,T746,T757,T762,T777,T778 $\succ\!\!-\!\!-$

TEOREMI 871-877.

871 $x \in o \rightarrow \varkappa(x) = 1.$

872 $\varkappa \in S_{surjekc}(G, N_{poz}).$

873 $x \in o \overset{u}{\text{———}} o \;\; \& \;\; u \in \{A, B, ba\} \rightarrow \varkappa(x) = 1.$

874 $(x \in (o \quad o) \;\; \vee \;\; x \in o \overset{ab}{\text{———}} o) \rightarrow \varkappa(x) = 2.$

875 $(\forall x \in G)(E_x = 0 \rightarrow \varkappa(x) = v(x)).$

876 $z \in (\; x \quad y \;) \rightarrow \varkappa(z) = \varkappa(x) + \varkappa(y).$

877 $z \in x \text{—} o \text{—} y \rightarrow \varkappa(z) = \varkappa(x) + \varkappa(y) - 1.$

DEFINICIJE 274-277

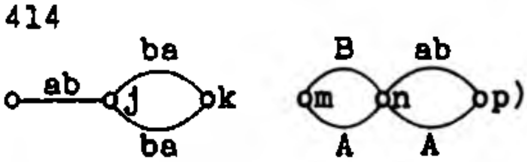
274 $(\forall x \in G)(\forall i \in V_x)(|i|_{a_x} = \{j\colon j \in V_x \;\; \& \;\; j \; a_x \; i\}).$

275 $(\forall x \in G)(\varkappa_a(x) = \kappa(\{Y\colon i \in V_x \;\; \& \;\; Y = |i|_{a_x}\}))$.

276 $(\forall x \in G)(\forall i \in V_x)(|i|_{b_x} = \{j\colon j \in V_x \;\; \& \;\; j \; b_x \; i\}).$

277 $(\forall x \in G)(\varkappa_b(x) = \kappa(\{Y\colon i \in V_x \;\; \& \;\; Y = |i|_{b_x}\}))$.

PRIMJER 414

(1) x∈(

) Sup.

1,D274,D276 $\succ\!\!-\!\!-$

(2) $|i|_{a_x}=\{i\},\; |j|_{a_x}=|k|_{a_x}=\{j,k\},\; |m|_{a_x}=|n|_{a_x}=|p|_{a_x}=\{m,n,p\}$ &

$|i|_{b_x}=\{i\},\; |j|_{b_x}=|k|_{b_x}=\{j,k\},\; |m|_{b_x}=|n|_{b_x}=\{m,n\}, |p|_{b_x}=\{p\}.$

1,2,D275,D277 $\succ\!\!-\!\!-$

(3) $\varkappa_a(x) = 3 \;\; \& \;\; \varkappa_b(x) = 4.$

D275,D277,D273,T017 $\succ\!\!-$ T019,T020.

TEOREM 019

$(\forall x,y\in G)(\varkappa_a(x) = \varkappa(y) \longleftrightarrow$ y rezultira iz x uklanjanjem svih grana kojima karakter ne završava sa a).

TEOREM 020

$(\forall x,y\in G)(\varkappa_b(x) = \varkappa(y) \longleftrightarrow$ y rezultira iz x uklanjanjem svih grana kojima karakter ne počinje sa b).

D275,D277,D185,D186,T777,T778,T840,T842 $\succ\!\!-$

TEOREMI 878-880

878 $x\in G \ \& \ E_x = 0 \ \rightarrow \ \varkappa_a(x) = \varkappa_b(x) = v(x).$

879 $z\in(\ x \quad y\ ) \ \rightarrow \ (\forall s\in\{a,b\})(\varkappa_s(z) = \varkappa_s(x) + \varkappa_s(y)).$

880 $z\in x\!-\!o\!-\!y \ \rightarrow \ (\forall s\in\{a,b\})(\varkappa_s(z) = \varkappa_s(x) + \varkappa_s(y) - 1).$

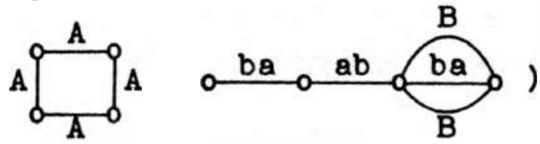
DEFINICIJA 278

$(\forall x\in G)(C(x) = \varepsilon(x) - v(x) + \varkappa(x)).$

DEFINICIJA 279

$(\forall x\in G)(R(x) = v(x) - \varkappa(x)).$

PRIMJER 415

(1) $x\in($ 

$)$ Sup.

1,D186,D189,D273 $\succ\!\!-$

(2) $\varepsilon(x) = 8 \quad \& \quad v(x) = 8 \quad \& \quad \varkappa(x) = 3.$

1,2,D278,D279 $\succ\!\!-$

(3) $C(x) = 8 - 8 + 3 = 3 \quad \& \quad R(x) = 8 - 3 = 5.$

Citat 3
"Definition 2-9. Nullity. The nullity of a graph with e edges, v vertices, and p maximal connected subgraphs is $\mu = e - v + p$. Nullity is also known by the names of cyclomatic number, connectivity, and first Betty number.

The fundamental system of circuits for an unconnected graph is obtained by taking the fundamental systems for each maximal connected subgraph.

Definition 2-10. Rank. The rank of a graph with e edges and p maximal connected subgraphs is v - p".
(Seshu-Reed, Linear graphs and electrical networks, 1960, str. 27).

D186,D189,D190,D273,D278,D279,Cit.3 >——

TEOREMI 021-022

021 $(\forall x \in G)$(C(x) je broj suvislosti, odnosno broj temeljnih petlji grafa od x).

022 $(\forall x \in G)$(R(x) je rang, odnosno broj temeljnih rezova grafa od x).

Citat 4
"Definition 2-11. Cut-set. A cut set is a set of edges of a connected graph G such that removal of these edges from G reduces the rank of G by one, provided that no proper subset of this set reduces the rank of G by one when it is removed from G".
(Seshu-Reed, Linear graphs and electr. networks, 1960, str. 28).

T847,T848,T015-T018,Cit.3 >——

TEOREMI 023-024

023 $(\forall jxk \in 2G)(l(jxk) = a \longleftrightarrow$ j i k nisu povezani putem u grafu od x ili egzistira rez (cut-set) u grafu od x i karakter svake grane reza počinje sa "a" i uklanjanjem grana reza rezultira graf u kojem j i k nisu povezani putem).

024 $(\forall jxk \in 2G)(d(jxk) = b \longleftrightarrow$ j i k nisu povezani putem u grafu od x ili egzistira rez (cut-set) u grafu od x i karakter svake grane reza završava sa "b" i uklanjanjem grana reza rezultira graf u kojem j i k nisu povezani putem).

DEFINICIJE 280-283

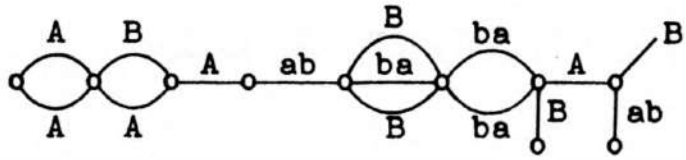
280 $(\forall x \in G)(R_a(x) = \varkappa_b(x) - 1)$.

281 $(\forall x \in G)(R_b(x) = \varkappa_a(x) - 1)$.

282 $(\forall x \in G)(C_a(x) = \varepsilon_A(x) + \varepsilon_{ba}(x) - v(x) + \varkappa_a(x))$.

283 $(\forall x \in G)(C_b(x) = \varepsilon_B(x) + \varepsilon_{ba}(x) - v(x) + \varkappa_b(x))$.

PRIMJER 416

(1) $x \in$ 

Sup.

1,D186,D188,D275,D277 >——

(2) $\pi_b(x) = 6$ & $\pi_a(x) = 4$ & $v(x) = 10$ &

$\varepsilon_{ba}(x) = 3$ & $\varepsilon_A(x) = 5$ & $\varepsilon_B(x) = 5$

2,D280-D283 >——

(3) $R_a(x) = 5$ & $R_b(x) = 3$ & $C_a(x) = 2$ & $C_b(x) = 4$.

D282,D283,T021 >——

TEOREMI 025-026

025 $(\forall x \in G)(C_a(x)$ je broj suvislosti grafa m-grafa koji rezultira iz x, uklanjanjem svih grana kojima karakter ne završava sa "a").

026 $(\forall x \in G)(C_b(x)$ je broj suvislosti grafa m-grafa koji rezultira iz x, uklanjanjem svih grana kojima karakter ne počinje sa "b").

D280-D283,D275,D277,T878-T880 >——

TEOREMI 881-888

881 $(\forall x \in G)((e \in E_x \ \& \ \omega_x(e) = ab) \rightarrow R_a(x) = R_b(x) = v(x) - 1)$.

882 $(\forall x \in G)((e \in E_x \ \& \ \neg\, d(\omega_x(e)) = a) \rightarrow C_a(x) = 0)$.

883 $(\forall x \in G)((e \in E_x \ \& \ \neg\, l(\omega_x(e)) = b) \rightarrow C_b(x) = 0)$.

884 $(\forall x \in G)(\forall s \in \{a,b\})(0 \leq C_s(x) \ \& \ 0 \leq R_s(x))$.

885 $z \in (\ x \quad y\ ) \rightarrow (\forall s \in \{a,b\})(R_s(z) = R_s(x) + R_s(y) + 1)$.

886 $z \in (\ x \quad y\ ) \rightarrow (\forall s \in \{a,b\})(C_s(z) = C_s(x) + C_s(y))$.

887 $z \in x$—o—$y \rightarrow (\forall s \in \{a,b\})(R_s(z) = R_s(x) + R_s(y))$.

888 $z \in x$—o—$y \rightarrow (\forall s \in \{a,b\})(C_s(z) = C_s(x) + C_s(y))$.

D275,D277,D280-D283,D186,D188,T749,T757,T760,T761,T809,T814 >——
TEOREMI 889-906

$$(\forall x\in G)(Y_x = (\pi_a(x),\pi_b(x),R_a(x),R_b(x),C_a(x),C_b(x)))$$

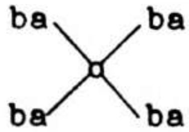
$\longrightarrow$

889	$x\in o$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 0,\ 0)$.
890	$x\in o\diagup^{ab}$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 0,\ 0)$.
891	$x\in o\diagup^{ba}$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 1,\ 1)$.
892	$x\in o\diagup^{A}$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 1,\ 0)$.
893	$x\in o\diagup^{B}$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 0,\ 1)$.
894	$x\in(o\quad o)$	$\longrightarrow$	$Y_x = (2,\ 2,\ 1,\ 1,\ 0,\ 0)$.
895	$x\in o\overset{ab}{\text{——}}o$	$\longrightarrow$	$Y_x = (2,\ 2,\ 1,\ 1,\ 0,\ 0)$.
896	$x\in o\overset{ba}{\text{——}}o$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 0,\ 0)$.
897	$x\in o\overset{A}{\text{——}}o$	$\longrightarrow$	$Y_x = (1,\ 2,\ 1,\ 0,\ 0,\ 0)$.
898	$x\in o\overset{B}{\text{——}}o$	$\longrightarrow$	$Y_x = (2,\ 1,\ 0,\ 1,\ 0,\ 0)$.
899	$x\in o\overset{ab}{\text{——}}o\overset{ab}{\text{——}}o$	$\longrightarrow$	$Y_x = (3,\ 3,\ 2,\ 2,\ 0,\ 0)$.
900	$x\in o\overset{A}{\text{——}}o\overset{A}{\text{——}}o$	$\longrightarrow$	$Y_x = (1,\ 3,\ 2,\ 0,\ 0,\ 0)$.
901	$x\in o\overset{A}{\text{——}}o\overset{B}{\text{——}}o$	$\longrightarrow$	$Y_x = (2,\ 2,\ 1,\ 1,\ 0,\ 0)$.
902	$x\in o\overset{B}{\text{——}}o\overset{B}{\text{——}}o$	$\longrightarrow$	$Y_x = (3,\ 1,\ 0,\ 2,\ 0,\ 0)$.
903	$x\in o\overset{ba}{\underset{ba}{=}}o$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 1,\ 1)$.
904	$x\in o\overset{A}{\underset{A}{=}}o$	$\longrightarrow$	$Y_x = (1,\ 2,\ 1,\ 0,\ 1,\ 0)$.
905	$x\in o\overset{B}{\underset{B}{=}}o$	$\longrightarrow$	$Y_x = (2,\ 1,\ 0,\ 1,\ 0,\ 1)$.
906	$x\in o\overset{B}{\underset{A}{=}}o$	$\longrightarrow$	$Y_x = (1,\ 1,\ 0,\ 0,\ 0,\ 0)$.

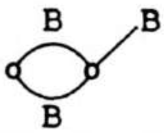
DEFINICIJA 284

$(\forall x\in G)(\lambda_s(x) = \varepsilon(x) - v(x) + 1 - C_a(x) - C_b(x))$.

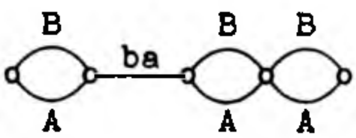
PRIMJERI

417 $x\in$ 

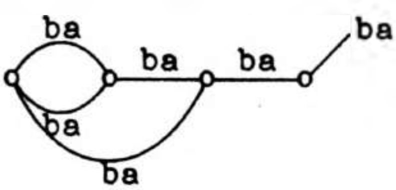
$\longrightarrow \lambda_s(x) = 4-1+1-4-4 = -4.$

418 $x\in$ 

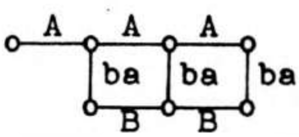
$\longrightarrow \lambda_s(x) = 3-2+1-0-2 = 0.$

419 $x\in$ 

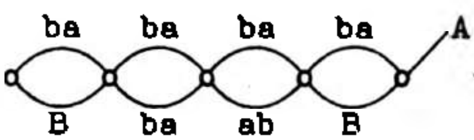
$\longrightarrow \lambda_s(x) = 7-5+1-0-0 = 3.$

420 $x\in$ 

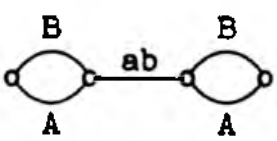
$\longrightarrow \lambda_s(x) = 6-4+1-3-3 = -3.$

421 $x\in$ 

$\longrightarrow \lambda_s(x) = 8-7+1-0-0 = 2.$

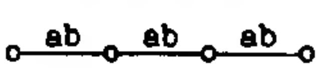
422 $x\in$ 

$\longrightarrow \lambda_s(x) = 8-5+1-2-3 = -1.$

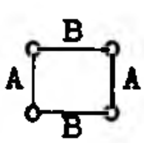
423 $x\in$ 

$\longrightarrow \lambda_s(x) = 4-4+1-0-0 = 1.$

424 $x\in$(o o o o) $\longrightarrow \lambda_s(x) = 0-4+1-0-0 = -3.$

425 $x\in$ 

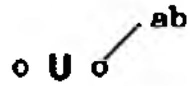
$\longrightarrow \lambda_s(x) = 0-4+1-0-0 = -3.$

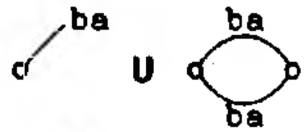
426 $x\in$(

) $\longrightarrow \lambda_s(x) = 4-4+1-0-0 = 1.$

D284,D186,D188,D189,T889-T906 $\succ\!\!-$

TEOREMI 907-915

907 $x \in ($  $)$ $\longrightarrow$ $\lambda_s(x) = 0.$

908 $x \in ($  $)$ $\longrightarrow$ $\lambda_s(x) = -1.$

909 $x \in ($  $)$ $\longrightarrow$ $\lambda_s(x) = 0.$

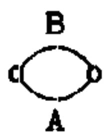
910 $x \in ((o \quad o) \cup o \overset{ab}{\text{———}} o)$ $\longrightarrow$ $\lambda_s(x) = -1.$

911 $x \in \cup\{o \overset{A}{\text{———}} o,\ o \overset{B}{\text{———}} o,\ o \overset{ba}{\text{———}} o\}$ $\longrightarrow$ $\lambda_s(x) = 0.$

912 $x \in (o \overset{ab}{\text{———}} o \overset{ab}{\text{———}} o \ \cup \ (o \quad o \quad o))$ $\longrightarrow$ $\lambda_s(x) = -2.$

913 $x \in o \overset{u}{\text{——}} o \overset{v}{\text{——}} o$ & $u, v \in \{A, B, ba\}$ $\longrightarrow$ $\lambda_s(x) = 0.$

914 $x \in ($  $)$ $\longrightarrow$ $\lambda_s(x) = 0.$

915 $x \in$  $\longrightarrow$ $\lambda_s(x) = 1.$

TEOREM 916

$z \in (x) \overset{k}{\text{—o—}} (y) \quad \longrightarrow \quad \lambda_s(z) = \lambda_s(x) + \lambda_s(y).$

D o k a z

(1) $x, y \in G$ & $V_x \cap V_y = \{k\}$ & $E_x \cap E_y = 0$ & $z = xy$ Sup.

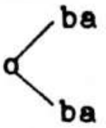
1,D186,D189,T888 $\succ\!\!-$

(2) $\varepsilon(z) = \varepsilon(x) + \varepsilon(y)$ & $v(z) = v(x) + v(y) - 1$ &

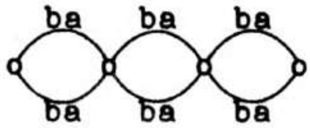
$C_a(z) = C_a(x) + C_a(y)$ & $C_b(z) = C_b(x) + C_b(y)$.

1,2,D284,T778 $\succ\!\!-$ T916.

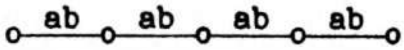
PRIMJERI

427 $x \in$ 

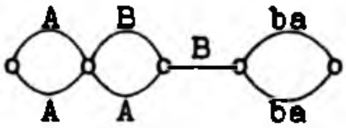
$\longrightarrow \lambda_s(x) = -1 - 1 = -2.$

428 $x \in$ 

$\longrightarrow \lambda_s(x) = -1 - 1 - 1 = -3.$

429 $x \in$ 

$\longrightarrow \lambda_s(x) = -1 - 1 - 1 - 1 = -4.$

430 $x \in$ 

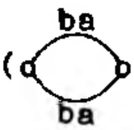
$\longrightarrow \lambda_s(x) = 0 + 1 + 0 - 1 = 0.$

D284,T777,T781,D189,T886 >——
TEOREM 917

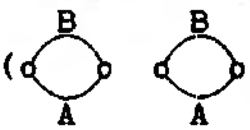
$z \in (x \quad y) \longrightarrow \lambda_s(z) = \lambda_s(x) + \lambda_s(y) - 1.$

PRIMJERI

431 $x \in (o \quad o) \longrightarrow \lambda_s(x) = 0 + 0 - 1 = -1.$

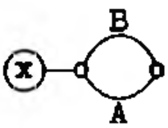
432 $x \in ($ 

$o \quad o) \longrightarrow \lambda_s(x) = -1 - 1 - 1 = -3.$

433 $x \in ($ 

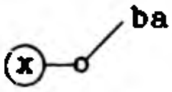
$) \longrightarrow \lambda_s(x) = 1 + 1 - 1 = 1.$

D245,T916,T915 >——
TEOREM 918

$y \in$ 

$\longrightarrow \lambda_s(y) = \lambda_s(x) + 1.$

D222,T916,T908 >——
TEOREM 919

$y \in$ 

$\longrightarrow \lambda_s(y) = \lambda_s(x) - 1.$

TEOREM 920

$\lambda_s \in S_{surjekc}(G,N)$.

D o k a z

(1) $\lambda_s \in S_{funkc}(G,N)$.	
(2) $H_1 = \{n\colon n\in N^+ \ \& \ (\exists x\in G)(\lambda_s(x) = n)\}$.	Def.
(3) $0\in H_1$	2,T907
(4) $n\in H_1$	Sup.
(5) $(\exists x\in G)(\lambda_s(x) = n)$	4,2
(6) $n\in H_1 \ \rightarrow\ (n+1)\in H_1$	4,5,T918
(7) $H_1 = N^+$	2,3,6
(8) $H_2 = \{n\colon n\in N^+ \ \& \ (\exists x\in G)(\lambda_s(x) = -n)\}$	Def.
(9) $n\in H_2 \ \rightarrow\ (n+1)\in H_2$	8,T919
(10) $H_2 = N^+$	8,2,3,9

2,7,8,10 >—— T920.

D284,D189,D280-D283 >——

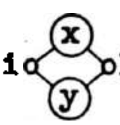
TEOREM 921

$(\forall x\in G)(\lambda_s(x) = v(x) - \varepsilon_{ba}(x) - R_a(x) - R_b(x))$.

DEFINICIJA 285

$(\forall i x j\in 2G)(h(ixj) = h(q(ixj)))$.

TEOREM 922

$z\in i$  $k \ \rightarrow\ \lambda_s(z) = \lambda_s(x)+\lambda_s(y)+h(ixk)+h(iyk)-h(izk)$.

D o k a z

(1) $x,y\in G \ \& \ V_x \cap V_y = \{i,k\} \ \& \ i\neq k \ \& \ E_x \cap E_y = 0 \ \&$
$z=xy \ \& \ q(ixk)=q(iyk)=ba$ Sup.

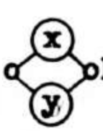
1,D284,D189,T783,D282,D283,D275,D277,T869,T221 >——

(2) $\lambda_s(z) = \varepsilon(x)+\varepsilon(y)-(v(x)+v(y)-2)+1-(C_a(x)+C_a(y)+1)-$
$-(C_b(x)+C_b(y)+1) \ \& \ q(izk) = ba$

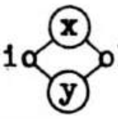
1,2,D284,D285,T256 >——

(3) $\lambda_s(z) = \lambda_s(x)+\lambda_s(y)-1 \ \& \ h(ixk)=h(iyk)=h(izk) = -1$

1,3,T779 >——

(4) $(z\in i$  $k \ \& \ q(ixk) = q(iyk) = ba) \ \rightarrow$
$\lambda_s(z)=\lambda_s(x)+\lambda_s(y)+h(ixk)+h(iyk)-h(izk)$.

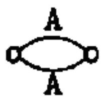
D284,T779,D189,T783,D282,D283,D275,D277,T869,T221,D285,T256 (analog 4) >——

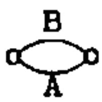
(5) ($z \in i$  k & $q(ixk)$, $q(iyk) \in \{ab, A, B\}$) $\rightarrow$

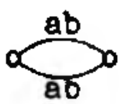
$$\lambda_s(z) = \lambda_s(x) + \lambda_s(y) + h(ixk) + h(iyk) - h(izk).$$

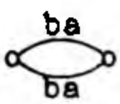
4,5,T38,T3 >—— T922.

PRIMJERI

434 $x \in$  $\rightarrow \lambda_s(x) = 0 + 0 + 0 + 0 + 0 = 0.$

435 $x \in$  $\rightarrow \lambda_s(x) = 0 + 0 + 0 + 0 + 1 = 1.$

436 $x \in$  $\rightarrow \lambda_s(x) = -1 - 1 + 1 + 1 - 1 = -1.$

437 $x \in$  $\rightarrow \lambda_s(x) = 0 + 0 - 1 - 1 + 1 = -1.$

4.8. m-P R O S T O R I

DEFINICIJA 286

$$(\forall x \in G)(\forall i, j \in V_x)(\vartheta(ixj) = Q^{\lambda_s(x)} q_s(ixj)).$$

D286,D270,T857,T920,T229 >——

TEOREM 923

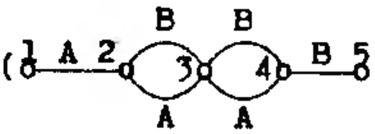
$\vartheta \in S_{surjekc}(2G, M).$

D286,T923,T832,D262 >——

TEOREM 924

$$(\forall x \in G)(\vartheta_x = \{((i,j),y) : i, j \in V_x \ \& \ \vartheta(ixj) = y\}).$$

PRIMJERI

438 (1) $x \in ($  $)$ Sup.

1,T911,T915,T916,T917 $\succ\!\!-\!\!-$

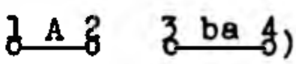
(2) $\lambda_s(x) = 1$

1,2,D286,T924,T866-T869,K7 $\succ\!\!-\!\!-$

(3)

θ_x	1	2	3	4	5	6
1	BA	ABA	ABA	ABA	ABAB	ABAB
2	ABA	BA	BA	BA	BAB	ABAB
3	ABA	BA	BA	BA	BAB	ABAB
4	ABA	BA	BA	BA	BAB	ABAB
5	ABAB	BAB	BAB	BAB	BA	ABAB
6	ABAB	ABAB	ABAB	ABAB	ABAB	BA

.

439 (1) $x \in ($  $)$ Sup.

1,T911,T917 $\succ\!\!-\!\!-$

(2) $\lambda_s(x) = -1$

1,2,D286,T924,T866-T869,K7 $\succ\!\!-\!\!-$

(3)

θ_x	1	2	3	4
1	bABa	aBa	ab	ab
2	aBa	bABa	ab	ab
3	ab	ab	bABa	bABa
4	ab	ab	bABa	bABa

.

TEOREM 925

$(\forall F \in \{1, d, q, q_s, q_p, h\})(\forall i x j \in 2G)(F(\theta(i x j)) = F(i x j)).$

D o k a z

(1) $i x j \in 2G$ Sup.

(2) $q(\theta(i x j)) = q(Q^{\lambda_s(x)} q_s(i x j))$ 1,D286

(3) $i x j \in 2G \rightarrow q(\theta(i x j)) = q(i x j).$ 1,2,T78,D270,T221

3,T217,T256,D267-D271,D285 $\succ\!\!-\!\!-$ T925.

DEFINICIJA 287

$(\forall F\in\{J_s,J_p,\lambda_s,\lambda_p,\lambda\})(\forall ixj\in 2G)(F(ixj) = F(\theta(ixj)))$.

DEFINICIJA 288

$(\forall F\in\{r,<_s,<_p,<\})(\forall ixj,kym\in 2G)(ixj\ F\ kym \longleftrightarrow \theta(ixj)\ F\ \theta(kym))$.

D286,D287,D49 $\succ\!\!-$

TEOREM 926

$(\forall ixj\in 2G)(\lambda_s(ixj) = \lambda_s(x))$.

D287,T850,T854,T926 $\succ\!\!-$

TEOREM 927

$(\forall x\in G)(\forall i,k\in V_x)(\theta(ixk) = \theta(kxi))$.

D286,T859,T863-T867,T907-T911 $\succ\!\!-$

TEOREMI 928-931

928 $x\in\overset{i}{\circ} \rightarrow \theta(ixi) = ba$.

929 $x\in i\circ\!\diagup^{u} \rightarrow \theta(ixi) = u\cap ba$.

930 $x\in(\overset{i}{\circ}\quad\overset{k}{\circ}) \rightarrow \theta(ixk) = ab$.

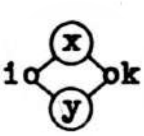
931 $x\in\overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ} \rightarrow \theta(ixk) = u$.

T868,T733,T75,T137 $\succ\!\!-$

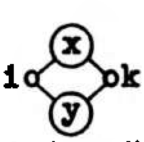
TEOREM 932

$z\in (x)\!-\!\overset{k}{\circ}\!-\!(y) \rightarrow (\forall i\in V_x)(\forall m\in V_y)(\theta(izm) = \theta(ixk)\cup\theta(kym))$.

TEOREM 933

z∈  $\rightarrow \theta(izk) = \theta(ixk)\cap\theta(iyk)$.

D o k a z

(1) z∈  Sup.

(2) $q_p(\theta(izk)) = q_p(\theta(ixk)\cap q_p(\theta(iyk))$ 1,T869,T985

(3) $\lambda_s(z)+h(izk) = \lambda_s(x)+h(ixk)+\lambda_s(y)+h(iyk)$ 1,T922

(4) $\lambda_s(\vartheta(izk))+h(\vartheta(izk)) =$
$=\lambda_s(\vartheta(ixk))+h(\vartheta(ixk))+\lambda_s(\vartheta(iyk))+h(\vartheta(iyk))$ 3,T925

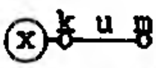
(5) $\lambda_p(\vartheta(izk) = \lambda_p(\vartheta(ixk)) + \lambda_p(\vartheta(iyk))$ 4,T923,D55

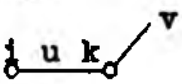
1,2,5,T226,T238 $\succ\!\!-\!\!-$ T933.

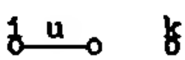
T929-T933,D223-D257 $\succ\!\!-\!\!-$

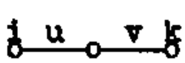
TEOREMI 934-945

934(y∊  & $i \in V_x$) → $\vartheta(iyk) = \vartheta(ixk) \cup (u \cap ba)$.

935(y∊  & $i \in V_x$) → $\vartheta(iym) = \vartheta(ixk) \cup u$.

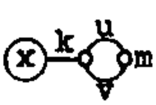
936 x∊  → $\vartheta(ixk) = u \cup (v \cap ba)$.

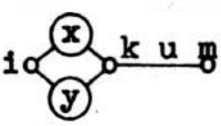
937 x∊() → $\vartheta(ixk) = u \cup ab$.

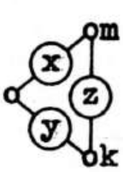
938 x∊() → $\vartheta(ixk) = u \cup v$.

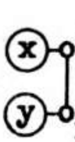
939 y∊() → $\vartheta(iyk) = \vartheta(ixk) \cap u$.

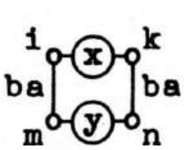
940 x∊  → $\vartheta(ixk) = u \cap v$.

941(y∊  & $i \in V_x$) → $\vartheta(iym) = \vartheta(ixk) \cup (u \cap v)$.

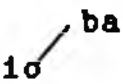
942 z∊  → $\vartheta(izm) = (\vartheta(ixk) \cap \vartheta(iyk)) \cup u$.

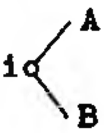
943 w∊  → $\vartheta(iwm) = \vartheta(ixm) \cap (\vartheta(iyk) \cup \vartheta(kzm))$.

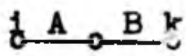
944(z∊  ba & $i \in V_x$ & $n \in V_y$) → $\vartheta(izn) = \vartheta(ixk) \cup \vartheta(myn)$.

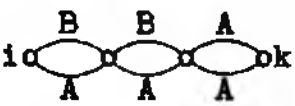
945 z∊  → $\vartheta(izn) = \vartheta(ixk) \cap \vartheta(myn)$.

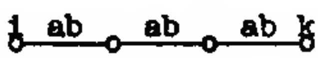
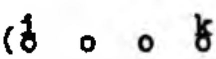
PRIMJERI

440 $x \in$  $\rightarrow \; \vartheta(ixi) = ba \cap ba = bABa.$

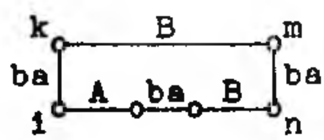
441 $x \in$  $\rightarrow \; \vartheta(ixi) = (A \cap ba) \cup (A \cap ba) = ba.$

442 $x \in$  $\rightarrow \; \vartheta(ixk) = A \cup B = AB.$

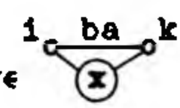
443 $x \in$  $\rightarrow \; \vartheta(ixk) = BA \cup BA \cup (A \cap A) = ABABA.$

444 $x \in($  $\cup$ ( $)) \rightarrow \; \vartheta(ixk) =$

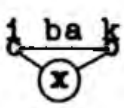
$= ab \cup ab \cup ab = aBABAb.$

445 $x \in$  $\rightarrow \; \vartheta(ixn) = \vartheta(kxm) = \vartheta(kxn) = \vartheta(ixm) = BAB.$

TEOREM 946

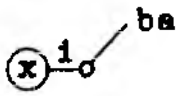
$y \in$  $\rightarrow \; \lambda_p(ixk) = \lambda_s(y).$

D o k a z

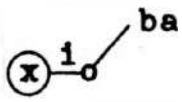
(1)	$y \in$ 	Sup.
(2)	$\vartheta(iyk) = \vartheta(ixk) \cap ba$	1,D233,T933,T931
(3)	$\lambda_s(y) = \lambda_s(ba \cap Q^{\lambda_p(ixk)} q_p(ixk))$	2,T923,T226
(4)	$\lambda_s(y) = \lambda_s(Q^{\lambda_p(ixk)} ba)$	3,T125,T217

1,4,T233,T227,T209 $\succ\!\!-$ T946.

TEOREM 947

$y \in$  $\rightarrow \lambda_p(ixi) = \lambda_s(y) = \lambda_s(x) - 1.$

D o k a z

(1) $y \in$  Sup.

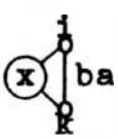
1,T919,T923,D55 $\succ\!\!-$

(2) $\lambda_s(y) = \lambda_s(x)-1 \;\;\&\;\; \lambda_p(\theta(ixi)) = \lambda_s(\theta(ixi))+h(\theta(ixi))$

1,2,D287,T925,T923,T859,T797,T254 $\succ\!\!-$ T947.

T946,T805,T947,T921 $\succ\!\!-$ T948,T949.

TEOREM 948

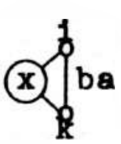
$y \in$  $\rightarrow \lambda_p(ixk) = v(x) - \varepsilon_{ba}(x) - 2 - R_a(y) - R_b(y).$

TEOREM 949

$(\forall x \in G)(\forall i \in V_x)(\lambda_p(ixi) = v(x) - \varepsilon_{ba}(x) - 2 - R_a(x) - R_b(x)).$

D284,T948,T949,T283 $\succ\!\!-$ T950,T951.

TEOREM 950

$y \in$  $\rightarrow \lambda(ixk) = \varepsilon(x)-\varepsilon_{ba}(x)-C_a(x)-C_b(x)-R_a(y)-R_b(y).$

TEOREM 951

$(\forall x \in G)(\forall i \in V_x)(\lambda(ixi) = \varepsilon(x)-\varepsilon_{ba}(x)-C_a(x)-C_b(x)-R_a(x)-R_b(x)).$

D263,D286,D284 $\succ\!\!-$

TEOREM 952

$(\forall x \in G)(\forall j \in V_x)(\forall k \in V \setminus V_x)(x^{j*k} = y \;\; \rightarrow \;\; (\forall i \in V_x)(\theta(ixj) = \theta(iyk)).$

TEOREM 953

$(F \in S_{funkc}(2G,M)$ & $((x \in \overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ} \to F(ixk) = u)$ &

$(x \in i\circ\!\diagup^{u} \to F(ixi) = u \cap ba)$ &

$(y \in (\ (x)\!<\!\overset{i}{\circ}\,|\,ab\,\underset{k}{\circ} \ \cup \ (x)\overset{i}{—}\circ\!\diagup^{ab}\ ) \to F_y = F_x)$ &

$(z \in i\circ\!<\!\begin{smallmatrix}(x)\\(y)\end{smallmatrix} \to (\forall i \in V_x)(\forall m \in V_y)(F(izm) = F(ixk) \cup F(kym))$ &

$(z \in i\circ\!<\!\begin{smallmatrix}(x)\\(y)\end{smallmatrix}\!>\!\circ k \to F(izk) = F(ixk) \cap F(iyk))$ &

$(x \in G$ & $i,j \in V_x \to q(F(ixj) = q(ixj))) \longrightarrow F = \theta.$

D o k a z

(1) (i) $F \in S_{funkc}(2G,M)$ &

(ii) $x \in \overset{i}{\circ}\overset{u}{\text{——}}\overset{k}{\circ} \to F(ixk) = u$ &

(iii) $x \in i\circ\!\diagup^{u} \to F(ixi) = u \cap ba$ &

(iv) $y \in (\ (x)\!<\!\overset{i}{\circ}\,|\,ab\,\underset{k}{\circ} \ \cup \ (x)\overset{i}{—}\circ\!\diagup^{ab}\ ) \to F_y = F_x$ &

(v) $z \in \begin{smallmatrix}(x)\\(y)\end{smallmatrix}\!>\!\circ k \to (\forall i \in V_x)(\forall m \in V_y)(F(izm) = F(ixk) \cup F(kym))$ &

(vi) $z \in i\circ\!<\!\begin{smallmatrix}(x)\\(y)\end{smallmatrix}\!>\!\circ k \to F(izk) = F(ixk) \cap F(iyk)$ &

(vii) $(x \in G$ & $i,j \in V_x) \to q(F(ixj)) = q(ixj).$ Sup.

1(iv),T797 $\succ$——

(2) $(x \in \circ$ & $y \in (x)\overset{i}{—}\circ\!\diagup^{ab}\ ) \to F_x = F_y$

2,1(iii) $\succ$——

(3) $x \in (\overset{i}{\circ} \cup {}^{i}\circ\!\diagup^{ab}\ ) \to F(ixi) = ab \cap ba = ba.$

1(v), T803, 1(ii) $\succ\!\!\!-\!\!-$

(4) $y \in \textcircled{x}\!\overset{k}{\circ}\!\overset{ba}{\text{———}}\!\overset{m}{\circ} \;\rightarrow\; (\forall i \in V_x)(F(iym) = F(ixk) \cup ba)$

4, T145, T803 $\succ\!\!\!-\!\!-$

(5) $y \in \textcircled{x}\!\overset{k}{\circ}\!\overset{ba}{\text{———}}\!\overset{m}{\circ} \;\rightarrow\; F(kym) = F(kxk)$

1(iv) $\succ\!\!\!-\!\!-$

(6) $y \in \textcircled{x}\!\overset{k}{\circ}\!\overset{ba}{\text{———}}\!\overset{m}{\circ} \;\&\; z \in \overbrace{i\circ\!-\!\textcircled{x}\!\overset{k}{\circ}\!\overset{ba}{\text{———}}\!\circ m}^{ab} \;\rightarrow\; F(kym) = F(kzm)$

6, 1(vi) $\succ\!\!\!-\!\!-$

(7) $y \in \textcircled{x}\!\overset{k}{\circ}\!\overset{ba}{\text{———}}\!\overset{m}{\circ} \;\rightarrow\; (\forall i \in V_x)(F(kym) = ba \cap (F(ixk) \cup ab))$

7, 5, T192 $\succ\!\!\!-\!\!-$

(8) $(\forall x \in G)(\forall i,k \in V_x)(F(kxk) = J_s(F(ixk) = F(ixi))$

8 $\succ\!\!\!-\!\!-$

(9) $(\forall x \in G)(\forall i,k,m,n \in V_x)(J_s(F(ixk) = J_s(F(mxn))$

9, T243 $\succ\!\!\!-\!\!-$

(10) $(\forall x \in G)(\forall i,k,m,n \in V_x)(\lambda_s(F(ixk) = \lambda_s(F(mxn)))$

(11) $(\forall x \in G)(\forall n \in N)(f(x)=n \;\longleftrightarrow\; (\exists i,j \in V_x)(\lambda_s(F(ixj)=n))$. Def.

11, 10 $\succ\!\!\!-\!\!-$

(12) $(\forall x \in G)(\forall i,j \in V_x)(f(x) = \lambda_s(F(ixj)))$.

10, 1(vi), T850, T854 $\succ\!\!\!-\!\!-$

(13) $(\forall x \in G)(\forall i,j \in V_x)(\lambda_s(F(ixj)) = \lambda_s(F(jxi)) \;\&$
$q_s(F(ixj)) = q_s(F(jxi)))$.

13, 1(i), T225 $\succ\!\!\!-\!\!-$

(14) $(\forall x \in G)(\forall i,j \in V_x)(F(ixj) = F(jxi))$.

12, 1(v), T237 $\succ\!\!\!-\!\!-$

(15) $z \in \textcircled{x}\!-\!\circ\!-\!\textcircled{y} \;\rightarrow\; f(z) = f(x) + f(y)$.

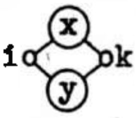
1(vi), T238 $\succ\!\!\!-\!\!-$

(16) $z \in i\circ\!\begin{smallmatrix}\textcircled{x}\\ \textcircled{y}\end{smallmatrix}\!\circ k \;\rightarrow\; \lambda_p(F(izk)) = \lambda_p(F(ixk)) + \lambda_p(F(iyk))$.

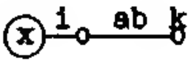
16, D55 $\succ\!\!\!-\!\!-$

(17) $z \in i\circ\!\begin{smallmatrix}\textcircled{x}\\ \textcircled{y}\end{smallmatrix}\!\circ k \;\rightarrow\; \lambda_s(F(izk)) + h(F(izk)) =$
$\lambda_s(F(ixk)) + h(F(ixk)) + \lambda_s(F(iyk)) + h(F(iyk))$.

17,12,1(vii),T256 $\succ\!\!-$

(18) $z\in i$  $k \;\rightarrow\; f(z) = f(x)+f(y)+h(ixk)+h(iyk)-h(izk).$

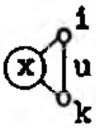
1(iv),12,T803 $\succ\!\!-$

(19) $y\in($  $)$ & $z\in$  $\rightarrow\; f(z) = f(y).$

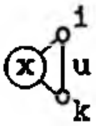
19,T795,1(v),D186,D188,D189 $\succ\!\!-$

(20) $y\in($  $) \;\rightarrow\; \varepsilon(y)=\varepsilon(x)$ & $v(y)=v(x)+1$ & $f(y)=f(x)-1.$

D186,D188,D189,18,12,1(ii),D285,T805,T861 $\succ\!\!-$

(21) $y\in$  & $u\in\{ba,A,B\} \rightarrow \varepsilon(y)=\varepsilon(x)+1$ & $v(y)=v(x)$ &
$f(y)=f(x)+f(u)+h(ixk)+h(u)-h(iyk)$

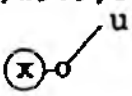
21,T922,12,1(ii) $\succ\!\!-$

(22) $y\in$  & $u\in\{ba,A,B\} \rightarrow \varepsilon(y)=\varepsilon(x)+1$ &
$f(y)=f(x)-\lambda_s(x)+\lambda_s(y).$

15,1(iii),T797,D189,12 $\succ\!\!-$

(23) $y\in$  & $u\in\{ba,A,B\} \rightarrow \varepsilon(y)=\varepsilon(x)+1$ &
$f(y)=f(x)+\lambda_s(u \cap ba)$

23,T908,T909,T916 $\succ\!\!-$

(24) $y\in$  & $u\in\{ba,A,B\} \rightarrow \varepsilon(y)=\varepsilon(x)+1$ &
$f(y)=f(x)-\lambda_s(x)+\lambda_s(y).$

(25) $H_1 = \{n\colon n\in N_{poz}$ & $((x\in G$ & $\varepsilon(x)=0$ & $v(x)=n) \rightarrow$
$f(x) = 1-v(x))\}$ Def.

25,12,3,T746 $\succ\!\!-$

(26) $1\in H_1$

25,20,T795 $\succ\!\!-$

(27) $n\in H_1 \;\rightarrow\; (n+1)\in H_1$

25,26,27 $\succ\!\!-$

(28) $(\forall x\in G)(\varepsilon(x) = 0 \;\rightarrow\; f(x) = 1-v(x))$

28,D284,D189,T882,T883 $\succ\!\!-$

(29) $(\forall x\in G)(\varepsilon(x) = 0 \;\rightarrow\; f(x) = \lambda_s(x)).$

$(30)\ H_2 = \{n\colon n \in N^+ \ \& \ ((x \in G \ \& \ \varepsilon(x)=n) \rightarrow f(x)=\lambda_s(x))\}.$ Def.

$30,29 \succ\!\!-\!\!-$

$(31)\ 0 \in H_2$

$30,22,24 \succ\!\!-\!\!-$

$(32)\ n \in H_2 \ \rightarrow \ n+1 \in H_2$

$30,31,32 \succ\!\!-\!\!-$

$(33)(\forall x \in G)(f(x) = \lambda_s(x)).$

$33,12,T923,1(vii),T221 \succ\!\!-\!\!-$

$(34)(\forall x \in G)(\forall i,j \in V_x)(\lambda_s(x)=\lambda_s(F(ixj)) \ \& \ q_s(ixj) = q_s(F(ixj)).$

$34,D286,T225 \succ\!\!-\!\!-$

$(35)(\forall x \in G)(\forall i,j \in V_x)(\vartheta(ixj) = Q^{\lambda_s(F(ixj))} q_s(F(ixj)) = F(ixj)).$

$1,35 \succ\!\!-\!\!-$ T953.

4.9. UVJET TROKUTA I DIJAMETAR m-PROSTORA

TEOREM 954

$(\forall x \in G)(\forall i,j,k \in V_x)(\theta(i x k) \leq_s \theta(i x j) \cup \theta(j x k)).$

D o k a z

D269,T849-T856 $\succ\!\!-$

(1) $x \in G \;\&\; i,j,k \in V_x \rightarrow$

$$\begin{array}{llll}
q(ixj) = ba \;\&\; q(jxk) = aa & \rightarrow & q(ixk) = aa & \& \\
q(ixj) = ba \;\&\; q(jxk) = ab & \rightarrow & q(ixk) = ab & \& \\
q(ixj) = ba \;\&\; q(jxk) = ba & \rightarrow & q(ixk) = ba & \& \\
q(ixj) = ba \;\&\; q(jxk) = bb & \rightarrow & q(ixk) = bb & \& \\
q(ixj) = aa \;\&\; q(jxk) = bb & \rightarrow & q(ixk) = ab & \& \\
q(ixj) = aa \;\&\; q(jxk) = aa & \rightarrow & q(ixk) \in \{aa,ba\} & \& \\
q(ixj) = aa \;\&\; q(jxk) = ab & \rightarrow & q(ixk) \in \{aa,bb\} & \& \\
q(ixj) = bb \;\&\; q(jxk) = ab & \rightarrow & q(ixk) \in \{aa,ab\} & \& \\
q(ixj) = bb \;\&\; q(jxk) = bb & \rightarrow & q(ixk) \in \{ba,bb\} & \& \\
q(ixj) = ab \;\&\; q(jxk) = ab & \rightarrow & q(ixk) \in \Omega. &
\end{array}$$

1,T183 $\succ\!\!-$

(2) $(\forall x \in G)(\forall i,j,k \in V_x)(q(ixk) \;\; r \;\; (q(ixj) \cup q(jxk))).$

1,2,T181,T327,T923 $\succ\!\!-$ T954.

TEOREM 955

$(\forall x \in G)(\forall i,j \in V_x)(\exists k,m \in V_x)(\vartheta(ixj) \leq_s \vartheta(kxm))$.

D o k a z

$(1)(\exists x \in G)(\exists f,g,i,j \in V_x)(\neg(\vartheta(fxg) \leq_s \vartheta(ixj))$ & $\neg(\vartheta(ixj) \leq_s \vartheta(fxg))$ Sup.

1,T327,T183 $\succ\!\!-$

$(2)(q_s(fxg)=A$ & $q_s(ixj)=B)$ v $(q_s(fxg)=B$ & $q_s(ixj)=A)$.

$(3)\neg(\exists m,n \in \{f,g,i,j\})(q_s(fxg) \leq_s q_s(mxn)$ & $q_s(ixj) \leq_s q_s(mxn))$ Sup.

3,T183 $\succ\!\!-$

$(4)\ \neg(\exists m,n \in \{f,g,i,j\})(q_s(mxn) = AB)$.

$(5)\ q_s(gxj) = ba$ Sup.

5,2,T851,T852 $\succ\!\!-$

$(6)\ q_s(gxi) = A$ v $q_s(gxi) = B$.

6,T855,T856 $\succ\!\!-$

$(7)\ q_s(fxi) = AB$ v $q_s(gxj) = AB$

4,5,7 $\succ\!\!-$

$(8)\ \neg(q_s(gxj) = ba))$.

4,8 $\succ\!\!-$

$(9)\ q_s(gxj) = A$ v $q_s(gxj) = B$

9,2,T855,T856 $\succ\!\!-$

$(10)\ q_s(gxi) = AB$ v $q_s(fxj) = AB$

4,9,10 $\succ\!\!-$

$(11)\ \neg(q_s(gxj) = A)$ & $\neg(q_s(gxj) = B)$

8,11 $\succ\!\!-$

$(12)\ q_s(gxj) = AB$

3,4,12 $\succ\!\!-$

$(13)(\exists m,n \in \{f,g,i,j\})(q_s(fxg) \leq_s q_s(mxn)$ & $q_s(ixj) \leq_s q_s(mxn))$

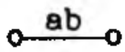
1,13 $\succ\!\!-$ T955.

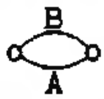
DEFINICIJA 289

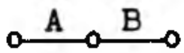
$(\forall x \in G)(\forall y \in M)(\delta(x) = y \longleftrightarrow (\forall i,j \in V_x)(\vartheta(ixj) \leq_s y$ & $(\exists k,m \in V_x)(y = \vartheta(kxm)))$.

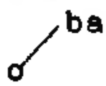
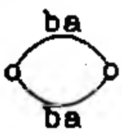
PRIMJERI

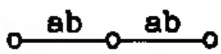
446 $x\in$ o $\longrightarrow$ $\delta(x) = ba$.

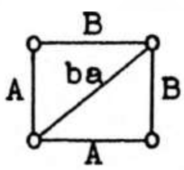
447 $x\in($ $\cup$ (o o)$)$ $\longrightarrow$ $\delta(x) = ab$.

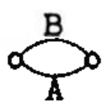
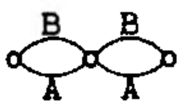
448 $x\in$  $\longrightarrow$ $\delta(x) = BA$.

449 $x\in$  $\longrightarrow$ $\delta(x) = AB$.

450 $x\in($ $\cup$ $)$ $\longrightarrow$ $\delta(x) = bABa$.

451 $x\in($ $\cup$ (o o o)$)$ $\longrightarrow$ $\delta(x) = aBAb$.

452 $x\in$  $\longrightarrow$ $\delta(x) = BABA$.

453 $x\in($ $)$ $\longrightarrow$ $\delta(x) = ABAB$.

DEFINICIJA 290
$(\forall x\in G)(\forall F(q_s,\lambda_s,J_s,q_p,\lambda_p,J_p,q,l,d,h,\lambda\})(F(x) = F(\delta(x)))$.

TEOREM 956
$(\forall x\in G)(\forall i,j\in V_x)(J_s(i\times j) = J_s(x))$.

D o k a z
(1) $x\in G$ & $i,j\in V_x$ Sup.
D289,D287,T923 >——
(2) $\lambda_s(\delta(x)) = \lambda_s(i\times j)$
2,D287,T244 >——
(3) $J_s(\delta(x)) = J_s(\theta(i\times j))$
1,3,D287,D290 >—— T956.

T956,T244,D286,T859 >——
TEOREM 957
$(\forall x\in G)(\forall i\in V_x)(\theta(i\times i) = J_s(x) = Q^{\lambda_s(x)}ba)$.

D289,T923 >——
TEOREM 958
$\delta\in S_{surjekc}(G,M)$.

4.10. TRANSFORMACIJE m-GRAFA UZ m-PROSTOR KAO INVARIJANTU TRANSFORMACIJE

4.10.1. KONGRUENTNOST m-GRAFOVA MODULO JEDAN

DEFINICIJA 291

$\Pi G = \{X\colon X \subseteq G\}$.

DEFINICIJA 292

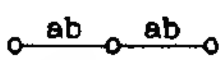
$(\forall X,Y\in\Pi G)(X\ \theta_1\ Y \longleftrightarrow ((\forall x\in X)(\exists y\in Y)\ \&\ (\forall y\in Y)(\exists x\in X))(q_x = q_y)\ \ v\ \ (X = 0\ \ \&\ \ Y = 0))$.

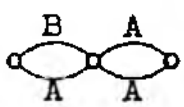
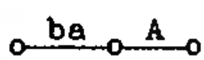
D292,D291,T857,T858 >——

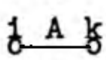
TEOREM 959

$\theta_1\in S_{rel.ekv.}(\Pi G)$.

PRIMJERI

454  θ_1 (o o o).

455  θ_1  .

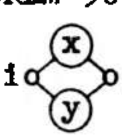
456  θ_1  $\rightarrow$ $\{i,k\} = \{m,n\}$.

D292 >——

TEOREM 960

$(\forall x,y\in G)(\{x\}\ \theta_1\ \{y\} \longleftrightarrow q_x = q_y)$.

TEOREM 961

$(z\in$  $\ \&\ \ ixj\ r\ iyj) \rightarrow (\forall k,m\in V_x)(q(kzm) = q(kxm))$.

D o k a z

(1) $x,y\in G\ \ \&\ \ V_x\cap V_y=\{i,j\}\ \ \&\ \ i\neq j\ \ \&\ \ E_x\cap E_y=0\ \ \&\ \ z=xy\ \ \&$
$ixj\ r\ iyj\ \ \&\ \ (\exists k,m\in V_x)(\neg\, q(kzm) = q(kxm))$. Sup.

1,D292,K8,T840,T842,T847,T848,(T023,T024) >——

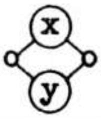
(2)$(l(ixj)=a\ \ \&\ \ l(iyj)=b)\ \ v\ \ (d(ixj)=b\ \ \&\ \ d(iyj)=a)$

2,T183,T159-T161 >——

(3) $\neg\,(ixj\ r\ iyj)$

1,3,T779 >—— T961.

TEOREM 962

$(w \in i$  j & $(\exists k \in V_x)(jxk \ r \ iyj)) \rightarrow$
$(\forall m,n \in V_x)(q(mwn) = q(m(xy^{j*k})n))$.

D o k a z

(1) $x,y \in G$ & $V_x \cap V_y = \{i,j\}$ & $i \neq j$ & $E_x \cap E_y = 0$ & $w = xy$ &
$(\exists k \in V_x)(jxk \ r \ iyj)$ & $z = xy^{j*k}$ & $(\exists m,n \in V_x)(\neg\, q(mwn) = q(mzn))$ Sup.

1,D292,T840,T842,T847,T848,(T023,T024) >——

(2) $\neg\,(q(iwj) = q(izj)) \ \ v \ \ \neg\,(q(iwk) = q(izk))$

2,D291 >——

(3) $(\neg\, l(iwj) = l(izj) \ \ v \ \ \neg\, d(iwj) = d(izj)) \ \ v$
$(\neg\, l(iwk) = l(izk) \ \ v \ \ \neg\, d(iwk) = d(izk))$.

1,3,T840,T842 >——

(4) $l(iyj) = b \ \ v \ \ d(iyj) = a$.

1,4,T840,T842 >——

(5) $(l(iwj)=b \ \& \ l(izk)=b) \ \ v \ \ (d(iwj)=a \ \& \ d(izk)=a)$.

3,5 >——

(6) $(l(izj)=a \ \ v \ \ l(iwk)=a) \ \ v \ \ (d(izj)=b \ \ v \ \ d(iwk)=b)$.

6,5,T855,T856 >——

(7) $(l(jzk)=a \ \ v \ \ l(jwk)=a) \ \ v \ \ (d(jzk)=b \ \ v \ \ (d(jwk)=b)$

7,T840,T842,T847,T848 >——

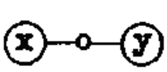
(8) $l(jxk)=a \ \ v \ \ d(jxk)=b$

8,4,T183,T159 >——

(9) $\neg\,(jxk \ r \ iyj)$

1,9,T779 >—— T962.

D292,T778,T777,T840,T842,T847,T848 >——
TEOREM 963

$z \in ($  U ()) $\rightarrow (\forall i,k \in V_x)(q(izk) = q(ixk))$.

4.10.2. KONGRUENTNOST m-GRAFOVA MODULO NULA

DEFINICIJA 293

$(\forall X,Y\in\sqcap G)(X\ \theta\ Y \longleftrightarrow ((\forall x\in X)(\exists y\in Y)\ \&\ (\forall y\in Y)(\exists x\in X))(\vartheta_x = \vartheta_y)\ \vee$
$(X = 0\ \ \&\ \ Y = 0)).$

D293,D291,T923,T924 >——

TEOREM 964

$\theta\in S_{\mathrm{rel.ekv.}}(\sqcap G).$

T964,D291 >——

TEOREM 965

$G\ \theta\ G\ \ \&\ \ 0\ \theta\ 0.$

D293,D291 >——

TEOREM 966

$(\forall x,y\in G)(\{x\}\ \theta\ \{y\} \longleftrightarrow \vartheta_x = \vartheta_y).$

D293,D286,T924,T858,D292 >——

TEOREM 967

$(\forall X,Y\in\sqcap G)(X\ \theta\ Y \longleftrightarrow (X\ \theta_1\ Y\ \ \&$
$((\forall x\in X)(\exists y\in Y)\&(\forall y\in Y)(\exists x\in X))(\lambda_s(x)=\lambda_s(y)))).$

T967,T858,D186 >——

TEOREM 968

$(\forall X,Y\in\sqcap G)(X\ \theta\ Y \longrightarrow ((\forall x\in X)(\exists y\in Y)\ \&\ (\forall y\in Y)(\exists x\in X))(v(x) = v(y))).$

T967,D292,T744 >——

TEOREM 969

$(\forall x,y,z\in G)((\{x\}\theta\{y\}\ \ \&\ \ E_x\cap E_z = 0\ \ \&\ \ E_y\cap E_z = 0) \rightarrow \{xz\}\theta\{yz\}).$

TEOREM 970

 ab θ $\{x\}$.

D o k a z

(1) $y\in$  ab (i, k) Sup.

1,T805,T865,T869,T961,D292 >——

(2) $\{y\}\ \theta_1\ \{x\}\ \ \&\ \ h(iyk) = h(ixk)$

1,T922,T910,T256 >——

(3) $\lambda_s(y) = \lambda_s(x) - 1 + h(ixk) + 1 - h(iyk)$

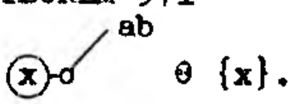
3,2 >——

(4) $\lambda_s(y) = \lambda_s(x)$

1,2,4,T967 >—— T970.

T797,T963,T907,T916,T967 >——

TEOREM 971

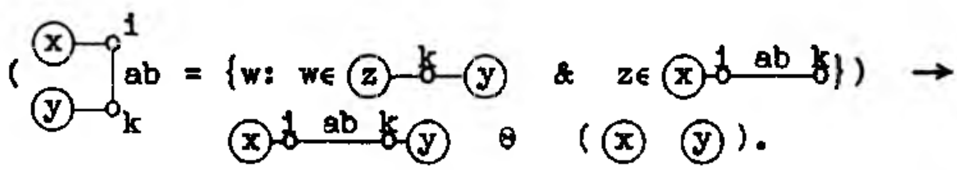


T803,T795,T970 >——

TEOREM 972

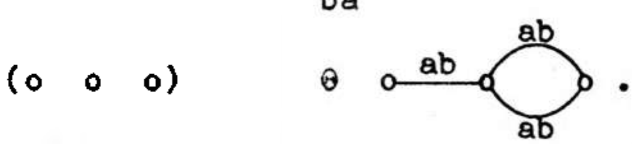
T972,T964,T803,T777,T778 >——

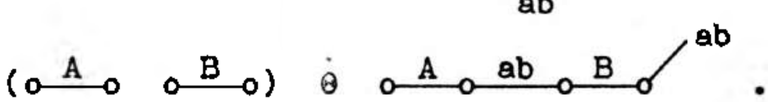
TEOREM 973



PRIMJERI

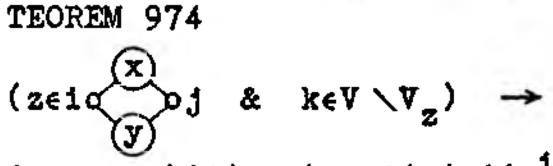
457 

458 

459 

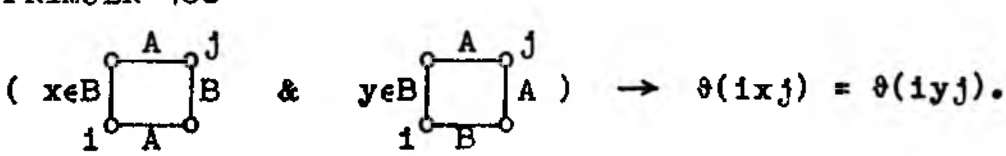
T967,T779,D263,T922,T961 >——

TEOREM 974



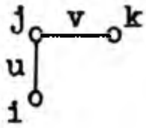
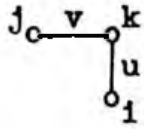
$(\forall m,n\in V_x)(\vartheta(mzn) = \vartheta(m(x((y^{i\to k})^{j\to i})^{k\to j})n))$.

PRIMJER 460

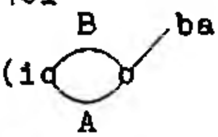
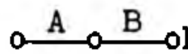


D293,T809,T932,T933,T930,T124,T125 $\succ\!\!-\!\!-$

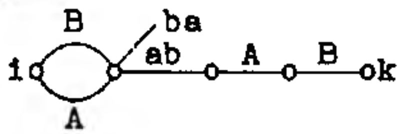
TEOREM 975

 θ  $\longleftrightarrow$ v r u.

PRIMJER 461

(1) X = ( ) Sup.

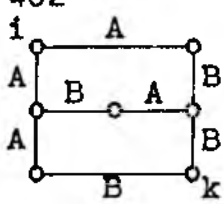
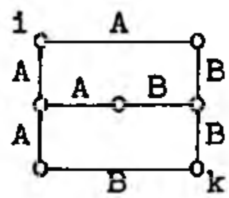
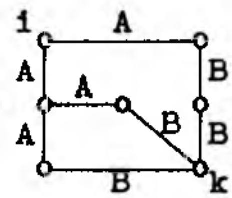
1,T970 $\succ\!\!-\!\!-$

(2) X θ 

2,D293,T932,T933,T929 $\succ\!\!-\!\!-$

(3) $x \in X \;\rightarrow\; \vartheta(ixk) = (B \cap A) \cup (ba \cap ba) \cup ab \cup A \cup B = aBAb.$

PRIMJER 462

(1) X =  & Y =  & Z =  Sup.

1,T974,T975,T969 $\succ\!\!-\!\!-$

(2) $(x \in X \;\&\; y \in Y \;\rightarrow\; \vartheta(ixk) = \vartheta(iyk)) \;\&\; Y \;\theta\; Z$

2,1,D293 $\succ\!\!-\!\!-$

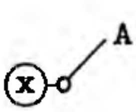
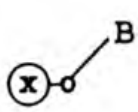
(3) $\vartheta(ixk) = \vartheta(izk)$

3,1,T932,T933 $\succ\!\!-\!\!-$

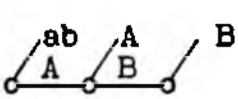
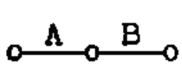
(4) $\vartheta(ixk) = (A \cup B \cup B) \cap (A \cup ((A \cup B) \cap (A \cup B)) = ABABAB.$

T967,T797,T778,T963,T909,T916 $\succ\!\!-\!\!-$

TEOREM 976

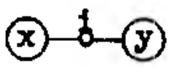
 θ  θ $\{x\}$.

PRIMJER 463

 θ  .

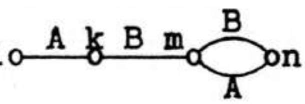
T967,T778,T963,T916,T952 ≻——

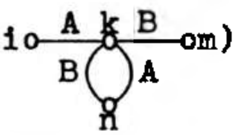
TEOREM 977

$z\in$ 

$\rightarrow \ (\forall j,k,m\in V_x)(\theta(kzm) = \theta(k(xy^{i*j})m))$.

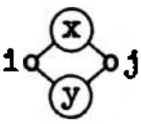
PRIMJER 464

$(x\in$ 

& $y\in$ 

$) \rightarrow \theta(ixm) = \theta(iym)$.

TEOREM 978

$(w\in$ 

& $(\exists k\in V_x)(jxk \ r \ iyj)) \rightarrow$

$(\forall m,n\in V_x)(\theta(mwn) = \theta(m(xy^{j*k})n))$.

D o k a z

(1) $x,y\in G$ & $V_x \cap V_y = \{i,j\}$ & $\neg\, i=j$ & $E_x \cap E_y = 0$ & $w=xy$ &
$(\exists k\in V_x)(jxk \ r \ iyj)$ & $z=xy^{j*k}$ & $\neg\, \lambda_s(w)=\lambda_s(z)$ Sup.

1,T922 ≻——

(2)$\neg\,(\lambda_s(x) + \lambda_s(y) + h(ixj) + h(iyj) - h(iwj) =$
$= \lambda_s(x) + \lambda_s(y^{j*k}) + h(ixk) + h(i(y^{j*k})k) - h(izk))$.

2,1,T952 ≻——

(3)$\neg\,(h(ixj) - h(iwj) = h(ixk) - h(iwk))$

3,T849-T856 ≻——

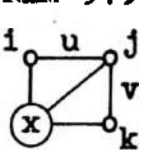
(4)$(l(jxk)=a$ & $l(iyj)=b)$ v $(d(jxk)=b$ & $d(iyj)=a)$

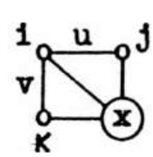
4,T183 ≻——

(5)$\neg\,(jxk \ r \ iyj)$

1,5,T967,T962,T779 ≻—— T978.

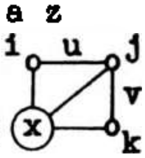
TEOREM 979

$(y\in$ 

& $u \ r \ v) \rightarrow \{y\} \ \theta$ 

.

D o k a z

(1) $y\in$ 

& $u \ r \ v$ Sup.

1,D255,T933,T931,T177,T175 >——

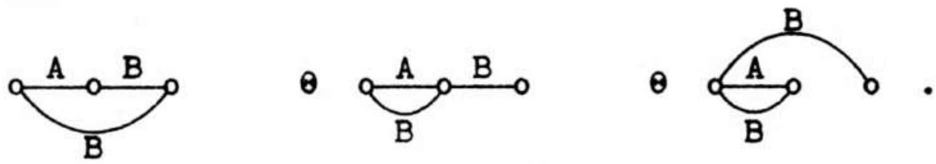
(2) θ(iyj) r u & $V_x = V_y$

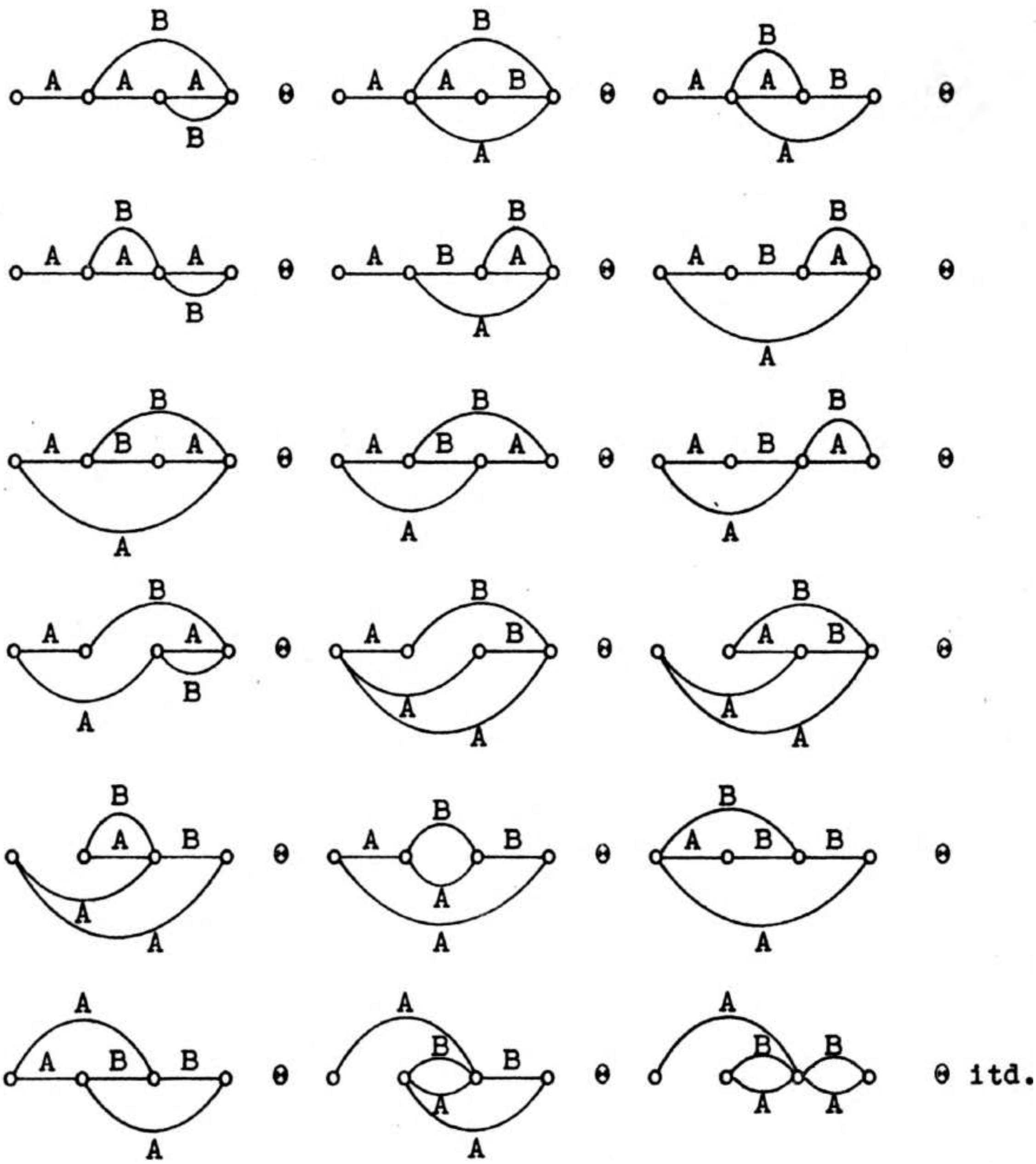
2,1,D288,T179 >——

(3) iyj r v

1,2,3,T931,T978 >—— T979.

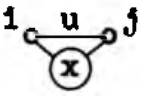
PRIMJERI

465 

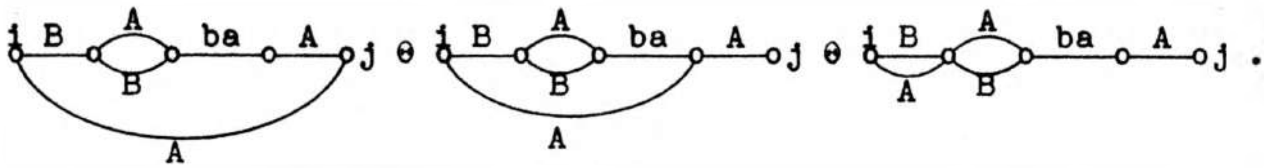
466 

T978,D288,T805,T931 $\succ\!\!-$

TEOREM 980

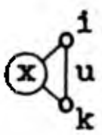
(y∈  & $(\exists k\in V_x)(jxk\ r\ u)$) → {y} θ  .

PRIMJER 467

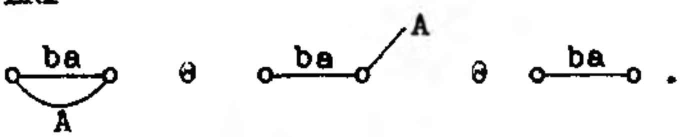


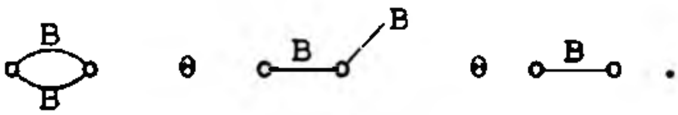
T980,D288,T797 $\succ\!\!-$

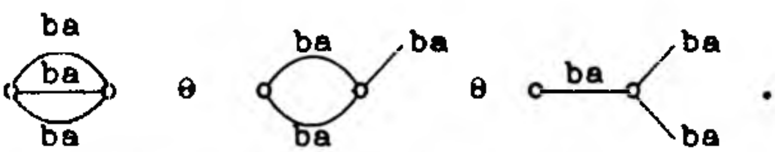
TEOREM 981

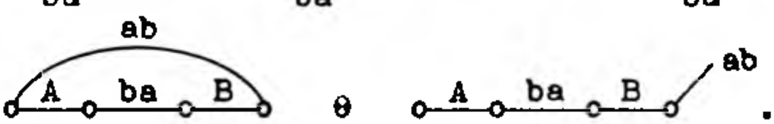
(y∈  & ixk r u) → {y} θ  .

PRIMJERI

468 

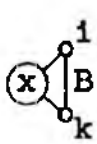
469 

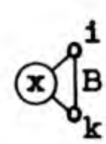
470 

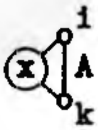
471 

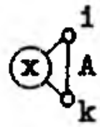
T981,T183,T925,T976 $\succ\!\!-$

TEOREMI 982-985

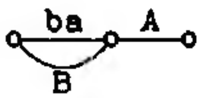
982 (y∈  & l(ixk) = b) → {y} θ {x}.

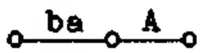
983 (x∈G & $(\exists i,k\in V_x)(l(ixk) = b)$) → {x} θ  .

984 (y∈  & d(ixk) = a) → {y} θ {x}.

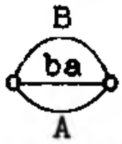
985 (x∈G & (∃i,k∈V_x)(d(ixk) = a)) → {x} θ  .

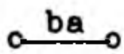
PRIMJERI

472 

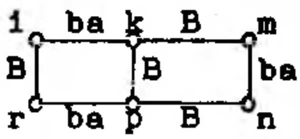
θ 

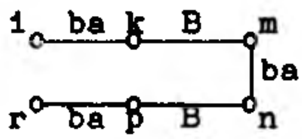
.

473 

θ 

.

474 

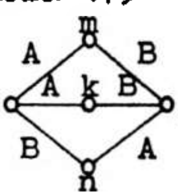
θ 

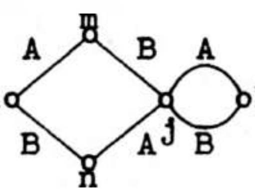
T978 >——

TEOREM 986

(z∈i  j & ixj r iyj) → (∀m,n∈V_x)(θ(mzn) = θ(m($xy^{i\to j}$)n)).

PRIMJER 475

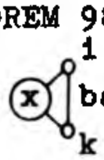
(x∈i 

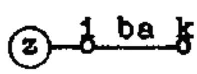
j & y∈i 

k) → θ(mxn) = θ(myn).

T986 >——

TEOREM 987

(y∈ 

& z = $x^{k\to i}$) → {y} θ 

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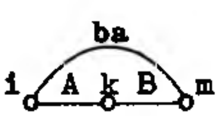
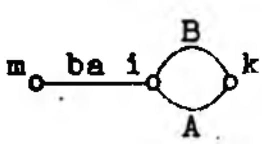
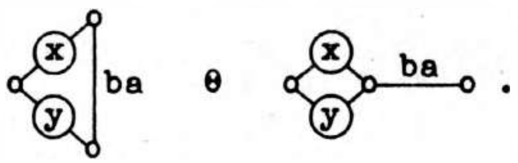
PRIMJER 476



θ 

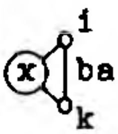
T987,T828 >——

TEOREM 988



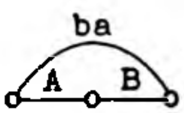
T986,T926,T911 >——

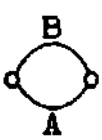
TEOREM 989

$y \in$ 

$\rightarrow \lambda_s(y) = \lambda_s(x^{i \to k})$.

PRIMJER 477

$(x \in$ 

& $y \in$ 

$) \rightarrow \lambda_s(x) = \lambda_s(y)$.

T946,T989 >——

TEOREM 990

$(\forall x \in G)(\forall i,k \in V_x)(\neg\, i=k \rightarrow \lambda_p(ixk) = \lambda_s(x^{i \to k}))$.

T990,T921,T836 >——

TEOREM 991

$(\forall x \in G)(\forall i,k \in V_x)(\neg\, i=k \rightarrow$

$\lambda_p(ixk) = v(x) - \varepsilon_{ba}(x) - 2 - R_a(x^{i \to k}) - R_b(x^{i \to k}))$.

T283,T991,D284 >——

TEOREM 992

$(\forall x \in G)(\forall i,k \in V_x)(\neg\, i=k \rightarrow$

$\lambda(ixk) = \varepsilon(x) - \varepsilon_{ba}(x) - C_a(x) - C_b(x) - R_a(x^{i \to k}) - R_b(x^{i \to k}))$.

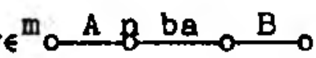
4.11. IZOMORFIZAM m-STRUKTURE I ALGEBRE KLASA OSTATAKA m-DVOPOLA MODULO NULA

DEFINICIJA 294

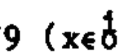
$(\forall x\in G)(\forall i,j\in V_x)(|ixj| = \{kym\colon\ kym\in 2G\ \ \&\ \ \theta(kym) = \theta(ixj)\})$.

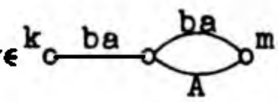
PRIMJERI

478 ($x\in$ 

& $y\in$ 

) $\rightarrow$ $|ixk| = |myn|$.

479 ($x\in$ 

& $y\in$ 

) $\rightarrow$ $|ixi| = |kym|$.

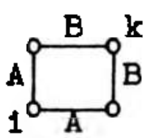
DEFINICIJA 295

$2G\theta = \{X\colon\ ixk\in 2G\ \ \&\ \ X = |ixk|\}$.

DEFINICIJA 296

$(\forall X\in 2G\theta)(\theta(X) = y \ \longleftrightarrow\ ixk\in X\ \ \&\ \ \theta(ixk) = y)$.

PRIMJER 480

$x\in$ 

$\rightarrow$ $\theta|ixk| = ABAB$.

D294-D296,T923 >——

TEOREM 993

$\theta\in S_{bijekc}(2G\theta,M)$.

D294,T927 >——

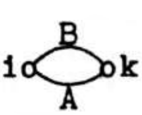
TEOREM 994

$(\forall ixk\in 2G)(|ixk| = |kxi|)$.

DEFINICIJA 297

$(\forall x\in M)(\theta_{2G}^{-1}(x) = Y \ \longleftrightarrow\ Y\in 2G\theta\ \ \&\ \ \theta(Y) = x)$.

PRIMJER 481

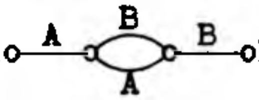
$x\in$ 

$\rightarrow$ $\theta_{2G}^{-1}(BA) = |ixk|$.

DEFINICIJA 298

$(\forall X\in 2G\theta)(\forall F\in\{1,d,\lambda,\lambda_p,\lambda_s\})(F(X) = F(\theta(X)))$.

PRIMJER 482

(x∈ik & X = |ixk|) → l(X) = a &

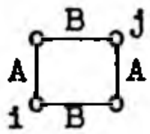
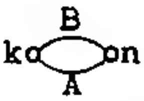
d(X) = b & λ(X) = 4 & λ_p(X) = 2 & λ_s(X) = 1.

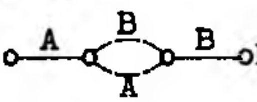
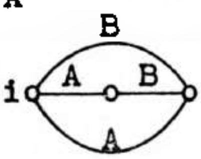
DEFINICIJA 299

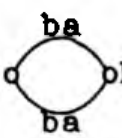
$(\forall X\in 2G\theta)(\forall F\in\{q,q_s,q_p,J_s,J_p,P_d,P_l,Q,D,K,I\})(F(X) = \theta_{2G}^{-1}(F(\theta(X)))$.

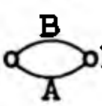
PRIMJERI

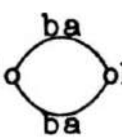
483 x∈($\overset{i}{o}$ o $\overset{k}{o}$ o) & y∈^{m}o—ab—o^n → q|ixk| = |myn|.

484 x∈  & y∈kn → J_s|ixj|= |kym|.

485 x∈ik & y∈i → D|ixk| = |iyk|.

486 x∈ik & y∈($\overset{i}{o}$ o $\overset{k}{o}$) → D|ixk| = |iyk|.

487 x∈ik & y∈($\overset{i}{o}$ o $\overset{k}{o}$) → K|ixk| = |iyk|.

488 x∈$\overset{i}{o}$—A—o—B—$\overset{k}{o}$ & y∈ik → K|ixk| = |iyk|.

489 x∈$\overset{i}{o}$—A—$\overset{k}{o}$ & y∈($\overset{i}{o}$—A—$\overset{k}{o}$ o) → I|ixk| = |iyk|.

DEFINICIJA 300

$(\forall X,Y\in 2G\theta)(\forall F\in\{r,\bar{r},<_s,<_p,<,\theta_n)(X\ F\ Y \longleftrightarrow \theta(X)\ F\ \theta(Y))$.

DEFINICIJA 301

$\Omega_{2G\theta} = \{\theta_{2G}^{-1}(A),\ \theta_{2G}^{-1}(B),\ \theta_{2G}^{-1}(ba),\ \theta_{2G}^{-1}(ab)\}$.

PRIMJERI

490 x∈$\overset{i}{o}$ → θ_{2G}^{-1}(ba) = |ixi|.

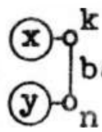
491 x∈($\overset{i}{o}$ $\overset{k}{o}$) → θ_{2G}^{-1}(ab) = |ixk|.

492(x∈$\overset{i}{o}$—u—$\overset{k}{o}$ & u∈{A,B}) → θ_{2G}^{-1}(u) = |ixk|.

D256,T778,T803,D293,D294 $\succ\!\!-\!\!-$

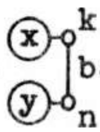
TEOREM 995

$(\forall X,Y\in 2G\theta)(\exists! Z\in 2G\theta)(ixk\in X \ \ \& \ \ i\neq k \ \ \& \ \ myn\in Y \ \ \& \ \ m\neq n \ \ \&$

ze  ba & $Z = |izm|$).

DEFINICIJA 302

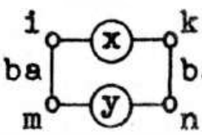
$(\forall X,Y\in 2G\theta)(X \cup Y = Z \ \longleftrightarrow \ (ixk\in X \ \ \& \ \ i\neq k \ \ \& \ \ myn\in Y \ \ \& \ \ m\neq n \ \ \&$

ze  ba & $Z = |izm|$)).

D257,T779,T803,D294,D295 $\succ\!\!-\!\!-$

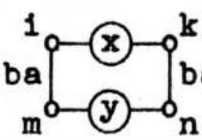
TEOREM 996

$(\forall X,Y\in 2G\theta)(\exists! Z\in 2G\theta)(ixk\in X \ \ \& \ \ myn\in Y \ \ \&$

ze ba  ba & $Z = |izn|$).

DEFINICIJA 303

$(\forall X,Y\in 2G\theta)(X \cap Y = Z \ \longleftrightarrow \ (ixk\in X \ \ \& \ \ myn\in Y \ \ \&$

ze ba  ba & $Z = |izn|$)).

T995,D301,T996,D303 $\succ\!\!-\!\!-$

TEOREM 997

$F\in\{\cup, \cap\} \ \rightarrow \ F\in S_{funkc}(2G\theta\times 2G\theta, 2G\theta).$

DEFINICIJA 304

$\mathscr{M}_{2G\theta} = (2G\theta, \ \Omega_{2G\theta}, \ \cup, \ \cap).$

D301,D303,D256,D257,D294-D296,T778,T779,T803,T932,T933,T931 $\succ\!\!-\!\!-$

TEOREM 998

$(\forall X,Y\in 2G\theta)(\theta(X \cup Y) = \theta(X) \cup \theta(Y) \ \ \& \ \ \theta(X \cap Y) = \theta(X) \cap \theta(Y)).$

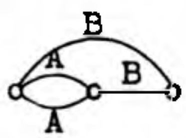
4.12. m_t-G R A F O V I

4.12.1. INTERPRETACIJA DULJINE IMENA KANONSKOG m_t-DVOPOLA TERMINIMA TEORIJE GRAFOVA

DEFINICIJA 305
$G_t = \{x:\ x\in G\ \ \&\ \ \varkappa(x)=1\ \ \&\ \ 1 < v(x)\ \ \&\ \ (\forall e\in E_x)(\omega_x(e)\in\{A,B\})\}.$

PRIMJERI

493 $u\in\{A,B\}\ \ \rightarrow\ \ \circ\!\overset{u}{\text{——}}\!\circ \subset G_t.$

494  $\subset G_t.$

495 $x\in\circ\ \ \rightarrow\ \ \neg\,(x\in G_t).$

496 $x\in(\circ\!\overset{A}{\text{——}}\!\circ\quad \circ\!\overset{B}{\text{——}}\!\circ)\ \ \rightarrow\ \ \neg\,(x\in G_t).$

D305,D188 >——
TEOREM 999
$(\forall x\in G_t)(\varepsilon_{ab}(x) = \varepsilon_{ba}(x) = 0).$

D305,D189,D186,D273 >——
TEOREM 1000
$x\in G_t\ \ \rightarrow\ \ 1 \leq \varepsilon(x).$

D305,D292,D293,D189 (tot.indukc.) >——
TEOREM 1001
$(\forall x,y\in G_t)(\{x\}\ \theta_1\ \{y\}\ \ \rightarrow\ \ \{x\}\ \theta\ \{y\}).$

D305,D289,D189,T309 (tot.indukc.) >——
TEOREM 1002
$(\forall x\in G_t)(\delta(x)\in M_{poz})\ \ \&\ \ (\forall x\in M_{poz})(\exists y\in G_t)(\delta(y) = x).$

D305,D284,D279,T921,T999 >——
TEOREM 1003
$(\forall x\in G_t)(\lambda_a(x) = C(x)-C_a(x)-C_b(x) = R(x)-R_a(x)-R_b(x)).$

DEFINICIJA 306

$2G_t = \{ixj:\ x\in G_t\ \ \&\ \ i,j\in V_x\}.$

D306,T999,T991 $\succ\!\!-$

TEOREM 1004

$(ixj\in 2G_t\ \ \&\ \ \neg\, i=j)\ \ \rightarrow\ \ \lambda_p(ixj)\ =\ v(x)-2-R_a(x^{i\rightarrow j})-R_b(x^{i\rightarrow j}).$

D306,T999,T992 $\succ\!\!-$

TEOREM 1005

$(ixj\in 2G_t\ \ \&\ \ \neg\, i=j)\ \ \rightarrow\ \ \lambda(ixj)\ =\ \varepsilon(x)-C_a(x)-C_b(x)-R_a(x^{i\rightarrow j})-$

$-R_b(x^{i\rightarrow j}).$

T1005,T884 $\succ\!\!-$

TEOREM 1006

$(\forall ixj\in 2G_t)(\lambda(ixj)\ \leq\ \varepsilon(x)).$

TEOREM 1007

$(\forall x\in G_t)(\forall i,j\in V_x)((\varepsilon(x)\ =\ \varepsilon_A(x)\ \ \&\ \ \neg\, i=j)\ \ \rightarrow\ \ \theta(ixj)\ =\ A).$

D o k a z

(1) $ixj\in 2G_t\ \ \&\ \ \varepsilon(x)\ =\ \varepsilon_A(x)\ \ \&\ \ \neg\, i=j$ Sup.

1,D188,D189,D278,D282,D283,D304 $\succ\!\!-$

(2) $C(x)\ =\ C_a(x)\ \ \&\ \ C_b(x)\ =\ 0$

1,2,D305,D284,D282,T999,T1000,D273,T847,T848 $\succ\!\!-$

(3) $\lambda_s(x)\ =\ 0\ \ \&\ \ q(ixj)\ =\ A$

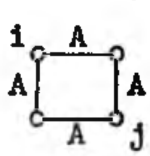
1,3,D286,T217 $\succ\!\!-$ T1007.

D305,D188,D189,D278,D282,D283,D284,T999,T1000,D273,D286,T217 $\succ\!\!-$

TEOREM 1008

$(\forall x\in G_t)(\forall i,j\in V_x)((\varepsilon(x)\ =\ \varepsilon_B(x)\ \ \&\ \ \neg\, i=j)\ \ \rightarrow\ \ \theta(ixj)\ =\ B).$

PRIMJER 497

$x\in$ 

$\rightarrow\ \ \theta(ixj)\ =\ A\ \ \&\ \ \theta(ixi)\ =\ ba.$

DEFINICIJA 307

$2G_{tkan}=\{ixj:\ ixj\in 2G_t\ \&\ (\exists kym\in 2G_t)(\theta(ixj)=\theta(kym)\ \rightarrow\ \varepsilon(x)\ \leq\ \varepsilon(y))\}.$

TEOREM 1009

$(\forall y\in M_{poz})(\exists ixj\in 2G_t)(\theta(ixj) = y \;\&\; \lambda(ixj) = \varepsilon(x) \;\&$
$\lambda_s(x) = C(x) \;\&\; \lambda_p(ixj) = v(x) - 2).$

D o k a z

T931,D189,D278,D305,D186 $\succ\!\!-$

$(1)(x\in \overset{i}{\circ}\overset{u}{-\!\!-}\overset{k}{\circ} \;\&\; u\in\{A,B\}) \rightarrow \theta(ixk) = u \;\&\; \varepsilon(x) = 1 = \lambda(ixk) \;\&$

$\lambda_s(x) = 0 = C(x) \;\&\; \lambda_p(ixk) = v(x) - 2 = 0.$

1,T933,T134,D15,T288,T281,D278,D189,D186,T805,T922 $\succ\!\!-$

$(2)(y\in x \overset{i}{\circ}\,|B\,\underset{k}{\circ} \;\&\; l(ixk) = a) \rightarrow \theta(iyk) = P\theta(ixk) \;\&$

$\lambda(iyk) = \lambda(ixk)+1 \;\&\; \lambda_s(y) = \lambda_s(x)+1 \;\&\; \lambda_p(iyk) = \lambda_p(ixk) \;\&$
$\varepsilon(y) = \varepsilon(x)+1 \;\&\; C(y) = C(x)+1 \;\&\; v(y) = v(x).$

1,T932,T133,T288,D284,T281,D278 $\succ\!\!-$

$(3)(y\in \overset{i}{\circ}\overset{A}{-\!\!-}\overset{k}{\circ}-x \;\&\; (\exists m\in V_x)(l(kxm) = b) \rightarrow \theta(iym) = P(\theta(kxm)) \;\&$
$\lambda(iym) = \lambda(kxm)+1 \;\&\; \lambda_s(y) = \lambda_s(x) \;\&\; \lambda_p(iym) = \lambda_p(kxm)+1 \;\&$
$\varepsilon(y) = \varepsilon(x)+1 \;\&\; C(y) = C(x) \;\&\; v(y) = v(x)+1.$

$(4)\ H = \{n: (y\in M_{poz} \;\&\; \lambda(y) = n) \rightarrow (\exists ixk\in 2G_t)(\theta(ixk) = y \;\&$
$\lambda(ixk) = \varepsilon(x) \;\&\; \lambda_s(x) = C(x) \;\&\; \lambda_p(ixk) = v(x) - 2)\}.$ Def.

4,1 $\succ\!\!-$

$(5)\ 1\in H$

4,2,3 $\succ\!\!-$

$(6)\ n\in H \rightarrow (n+1)\in H$

4,5,6 $\succ\!\!-$

$(7)\ H = N_{poz}.$

4,7,T309 $\succ\!\!-$ T1009.

TEOREM 1010

$(\forall ixk\in 2G_t)(ixk\in 2G_{tkan} \leftrightarrow \lambda(ixk) = \varepsilon(x)).$

D o k a z

$(1)\ ixk\in 2G_{tkan} \;\&\; \lambda(ixk) < \varepsilon(x).$ Sup.

1,T1009,D307 $\succ\!\!-$

$(2)(\exists myn\in 2G_t)(\theta(myn) = \theta(ixk) \;\&\; \lambda(myn) = \varepsilon(y))$

1,2 $\succ\!\!-$

$(3)(\exists myn\in 2G_t)(\theta(myn) = \theta(ixk) \;\&\; \varepsilon(y) < \varepsilon(x))$

3,1,D307,T1006 >——

$(4)(\forall ixk \in 2G_t)(ixk \in 2G_{tkan} \rightarrow \lambda(ixk) = \varepsilon(x))$.

$(5)(\exists ixk \in 2G_t)(\lambda(ixk) = \varepsilon(x) \ \& \ \neg(ixk \in 2G_{tkan})$. Sup.

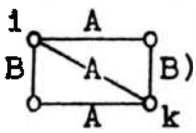
$(6)(\exists myn \in 2G_t)(\vartheta(myn) = \vartheta(ixk) \ \& \ \varepsilon(y) < \varepsilon(x))$ 5,D307

$(7)(\exists myn \in 2G_t)(\varepsilon(y) < \lambda(myn))$ 5,6

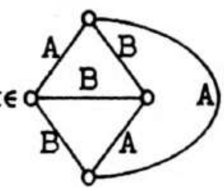
$(8)(\forall ixk \in 2G_t)(\lambda(ixk) = \varepsilon(x) \rightarrow ixk \in 2G_{tkan})$. 5,7,T1006

4,8 >—— T1010.

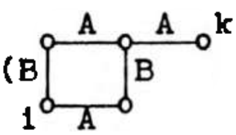
PRIMJERI

498 $x \in ($ 

$)$ $\longrightarrow (\vartheta(ixk) = ABABA \ \& \ \varepsilon(x) = 5) \rightarrow (\lambda(ixk) = \varepsilon(x) \longrightarrow ixk \in 2G_{tkan}$.

499 $x \in$ 

$\& \ i,k \in V_x \longrightarrow (\vartheta(ixk) = BABABA \ \& \ \varepsilon(x) = 6) \rightarrow (\lambda(ixk) = \varepsilon(x)) \longrightarrow ixk \in 2G_{tkan}$.

500 $x \in ($ 

$)$ $\longrightarrow (\vartheta(ixk) = ABAB \ \& \ \varepsilon(x) = 5) \rightarrow (\lambda(ixk) = 4 \neq \varepsilon(x)) \longrightarrow \neg(ixk \in 2G_{tkan})$.

TEOREM 1011

$$(\forall ixk \in 2G_t)(ixk \in 2G_{tkan} \leftrightarrow ((C_a(x) = 0 \ \& \ R_a(x^{i*k}) = 0 \ \& \ (C_b(x) = 0 \ \& \ R_b(x^{i*k}) = 0))).$$

D o k a z

T1010,T992,T994,T884 >——

$$(1)(ixk \in 2G_t \ \& \ \neg i = k) \rightarrow (ixk \in 2G_{tkan} \leftrightarrow C_a(x)=0 \ \& \ C_b(x)=0 \ \& \ R_a(x^{i*k})=0 \ \& \ R_b(x^{i*k})=0)).$$

T1010,T281,T859,T254 >——

$$(2)(\forall ixi \in 2G_t)(ixi \in 2G_{tkan} \leftrightarrow \varepsilon(x) = 2\lambda_a(x)).$$

2,T1003 >——

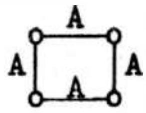
$$(3)(\forall ixi \in 2G_t)(ixi \in 2G_{tkan} \leftrightarrow \varepsilon(x) = C(x)+R(x)-C_a(x)-C_b(x)-R_a(x)-R_b(x)).$$

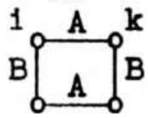
3,D278,D279,T884 >——

$$(4)(\forall ixi\in 2G_t)(ixi\in 2G_{tkan} \longleftrightarrow C_a(x) = 0 \ \& \ C_b(x) = 0 \ \& \ R_a(x) = 0 \ \& \ R_b(x) = 0)\ .$$

1,4,T834 >—— T1011.

PRIMJERI

501 $x\in$  $\rightarrow C_a(x) = 1 \rightarrow (\forall i,k\in V_x)\ \neg(ixk\in 2G_{tkan}).$

502 $x\in$  $\rightarrow R_b(x^{i*k}) = 1 \rightarrow \neg(ixk\in 2G_{tkan}).$

TEOREM 1012

$$(\forall ixk\in 2G_t)(ixk\in 2G_{tkan} \longleftrightarrow \lambda_s(x) = C(x) \ \& \ \lambda_p(ixk) = v(x) - 2).$$

D o k a z

T1011,T1003,T1004 >——

$$(1)(\forall ixk\in 2G_t \ \& \ \neg i=k) \rightarrow (ixk\in 2G_{tkan} \longleftrightarrow \lambda_s(x) = C(x) \ \& \ \lambda_p(ixk) = v(x) - 2\ )).$$

T1011,T1003,T949 >——

$$(2)(\forall ixi\in 2G_t)(ixi\in 2G_{tkan} \longleftrightarrow \lambda_s(x) = C(x) \ \& \ \lambda_p(ixi) = v(x) - 2)$$

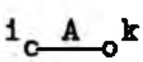
1,2 >—— T1012.

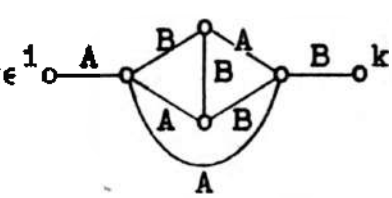
T1012,D279,T836 >——

TEOREM 1013

$$(ixk\in 2G_t \ \& \ \neg i=k) \rightarrow (ixk\in 2G_{tkan} \longleftrightarrow \lambda_s(x) = C(x) \ \& \ \lambda_p(ixk) = R(x^{i*k})).$$

PRIMJERI

503 $x\in$  $\rightarrow \theta(ixk) = A \ \& \ \lambda_s(x) = C(x) = 0 \ \& \ \lambda_p(ixk) = v(x) - 2 = 0 \ \& \ \lambda(ixk) = \varepsilon(x) = 1.$

504 $x\in$  $\longrightarrow$

$$\theta(ixk) = ABABABAB \ \&$$
$$\lambda_s(x) = C(x) = 3 \ \&$$
$$\lambda_p(ixk) = v(x) - 2 = 4 \ \&$$
$$\lambda(ixk) = \varepsilon(x) = 8\ .$$

4.12.2. IZOMORFIZAM ALGEBRE KLASA OSTATAKA m_t-DVOPOLA MODULO NULA I m_t-STRUKTURE

DEFINICIJA 308

$$(ixk \in 2G_t \ \& \ i \neq k) \longrightarrow |ixk|_t = \{myn: myn \in 2G_t \ \& \ m \neq n \ \& \ \vartheta(myn) = \vartheta(ixk)\} .$$

DEFINICIJA 309

$$2G_t\theta = \{X: ixk \in 2G_t \ \& \ i \neq k \ \& \ X = |ixk|_t\}.$$

DEFINICIJA 310

$$A_t = X \longleftrightarrow (x \in \overset{i \ \ A \ \ k}{\delta\!-\!\!-\!\!-\!\delta} \ \& \ X = |ixk|_t).$$

DEFINICIJA 311

$$B_t = X \longleftrightarrow (x \in \overset{i \ \ B \ \ k}{\delta\!-\!\!-\!\!-\!\delta} \ \& \ X = |ixk|_t).$$

DEFINICIJA 312

$$(\forall X,Y \in 2G_t\theta)(X \cup Y = Z \longleftrightarrow ixk \in X \ \& \ myn \in Y \ \& \ \neg((x \ \ y)=0) \ \& \ Z = |i(x^{k \to n})(y)m|_t).$$

DEFINICIJA 313

$$(\forall X,Y \in 2G_t\theta)(X \cap Y = Z \longleftrightarrow ixk \in X \ \& \ myn \in Y \ \& \ \neg((x \ \ y))=0 \ \& \ Z = |i(x^{i \to m})(y^{n \to k})n|_t).$$

A3,T484-T486,D308-D313,T777,D263,T931-T933,T923 $\succ\!\!-\!\!-$

TEOREM 1014

$$(\exists! F \in S_{bijekc}(2G_t\theta, M_t))(F(A_t) = A \ \& \ F(B_t) = B \ \& \ (\forall X,Y \in 2G_t\theta)(F(X \cup Y) = F(X) \cup F(Y) \ \& \ F(X \cap Y) = F(X) \cap F(Y))) .$$

4.13. m_ζ-G R A F O V I

4.13.1. INTERPRETACIJA m-TOPOLOGIJE m_ζ-GRAFOVA TERMINIMA TEORIJE GRAFOVA

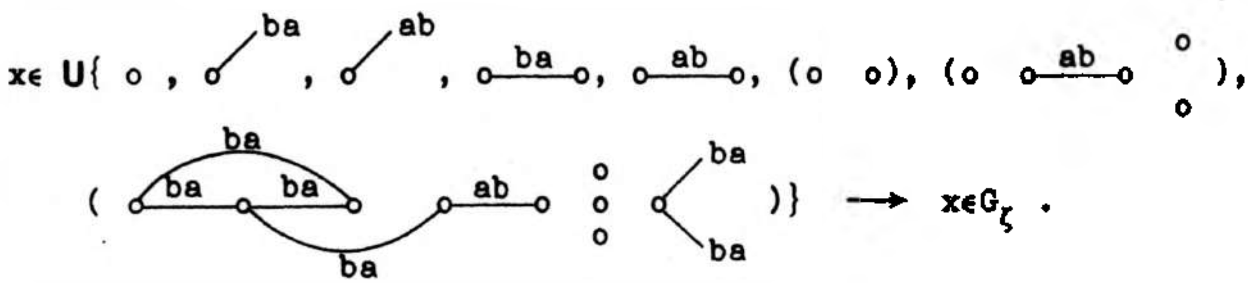
DEFINICIJA 314

$G_\zeta = \{x\colon x\in G \;\; \& \;\; (E_x = 0 \;\; \vee \;\; (\forall e\in E_x)(\omega_x(g)\in\{ba, ab\})))\}.$

DEFINICIJA 315

$2G_\zeta = \{i x j\colon x\in G_\zeta \;\; \& \;\; i, j\in V_x\}.$

PRIMJER 505



D314,D189 >——
TEOREM 1015
$(\forall x\in G_\zeta)(\varepsilon(x) = \varepsilon_{ba}(x)).$

D314,D275,D276,D273 >——
TEOREM 1016
$(\forall x\in G_\zeta)(\varkappa_a(x) = \varkappa_b(x) = \varkappa(x)).$

T1016,D280,D281 >——
TEOREM 1017
$(\forall x\in G_\zeta)(R_a(x) = R_b(x) = \varkappa(x) - 1).$

T1016,T1015,D282,D283 >——
TEOREM 1018
$(\forall x\in G_\zeta)(C_a(x) = C_b(x) = C(x)).$

T921,T1015,T1017,D278 >——
TEOREM 1019
$(\forall x\in G_\zeta)(-\lambda_a(x) = C(x) + \varkappa(x) - 1).$

T1019 >——

TEOREM 1020

$(\forall x \in G_\zeta)(\pi(x) = 1 \rightarrow -\lambda_s(x) = C(x)).$

D314,T878,D278,D279 >——

TEOREM 1021

$(\forall x \in G)(\varepsilon(x) = 0 \rightarrow x \in G_\zeta \ \& \ \pi(x) = v(x) \ \& \ C(x) = 0 \ \& \ R(x)=0).$

T1021,T1019,T1017 >——

TEOREM 1022

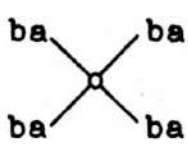
$(\forall x \in G)(\varepsilon(x) = 0 \rightarrow -\lambda_s(x) = v(x) - 1 = R_a(x) = R_b(x)).$

PRIMJERI

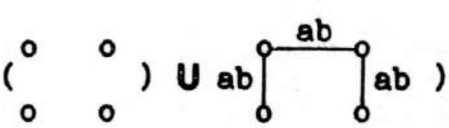
506 x∈ o $\longrightarrow \lambda_s(x) = -(1 - 1) = 0.$

507 x∈ 

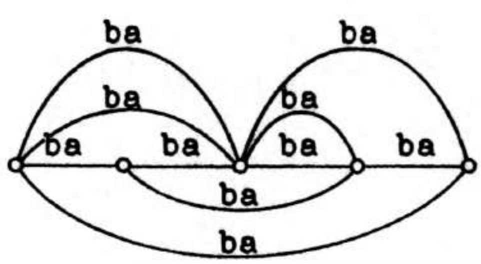
$\longrightarrow \lambda_s(x) = -(1 - 1) = 0.$

508 x∈ 

$\longrightarrow \lambda_s(x) = -(4 + 1 - 1) = -4.$

509 x∈ 

$\longrightarrow \lambda_s(x) = -(0 + 4 - 1) = -3.$

510 x∈ 

$\longrightarrow \lambda_s(x) = -(6 + 1 - 1) = -6.$

T947-T951,D278,T1015-T1020 >——

TEOREM 1023

$(\forall i x k \in 2G_\zeta(\pi(x) = 1 \rightarrow \lambda_p(ixk) = \lambda_s(x) - 1 \ \& \ \lambda(ixk) = 2\lambda_s(x)).$

T950,T1015,T1017,T1021,T1022 >——

TEOREM 1024

$(\forall i x k \in 2G)((\varepsilon(x) = 0 \ \& \ \neg\, i=k) \rightarrow \lambda_p(ixk) = \lambda_s(x) + 1 \ \&$

$\lambda(ixk) = 2\lambda_p(ixk))$.

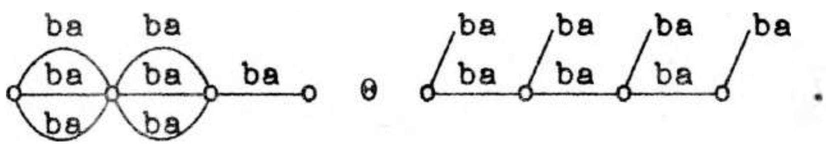
T967,T1020,D278,T966 $>\!\!-\!\!-$

TEOREM 1025

$(\forall x,y\in G_\zeta)(\pi(x) = \pi(y) = 1 \longrightarrow$

$(\{x\}\ \theta\ \{y\} \longleftrightarrow V_x = V_y\ \&\ \varepsilon(x) = \varepsilon(y)))$.

PRIMJER 511



T1025,D278,T1020,T966 $>\!\!-\!\!-$

TEOREM 1026

$(\forall ixj,kym\in 2G_\zeta)(\pi(x) = \pi(y) = 1 \longrightarrow$

$(\theta(ixj) = \theta(kym) \longleftrightarrow C(x) = C(y)))$.

T1026 $>\!\!-\!\!-$

TEOREM 1027

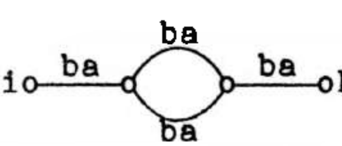
$(\forall x\in G\ )(\pi(x) = 1 \longrightarrow (\forall i,j,k,m\in V_x)(\theta(ixj) = \theta(kym)))$.

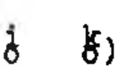
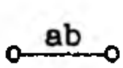
T1022,T967 $>\!\!-\!\!-$

TEOREM 1028

$(\forall x,y\in G)(\varepsilon(x) = \varepsilon(y) = 0 \longrightarrow (\{x\}\ \theta\ \{y\} \longleftrightarrow V_x = V_y))$.

PRIMJERI

512 $x\in i$  $k \longrightarrow \theta(ixi) = \theta(ixk)$.

513$(x\in$  $)\ \&\ y\in$  $\&\ \{x\}\ \theta\ \{y\}) \longrightarrow V_y = \{i,k\}$.

4.13.2. IZOMORFIZAM ALGEBRE KLASA OSTATAKA m_ζ-DVOPOLA MODULO JEDAN I ALGEBRE SUDOVA

DEFINICIJE 316-319

316 $(\forall ixj \in 2G_\zeta)(|ixj|_{\zeta 1} = \{myn\colon myn \in 2G_\zeta \;\&\; q(myn) = q(ixj)\})$.

317 $2G_\zeta\theta_1 = \{X\colon ixj \in 2G_\zeta \;\&\; X = |ixj|_{\zeta 1}\}$.

318 $T_1 = X \longleftrightarrow (x \in i\circ\!\xrightarrow{\ ba\ }\!\circ j \;\&\; X = |ixj|_{\zeta 1})$.

319 $\perp_1 = X \longleftrightarrow (x \in i\circ\!\xrightarrow{\ ab\ }\!\circ j \;\&\; X = |ixj|_{\zeta 1})$.

D316-D319,T861,D267,D268,T862 $>\!\!-\!\!-$
TEOREMI 1029-1035

1029 $T_1 = \{ixj\colon ixj \in 2G_\zeta \;\&\; q(ixj) = ba\}$.

1030 $\perp_1 = \{ixj\colon ixj \in 2G_\zeta \;\&\; q(ixj) = ab\}$.

1031 $(ixj \in 2G_\zeta \;\&\; \pi(x) = 1) \longrightarrow ixj \in T_1$.

1032 $x \in G_\zeta \longrightarrow (\forall i \in V_x)(ixi \in T_1)$.

1033 $(\forall x,y \in G_\zeta)(z \in (\,x \quad y\,) \longrightarrow (\forall i \in V_x)(\forall j \in V_y)(ixj \in \perp_1))$.

1034 $T_1 \cap \perp_1 = 0$.

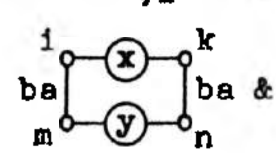
1035 $T_1 \cup \perp_1 = 2G_\zeta\theta_1$.

DEFINICIJE 320-324

$X, Y \in 2G_\zeta\theta_1 \longrightarrow$

320 $\neg_1 X = Y \longleftrightarrow (ixj \in X \;\&\; kym \in Y \;\&\; V_x \cap V_y = 0 \;\&\; q(ixj) = Dq(kym))$

321 $X \;\&_1\; Y = Z \longleftrightarrow (ixk \in X \;\&\; myn \in Y \;\&\; i \neq k \;\&\; m \neq n \;\&$
$z \in x \xrightarrow{k}\circ\!\xrightarrow{\ ba\ }\!\circ\xrightarrow{n} y \;\&\; Z = |izm|_{\zeta 1})$.

322 $X \;v_1\; Y = Z \longleftrightarrow (ixk \in X \;\&\; myn \in Y \;\&\; z \in$  $\&$
$Z = |izn|_{\zeta 1})$.

323 $X \longrightarrow_1 Y = (\neg_1 X) \; v_1 \; Y$.

324 $X \longleftrightarrow_1 Y = (X \rightarrow_1 Y) \;\&_1\; (Y \rightarrow_1 X)$.

D316-D324,D269,D256,D257 >——

TEOREMI 1036-1040

1036 $\neg_1(\top_1) = \bot_1 \;\; \& \;\; \neg_1(\bot_1) = \top_1.$

1037 $X \;\&_1\; Y = \top_1 \;\longleftrightarrow\; (X = \top_1 \;\&\; Y = \top_1).$

1038 $X \;\vee_1\; Y = \bot_1 \;\longleftrightarrow\; (X = \bot_1 \;\&\; Y = \bot_1).$

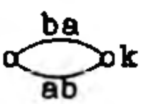
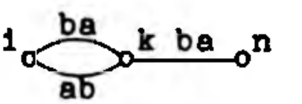
1039 $X \longrightarrow_1 Y = \bot_1 \;\longleftrightarrow\; (X = \top_1 \;\&\; Y = \bot_1).$

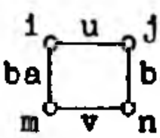
1040 $X \longleftrightarrow_1 Y = \top_1 \;\longleftrightarrow\; X = Y.$

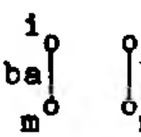
PRIMJERI

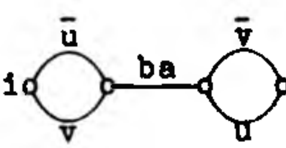
514 $(x \in i\circ\!\overset{u}{\text{——}}\!\circ j \;\&\; y \in k\circ\!\overset{\bar{u}}{\text{——}}\!\circ m \;\&\; V_x \cap V_y = 0 \;\&\; u \in \Omega_o) \longrightarrow$
$\neg_1 |ixj|_{\zeta 1} = |kym|_{\zeta 1}\,.$

515 $x \in ({}^{i}\circ\!\overset{ba}{\text{——}}\!\circ \quad \circ^{j}) \longrightarrow \neg_1 |ixj|_{\zeta 1} = \top_1\,.$

516 (xεi  k & yε(^{m}o o^n) & zε(^{i}  ^{m})) ⟶
$|ixk|_{\zeta 1} \;\&_1\; |myn|_{\zeta 1} = |izm|_{\zeta 1} = \bot_1.$

517 $(x \in i\circ\!\overset{u}{\text{——}}\!\circ j \;\&\; y \in m\circ\!\overset{v}{\text{——}}\!\circ n$ & zε(ba  ba) & $u,v \in \Omega_o) \longrightarrow$
$|ixj|_{\zeta 1} \;\vee_1\; |myn|_{\zeta 1} = |izn|_{\zeta 1}.$

518 $(x \in (i\circ\!\overset{ba}{\text{——}}\!\circ j \;\&\; y \in (m\circ \quad \circ n))$ & zε(ba  ba)) ⟶
$|ixj|_{\zeta 1} \longrightarrow_1 |myn|_{\zeta 1} = |izn|_{\zeta 1}.$

519 $(x \in i\circ\!\overset{u}{\text{——}}\!\circ j \;\&\; y \in m\circ\!\overset{v}{\text{——}}\!\circ n$ & zεi  n & $u,v \in \Omega_o) \longrightarrow$
$|ixj|_{\zeta 1} \longleftrightarrow_1 |myn|_{\zeta 1} = |izn|_{\zeta 1}.$

D316-D324,D256,D257,T1035-T1040,D98,T433 $\succ\!\!-\!\!-$

TEOREM 1041

$(2G_\zeta\theta_1,\ \neg_1,\ \&_1,\ \vee_1,\ \longrightarrow_1,\ \longleftrightarrow_1\ )\in S_{alg.sud.}$.

4.13.3. IZOMORFIZAM ALGEBRE KLASA OSTATAKA m_ζ-DVOPOLA MODULO NULA I m_ζ-STRUKTURE

DEFINICIJE 325-330

325 $(\forall ixj\in 2G_\zeta)(|ixj|_\zeta\ =\ \{kym:\ kym\in 2G_\zeta\ \ \&\ \ \theta(kym)\ =\ \theta(ixj)\})$.

326 $2G_\zeta\theta\ =\ \{X:\ ixj\in 2G_\zeta\ \ \&\ \ X\ =\ |ixj|_\zeta\}$.

327 $(ba)_\zeta\ =\ X\ \ \longleftrightarrow\ \ (x\in\ \overset{i}{\circ}\!\!\overset{ba}{\overline{\qquad}}\!\!\overset{k}{\circ}\ \ \&\ \ X\ =\ |ixk|_\zeta)$.

328 $(ab)_\zeta\ =\ X\ \ \longleftrightarrow\ \ (x\in\ \overset{i}{\circ}\!\!\overset{ab}{\overline{\qquad}}\!\!\overset{k}{\circ}\ \ \&\ \ X\ =\ |ixk|_\zeta)$.

329 $(\forall X,Y\in 2G_\zeta\theta)(X\cup Y\ =\ Z\ \ \longleftrightarrow\ \ (ixk\in X\ \ \&\ \ i\neq k\ \ \&\ \ myn\in Y\ \ \&$

$m\neq n\ \ \&\ \ z\in x\!-\!\overset{k}{\circ}\!\!\overset{ba}{\overline{\qquad}}\!\!\overset{n}{\circ}\!-\!y\ \ \&\ \ Z\ =\ |izm|_\zeta))$.

330 $(\forall X,Y\in 2G_\zeta\theta)(X\cap Y\ =\ Z\ \ \longleftrightarrow\ \ (ixk\in X\ \ \&\ \ myn\in Y\ \ \&$

$$z\in\ \begin{tikzcd} i \arrow[r, dash, "x"] \arrow[d, dash, "ba"'] & k \arrow[d, dash, "ba"] \\ m \arrow[r, dash, "y"'] & n \end{tikzcd}\ \ \&\ \ Z\ =\ |izn|_\zeta))\text{.}$$

D325-D330,D256,D257,T923,T932,T933,D99,T462 $\succ\!\!-\!\!-$

TEOREM 1042

$(\exists! F\in S_{bijekc}(2G_\zeta\theta,M_\zeta))(F((ba)_\zeta)\ =\ ba\ \ \&\ \ F((ab)_\zeta)\ =\ ab\ \ \&$

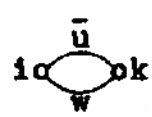
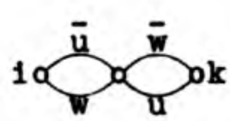
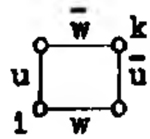
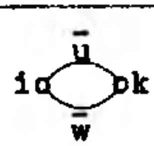
$(\forall X,Y\in 2G_\zeta\theta)(F(X\cup Y)\ =\ F(X)\cup F(Y)\ \ \&$

$F(X\cap Y)\ =\ F(X)\cap F(Y)))$.

4.13.4. INTERPRETACIJA RIJEČI m-TOPOLOGIJE FUNKCIJAMA ALGEBRE SUDOVA

D204-D207,D29,T809,T814,D247,D248,T925,T861,T868,T869 >——

TABLICA KORESPODENCIJE FUNKCIJA ALGEBRE SUDOVA I LJUSKE m_ζ-DVOPOLA OBUHVAĆENIH RIJEČJU m-TOPOLOGIJE

	FUNKCIJA ALGEBRE SUDOVA	FUNKCIJA LJUSKE m_ζ-DVOPOLA			
		x∈	u	w	q(ixk)
1.	Konstanta: T.	i ba k	-	-	ba
2.	Konstanta: ⊥.	i ab k	-	-	ab
3.	Identičnost: F(u) = u.	i u k	ba ab	- -	ba ab
4.	Negacija: F(u) = ¬ u.	i $\bar{u}$ k	ba ab	- -	ab ba
5.	Konjukcija: F(u,w) = (u & w).	i u ○ w k	ba ba ab ab	ba ab ba ab	ba ab ab ab
6.	Disjunkcija: F(u,w) = (u v w).		ba ba ab ab	ba ab ba ab	ba ba ba ab
7.	Implikacija: F(u,w) = (u → w).		ba ba ab ab	ba ab ba ab	ba ab ba ba
8.	Ekvivalencija: F(u,w) = (u ↔ w).		ba ba ab ab	ba ab ba ab	ba ab ab ba
9.	Ekskluzivna disjunkcija: F(u,w) = (u ↮ w).		ba ba ab ab	ba ab ba ab	ab ba ba ab
10.	Shefferova operacija: F(u,w) = (u | w).		ba ba ab ab	ba ab ba ab	ab ba ba ba
11.	Lukasiewiczeva operacija: F(u,w) = (u ↓ w).	i $\bar{u}$ ○ $\bar{w}$ k	ba ba ab ab	ba ab ba ab	ab ab ab ba

Peti dio

ELEKTRIČKE MREŽE I m-BROJEVI

5.1. UVOĐENJE SIMBOLA ZA OSNOVNE TERMINE TEORIJE ELEKTRIČKIH MREŽA

Citat 5

"The main purpose of this chapter is to provide a rigorous mathematical foundation for the discipline of electrical network theory, justifying many of the familiar statements and procedures of network analysis. A general familiarity with network analysis, including the Laplace transformation technique, is assumed in this chapter. Therefore no time is devoted to the "physical aspects" or to the relationships to the other disciplines, e.g., the equations of Maxwell and those of Lagrange.

6-1. Kirchhoff's laws. Since the purpose here is to "prove" some properties of Kirchhoff's current and voltage equations, it is necessary to begin with a precise (postulational) formulation of Kirchhoff's laws. A very brief discussion of the concept of a reference is given first, to allow for the correlation of the present formulation with the conventional ones.

Electrical network theory is formulated in terms of two variables, current and voltage, associated with each network element (branch, in conventional terminology). As is any other physical science, these quantities are correlated with the reading of certain instruments, which in this case are called ammeters and voltmeters. Since our discussion here is concerned with current and voltage as real function of time, $i(t)$ and $v(t)$, the meters should be of the "instantaneous-value" kind. They might be centerzero D'Arsonval meters or oscilloscopes, for instance. As is well known the sign of the reading depends on the way in which the instrument is connected in the network; reversing the terminals changes the reading from positive to negative or vice versa. Hence, for unique correlation of theory with experiment, it is necessary to specify, on the network diagram, how these quantities are to be measured. Such a specification is done by means of current and voltage references.

Definition 6-1. Electrical network. An electrical network is a directed linear graph with two real-valued functions v and i of the real variable t, which are of bounded variation, associated with each edge, satisfying the three postulates N_1, N_2, and N_3 below.

Postulate N_1 Kirchhoff's current law:

$$A_a i(t) = 0, \tag{6-5}$$

where A_a is the vertex matrix of the directed graph

$$i(t) = \begin{bmatrix} i_1(t) \\ i_2(t) \\ \vdots \\ i_e(t) \end{bmatrix} \tag{6-6}$$

and $i_j(t)$ is associated with edge j.

Postulate N_2 Kirchhoff's voltage law:

$$B_a v(t) = 0 \tag{6-7}$$

where B_a is the circuit matrix of the directed graph

$$v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_e(t) \end{bmatrix} \tag{6-8}$$

and $v_j(t)$ is associated with edge j.

Since the properties of incidence and circuit matrices are known, it suffices to restate these results as properties of Kirchhoff's current and voltage equations.

6-3 The third postulate. Postulates N_1 and N_2 are concerned only with the way in which the network elements are interconnected. The character of the network elements (resistor,inductor,capacitor,generator,etc.) does not enter the discussion of Kirchhoff's laws in any way. Kirchhoff's laws are associated purely with the topology of the network.

On the other hand, the character of the individual network element(whether it is a resistor or inductor or generator)is clearly independent of where in the network the element happens to be located.The network element is characterized by the relationship between voltage and current. Postulate N_3 concerns this relationship. The independence of the two aspects of a network (the geomatry,or interconnection aspect,and the character of the network elements) must be clearly borne in mind.

The functions associated with each element of a network, in addition to satisfying Kirchhoff's laws, are required to satisfy a system of integrodifferential equations. These element equations have the general form

$$v(t) = L\frac{di(t)}{dt} + Ri(t) + D\int_0^t i(x)dx + e(t) + v_C(0+). \tag{6-35}$$

The entries in the matrices L, R, and D characterize the equations and the network.

(a) If the matrices are symmetric, the network is bilateral, or reciprocal; otherwise it is nonreciprocal.
(b) If the entries in the matrices L, R, and D are independent of the dependent variables v_j, and i_j, then network is linear; otherwise it is nonlinear.
(c) If the entries in the matrices L, R, and D are functions of the independent variables t, but not of i_j and v_j, the network is linear time-variable network.
(d) If the matrices L, R, and D are positive semidefinite or definite, and if e(t) = 0, the network id passive.
(e) If the matrices R, L, and D contain only constants, the network is linear time-invariant.

The general principles of the discussions in this chapter are applicable to all linear time-invariant networks. The discussions up to this point are applicable to all lumped networks. However, for the major theorems in the rest of this chapter, a linear, reciprocal, time-invariant network with positive semidefinite matrices is assumed. The reason for this restriction is given by means of an example at the end of Section 6-4. The type of network to be considered is characterized by the third postulate.

Postulate N_3. The functions v(t) and i(t) are related by

$$v(t) = L\frac{d}{dt}i(t) + Ri(t) + D\int_0^t i(x)dx + v_C(0) + e(t), \qquad (6\text{-}36)$$

where R and D are real diagonal matrices with nonnegative entries on the main diagonal, and L is real symmetric, with the nonzero rows and columns of L constituting a positive definite submatrix.

(Linear graphs and electrical networks, 1961, str. 117 - 128).

KONVENCIJA 8

Termini "pasivna električka mreža, grana (edge) i čvorište (vertex)", u definicijama 331 i 332 imaju značenje koje proizlazi iz konteksta citata 5.

DEFINICIJA 331

$S_{el.mreža}$ = {x: x je pasivna električka mreža }.

DEFINICIJA 332

$(\forall x \in S_{el.mreža})((V'_x$ je skup čvorišta u x) & (E'_x je skup grana u x & (I'_x je funkcija incidencije u x)).

DEFINICIJA 333

$N = \{(x_1,x_2,x_3): x_1 \in S_{el.mreža}$ & $x_2 \in S_{injekc}(V'_x,V)$ & $x_3 \in S_{injekc}(E'_x,E)$ }.

DEFINICIJA 334

$(\forall x\in N)(F_{\text{mreža}}(x) = x_1 \ \& \ F_{\text{ime čvorišta}}(x) = x_2 \ \&$
$F_{\text{ime grane}}(x) = x_3) \longleftrightarrow x = (x_1,x_2,x_3)).$

DEFINICIJA 335

$(\forall x\in N)(V_x = \text{Adom}(F_{\text{ime čvorišta}}(x) \ \& \ E_x = \text{Adom}(F_{\text{ime grane}}(x))).$

DEFINICIJA 336

$(\forall x\in N)(I_x = \{(e,\{i,k\}):\ e\in E_x \ \& \ i,k\in V_x \ \& \ ((F_{\text{mreža}}(x) = y \ \&$
$F^{-1}_{\text{ime grane}(y)}(e) = e' \ \& \ F^{-1}_{\text{ime čvorišta}(y)}(i) = i' \ \&$
$F^{-1}_{\text{ime čvorišta}(y)}(k) = k') \rightarrow I'_y(e') = \{i,k\})\}.$

DEFINICIJA 337

$(\forall x\in N)(\forall i,k\in V_x)(ixk = (x,(i,k))).$

DEFINICIJA 338

$2N = \{z:\ x\in N \ \& \ i,k\in V_x \ \& \ z = ixk\}.$

Citat 6

"Definition 6-4.Driving-point impedance and driving-point admittance. Let N be an electrical network not containing any (independent) generators, and let two vertices(1,1')of N be designated as input vertices. Then the ratio of the transform of $v_1(t)$ to the transform of $i_1(t)$,with references as shown in Fig. 6-10, under zero initial conditions, is the driving-point impedance at (1,1'):

$$Z_d(s) = \left.\frac{V_1(s)}{I_1(s)}\right|_{\text{all initial conditions zero}} \qquad (6\text{-}110)$$

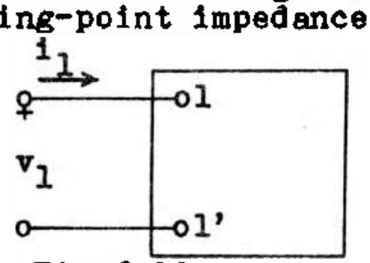


Fig.6-10

The reciprocal of $Z_d(s)$ is the driving-point admittance:

$$Y_d(s) = \left.\frac{I_1(s)}{V_1(s)}\right|_{\text{all initial conditions zero}} \qquad (6\text{-}111)$$

(It may happen that for the given $v_1(t)$there is no solution $i_1(t)$ under zero initial conditions.The definition refers merely to the formal procedure)".(Seshu-Reed,Linear Graphs and Electrical networks,1961,str.149)

KONVENCIJA 9

Termini "impedancija od x u (1,1')" (driving-point impedance at (1,1'), resp."admitancija od x u (1,1')" (driving-point admittance at (1,1')) u definiciji 339, resp. 340 imaju značenje iz definicije 6-4 u citatu 6.

DEFINICIJA 339

$s\in\Xi \;\rightarrow\; (\forall i x k\in 2N)(Z(ixk) = (\Lambda s)(z) \;\longleftrightarrow\; (F_{\text{mreža}}(x) = x' \;\;\&$

z je impedancija od x' u $(1,1')$ & $F_{\text{ime čvorišta}(x')}(1) = i$ &

$F_{\text{ime čvorišta}(x')}(1') = k))$.

DEFINICIJA 340

$s\in\Xi \;\rightarrow\; (\forall i x k\in 2N)(Y(ixk) = (\Lambda s)(z) \;\longleftrightarrow\; (F_{\text{mreža}}(x) = x' \;\;\&$

z je admitancija od x' u $(1,1')$ & $F_{\text{ime čvorišta}(x')}(1) = i$ &

$F_{\text{ime čvorišta}(x')}(1') = k))$.

D339,D340,K9 $\succ\!\!-\!\!-$

TEOREM 1043

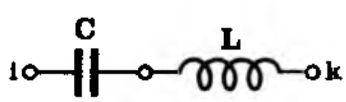
$(\forall i x k\in 2N)(Z(ixk) = Z(kxi) \;\;\&\;\; Y(ixk) = Y(kxi))$.

KONVENCIJA 10

(i) $(\forall i x k\in 2N)(Z(ixk) = (\Lambda s)(z) \frown Z(ixk) = z)$.

(ii) $(\forall i x k\in 2N)(Y(ixk) = (\Lambda s)(z) \frown Y(ixk) = z)$.

PRIMJERI

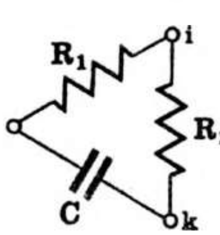
520 (1) $x\in$  Sup.

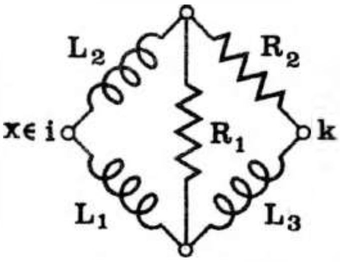
1,D339,Cit.6 $\succ\!\!-\!\!-$

(2) $Z(ixk) = (\Lambda s)(\frac{1 + LCs^2}{Cs})$

2,K10 $\succ\!\!-\!\!-$

(3) $Z(ixk) = \frac{1 + LCs^2}{Cs}$

521 $x\in$  $\longrightarrow\; Z(ixk) = \dfrac{R_1R_2 + R_1^2R_2Cs}{R_1 + R_1(R_1 + R_2)Cs}$.

522 

$\longrightarrow$

$$Z(ixk)=\frac{R_1R_2(L_1+L_3)s+((R_1L_2(L_1+L_3)(R_2(L_1L_2+L_1L_3+L_2L_3))s^2+L_1L_2L_3s^3}{R_1R_2+(R_1(L_1+L_2+L_3)+R_2(L_1+L_2))s+L_3(L_1+L_2)s^2}.$$

Citat 7

"7-2 Driving-point and transfer admittance. Figure 7-4 shows a one terminal-pair network not containing any generators. V_{11}, and I_1 denote the transforms of the voltage current respectivly,with references as shown. By Definition 6-4, the driving-point admittance at terminals (1,1') is

$$Y_d(s) = \frac{I_1(s)}{V_{1,1'}(s)} , \tag{7-32}$$

with all initial conditions equal to zero. If node equations are written with 1' as the reference node, then,

$$Y_d(s) = \frac{\Delta}{\Delta_{11}} , \tag{7-33}$$

where Δ and Δ_{11} are the determinant and cofactor (1,1) respectively, of the node-admittance matrix, as in Eq. (6-116b). It is important to note that the node-admittance matrix of the network of Fig.7-4 (including I_1) is the same as the matrix $Y_n(s)$for the one terminal-pair alone, without $I_1(s)$.(If loop equations are used, the matrices with and without the driver are different)." (Seshu-Reed,Linear graphs and electrical networks.1961,str.165).

KONVENCIJA 11

Termin "matrica admitancije čvorišta"(node-admittance matrix), u definiciji 341 ima značenje u smislu konteksta citata 7.

DEFINICIJA 341

(i)$(\forall ixk\in 2N)(\Delta(ixk) = z \longleftrightarrow (F_{\text{mreža}}(x) = x'$ & (z je determinanta matrice admitancije čvorišta od x' sa 1' kao referentnim čvorištem) & $F_{\text{ime čvorišta}(x')}(1') = i))$.

(ii)$(\forall ixk\in 2N)(\Delta_{11}(ixk)$ je kofaktor (1,1) determinante $\Delta(ixk))$.

D341,K11 $\succ$——

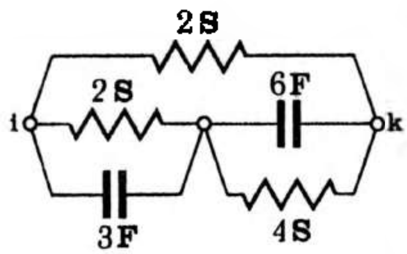
TEOREM 1044

$(\forall ixk\in 2N)(Y(ixk) = \frac{\Delta(ixk)}{\Delta_{11}(ixk)})$.

DEFINICIJA 342

$(\forall ixk \in 2N)(El_{\Delta}(ixk) = El(\Delta(ixk),\ \Delta_{11}(ixk))).$

PRIMJERI

523 (1) $x \in$ 

Sup.

1,D341 $\succ\!\!\!-$

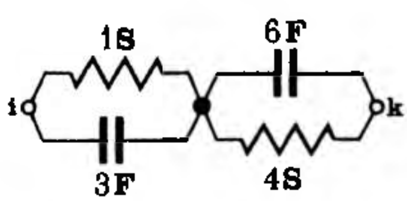
(2) $\Delta(ixk) = \begin{vmatrix} (4+2+6s) & -(4+6s) \\ -(4+6s) & (2+4+3s+6s) \end{vmatrix} = 18s^2 + 42s + 20$

2,D341 $\succ\!\!\!-$

(3) $\Delta_{11}(ixk) = 6 + 9s$

2,3,D342 $\succ\!\!\!-$

(4) $El_{\Delta}(ixk) = \begin{vmatrix} 18 & 42 & 20 \\ 9 & 6 & 0 \\ 0 & 9 & 6 \end{vmatrix} = 0.$

524 (1) $x \in$ 

Sup.

1,D341 $\succ\!\!\!-$

(2) $\Delta(ixk) = \begin{vmatrix} (4+6s) & -(4+6s) \\ -(4+6s) & (5+9s) \end{vmatrix} = 18s^2 + 18s + 4.$

2,D341 $\succ\!\!\!-$

(3) $\Delta_{11}(ixk) = 9s + 5$

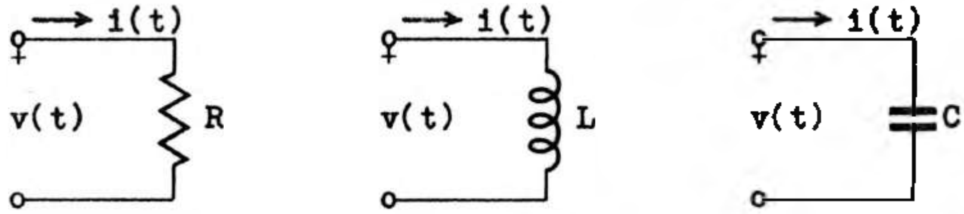
2,3,D342 $\succ\!\!\!-$

(4) $El_{\Delta}(ixk) = \begin{vmatrix} 18 & 18 & 4 \\ 9 & 5 & 0 \\ 0 & 9 & 5 \end{vmatrix} = -36.$

Citat 8

1.3 Network Elements

We define three network elements in our model: resistance, inductance, and capacitance. The diagrammatic representations are shown in Fig.7.



$v(t) = Ri(t)$ (a) $v(t) = L\frac{di}{dt}$ (b) $i(t) = C\frac{dv}{dt}$ (c)

Fig.7 Network elements

The precise definitions of the elements are given in terms of the relationships between the current and the voltage at the element terminals. These definitions are shown in Fig. 7 and are tabulated in Table 1.

Table 1

Element	Symbol	Voltage-current-relationships
Resistance	R	$v = Ri$
		$i = \frac{1}{R} v = Gv$
Inductance	L	$v = L \frac{di}{dt}$
		$i(t) = \frac{1}{L}\int_0^t v(x)dx + i(0)$
Capacitance	C	$i = C \frac{dv}{dt}$
		$v(t) = \frac{1}{C}\int_0^t i(x)dx + v(0)$

Note that these expressions are valid only for the voltage and current references shown on the diagrams.Reversing either a current or a voltage reference will reverse the sign of the corresponding expression.(Seshu-Balabanian,Linear network analysis, 1959,str.10-12)

KONVENCIJA 12

Termini "kapacitet", "otpor" i "induktivitet" u definicijama 343-345 znače grane, odnosno elemente pasivnih električkih mreža u smislu konteksta citata 8.

DEFINICIJE 343-345

343 $(\forall x \in S_{el.mreža})(E'_C(x) = \{z\colon z \in E'(x) \ \& \ \text{"z je kapacitet"}\})$.

344 $(\forall x \in S_{el.mreža})(E'_R(x) = \{z\colon z \in E'(x) \ \& \ \text{"z je otpor"}\})$.

345 $(\forall x \in S_{el.mreža})(E'_L(x) = \{z\colon z \in E'(x) \ \& \ \text{"z je induktivitet"}\})$.

DEFINICIJE 346-348

346 $N_{\bar{1},1} = \{x\colon x \in N \ \& \ (F_{mreža}(x) = y \rightarrow E'_y = E'_C(y) \cup E'_L(y))\}$.

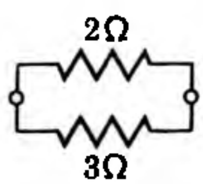
347 $N_{\bar{1},0} = \{x\colon x \in N \ \& \ (F_{mreža}(x) = y \rightarrow E'_y = E'_C(y) \cup E'_R(y))\}$.

348 $N_{0,1} = \{x\colon x \in N \ \& \ (F_{mreža}(x) = y \rightarrow E'_y = E'_R(y) \cup E'_L(y))\}$.

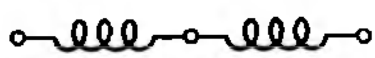
PRIMJERI

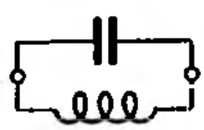
525 $x \in$ 

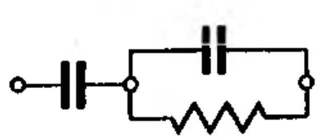
$\longrightarrow$ $x \in (N_{\bar{1},1} \cup N_{\bar{1},0})$.

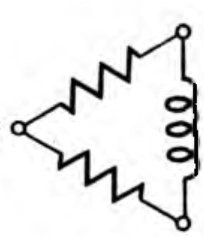
526 $x \in$ 

$\longrightarrow$ $x \in (N_{\bar{1},0} \cup N_{0,1})$.

527 $x \in$  $\longrightarrow$ $x \in (N_{0,1} \cup N_{\bar{1},1})$

528 $x \in$  $\longrightarrow$ $x \in N_{\bar{1},1}$.

529 $x \in$  $\longrightarrow$ $x \in N_{\bar{1},0}$.

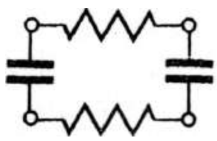
530 $x \in$  $\longrightarrow$ $x \in N_{0,1}$.

Citat 9
"Definicija 5. Otvoren put je jednostavan ako su u njemu svaka dva čvorišta različita". (D. Blanuša, Viša matematika I, 1963, str.194)

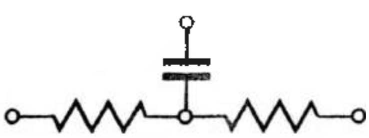
DEFINICIJA 349
$(\forall(e,f)\in\{(\bar{1},1),(\bar{1},0),(0,1)\})(2N_{e,f} = \{z\colon x\in N_{e,f}$ & $j,k\in V_x$ & $z = jxk$ & (svaka grana u grafu od x nalazi se bar u jednom jednostavnom putu (cit.9) koji povezuju čvorišta "j" i "k")}).

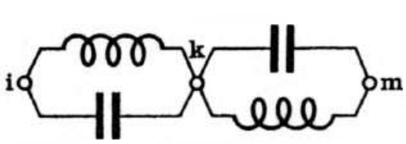
KONVENCIJA 13
"$(\forall(e,f)\in\{(\bar{1},0),(\bar{1},1),(0,1)\})(\forall ixk\in 2N_{e,f})X \longleftrightarrow (\forall xik\in 2N_{ef})X$.

PRIMJERI

531 (x∈  & $i,j\in V_x$ & $i\neq j$) $\longrightarrow$ $ixj\in 2N_{\bar{1},0}$.

532 (x∈  & $i,j\in V_x$) $\longrightarrow$ $\neg\, ixj\in 2N_{0,1}$.

533 (x∈  & $i,j\in V_x$) $\longrightarrow$ $\neg\, ixj\in 2N_{\bar{1},0}$.

534 (x∈ 

$\longrightarrow$ $\neg\, ixk\in 2N_{\bar{1},1}$.

535 (x∈( ) & $i,j\in V_x$) $\longrightarrow$ $\neg\, ixj\in 2N_{\bar{1},0}$.

Citat 10
"Definition 3-1. A graph G is nonseparable if every subgraph of G has at least two vertices in common with its complement. All other graphs are separable.
Definition 3-13. A one terminal-pair graph G is nonseparable if the graph obtained by adding an edge between the terminals is nonseparable". (Seshu-Reed, Lin.Graphs, and electric.netw. 1961, str. 35 i str. 53).

D349,Cit.10 >——

TEOREM 027

$(\forall ixk\in 2N_{ef})$(ixk je "neseparabilan dvopol" ("a nonseparable one terminal-pair graph")).

5.2. m-BROJEVI KAO TEMELJNE KARAKTERISTIKE DVOGENERATORSKIH ELEKTRIČKIH DVOPOLA

5.2.1. m-GRAFOVI DVOGENERATORSKIH ELEKTRIČKIH DVOPOLA

DEFINICIJA 350

$(\forall x\in N_{\bar{1},1})(\omega_x = \{(e,u):\ e\in E_x\ \ \&\ \ (F^{-1}_{ime}\ {}_{grane(x)}(e)\in E'_C(x)\ \rightarrow\ u = A)\ \ \&\ \ F^{-1}_{ime}\ {}_{grane(x)}(e)\in E'_L(x)\ \rightarrow\ u = B)\}).$

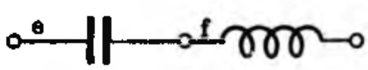
DEFINICIJA 351

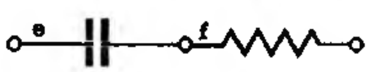
$(\forall x\in N_{\bar{1},0})(\omega_x = \{(e,u):\ e\in E_x\ \ \&\ \ (F^{-1}_{ime}\ {}_{grane(x)}(e)\in E'_C(x)\ \rightarrow\ u = A)\ \ \&\ \ F^{-1}_{ime}\ {}_{grane(x)}(e)\in E'_R(x)\ \rightarrow\ u = B)\}).$

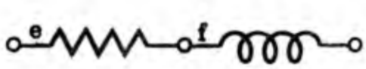
DEFINICIJA 352

$(\forall x\in N_{0,1})(\omega_x = \{(e,u):\ e\in E_x\ \ \&\ \ (F^{-1}_{ime}\ {}_{grane(x)}(e)\in E'_R(x)\ \rightarrow\ u = A)\ \ \&\ \ (F^{-1}_{ime}\ {}_{grane(x)}(e)\in E'_L(x)\ \rightarrow\ u = B)\}).$

PRIMJERI

536 $x\in$  $\longrightarrow$ $\omega_x = \{(e,A),(f,B)\}$.

537 $x\in$  $\longrightarrow$ $\omega_x = \{(e,A),(f,B)\}$.

538 $x\in$  $\longrightarrow$ $\omega_x = \{(e,A),(f,B)\}$.

D350-352,T18 >——

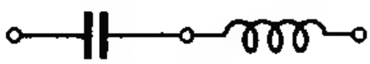
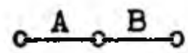
TEOREM 1045

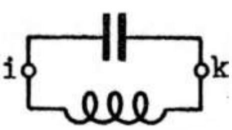
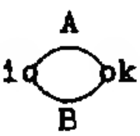
$(\forall(e,f)\in\{(\bar{1},0),(\bar{1},1),(0,1)\})(\forall x\in N_{e,f})(\omega_x\in S_{funkc}(E_x,\Omega_t))$.

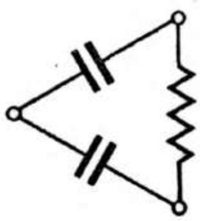
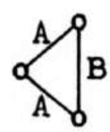
DEFINICIJA 353

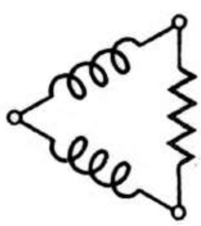
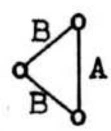
$(\forall(e,f)\in\{(1,1),(1,0),(0,1)\})(\forall x\in N_{ef})(O_{ef}(x) = y \longleftrightarrow y\in G \;\&\; V_y=V_x \;\&\; E_y=E_x \;\&\; I_y=I_x \;\&\; \omega_y=\omega_x)$.

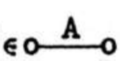
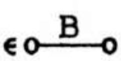
PRIMJERI

539 $x\in$  $\longrightarrow O_{\bar{1},1}(x)\in$  .

540 $x\in$  $\longrightarrow O_{\bar{1},1}(x)\in$ 

541 $x\in$  $\longrightarrow O_{\bar{1},0}(x)\in$ 

542 $x\in$  $\longrightarrow O_{0,1}(x)\in$ 

543 $x\in$  $\longrightarrow (O_{0,1}(x)\in$  $\&\; O_{\bar{1},0}(x)\in$ ).

D353,D333,D305 >——

TEOREM 1046

$(\forall(e,f)\in\{(\bar{1},1),(\bar{1},0),(0,1)\})(O_{e,f}\in S_{funkc}(N_{ef},G_t))$.

KONVENCIJA 14

$O_{e,f} \frown O$.

DEFINICIJA 354

$2G_N = \{ixk:\ ixk\in 2G \;\&\; (\exists iyk\in 2N_{ef})(ixk = iO(y)k)\}$.

5.2.2. IME IMPEDANCIJE I ADMITANCIJE DVOGENERATORSKIH ELEKTRIČKIH DVOPOLA

TEOREM 1047

$s \in \Xi \rightarrow$ $(((\Lambda s)(F)$ je reaktancija (reactance function, Reaktanz-Funktion)) $\longleftrightarrow$ $(\Lambda s)(F) \in T_{\overline{1},1})$.

D o k a z

(1)"Theorem 9. A real rational function $\psi(s)$ is a reactance function if and only if (a) all of its poles are simple and lie on the $j\omega$-axis; (b) the residues are all real and positive; (c) the function has either a pole or a zero at $s=0$ and $s=\infty$; and (d) Re $(j\omega)=0$ for some ω".(Seshu-Balabanian,Linear Network Analysis, 1959,str.368),

1 >——

(2) funkcija kompleksne varijable F(s) je reaktancija,ako i samo ako egzistiraju nenegativni realni brojevi g,m, i egzistira 2n pozitivnih realnih brojeva $h_1..h_n,k_1..k_n$, (pri čemu n može biti i nula), takvih da je

$$F(s) = \frac{g}{s} + \sum_{i=1}^{n} \left(\frac{h_i s}{k_i + s^2}\right) + ms.$$

2,D137,D130,T670,T671 >—— T1047.

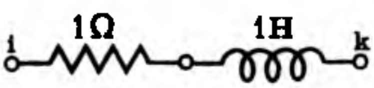
DEFINICIJA 355

$(\forall(e,f) \in \{(\overline{1},1),(\overline{1},0),(0,1)\})(Z_{e,f} = \{(ixk,z): ixk \in 2N_{e,f}$ & $Z(ixk) = z\})$.

DEFINICIJA 356

$(\forall(e,f) \in \{(\overline{1},1),(\overline{1},0),(0,1)\})(Y_{e,f} = \{(ixk,z): ixk \in 2N_{e,f}$ & $Y(ixk) = z\})$.

PRIMJERI

544 x∈ 

$\longrightarrow$ $(ixk,1+s) \in Z_{0,1}$.

545 x∈ 

$\longrightarrow$ $(ixk,1+s) \in Y_{0,1}$.

TEOREM 1048

$$(\forall e,f\in\{(\bar{1},1),(\bar{1},0),(0,1)\})(Z_{e,f}\in S_{surjekc}(2N_{e,f},T_{e,f})\ \&\ Y_{e,f}\in S_{surjekc}(2N_{e,f},T_{\bar{f},\bar{e}}))\ .$$

D o k a z

(1)"Theorem 11. A rational function of s is a reactance function if and only if it is the driving-point impedance or admittance of a lossless network.

Theorem 14. A rational function F(s) is the driving-point impedence of an RC network or the admittance of an RL network if and only if all of its poles are simple and are restricted to the finitive negative real axis (including s=0 but excluding s=∞) with real positive residues at all poles and with F(∞) real and nonnegative.

Theorem 16. A rational function F(s) is the driving-point impedance of an RL network or the admittance of an RC network if and only if all of its poles are simple and are restricted to the negative real axis, excluding the point s=0 (but including infinity) with F(0) real and nonnegative, and with all the residues of $\frac{F(s)}{s}$ real and positive".

(Seshu-Balabanian, Linear Network Analysis, 1959, str. 370-375).

1(Th11),T1047,D339,D340,D346,T670,T671 $\succ\!\!-$

$$(2)s\in\Xi\ \rightarrow\ ((\Lambda s)(F)\in T_{\bar{1},1}\ \longleftrightarrow\ (\exists ixj\in 2N_{\bar{1},1})(Z(ixj)\ =\ (\Lambda s)(F))\ \&$$
$$(\Lambda s)(F)\in T_{\bar{1},1}\ \longleftrightarrow\ (\exists ixj\in 2N_{\bar{1},1})(Y(ixj)\ =\ (\Lambda s)(F))).$$

1(Th14),D347,D348,D355,D356 $\succ\!\!-$.

$$(3)(ixk,F(s))\in(Z_{\bar{1},0}\cap Y_{0,1})\ \longleftrightarrow\ (\exists g,m\in Re^{+})(F(s)\ =\ \frac{g}{s}+m\ \ \vee$$
$$(\exists h_1,..h_n,k_1,..k_n\in Re_{poz})(F(s)\ =\ \frac{g}{s}\ +\ \sum_{i=1}^{n}\ (\frac{h_i}{k_i+s})\ +\ m)).$$

3,D137,D130,T576,T577,T670,T671,D339,D340 $\succ\!\!-$

$$(4)s\in\Xi\ \rightarrow\ ((\Lambda s)(F)\in T_{\bar{1},0}\ \longleftrightarrow\ (\exists ixj\in 2N_{\bar{1},0})(Z(ixj)\ =\ (\Lambda s)(F))\ \&$$
$$(\Lambda s)(F)\in T_{\bar{1},0}\ \longleftrightarrow\ (\exists ixj\in 2N_{0,1})(Y(ixj)\ =\ (\Lambda x)(F))).$$

1(Th16),D347,D348,D355,D356 $\succ\!\!-$.

$$(5)(ixk,F(s))\in(Z_{0,1}\cap Y_{\bar{1},0})\ \longleftrightarrow\ (\exists g,m\in Re^{+})(F(s)\ =\ g\ +\ ms\ \ \vee$$
$$(\exists h_1,..h_n,k_1,..k_n\in Re_{poz})(F(s)\ =\ g\ +\ \sum_{i=1}^{n}\ (\frac{h_i s}{k_i+s}\ )\ +\ ms)).$$

5,D137,D130,T576,T577,T670,T671,D339,D340 $\succ\!\!-$

$$(6)s\in\Xi\ \rightarrow\ ((\Lambda s)(F)\in T_{0,1}\ \longleftrightarrow\ (\exists ixj\in 2N_{0,1})(Z(ixj)\ =\ (\Lambda s)(F))\ \&$$
$$(\Lambda s)(F)\in T_{0,1}\ \longleftrightarrow\ (\exists ixj\in 2N_{\bar{1},0})(Y(ixj)\ =\ (\Lambda s)(F))).$$

$2,4,6 \succ\!\!-$

$(7)s\in\Xi \;\rightarrow\; (\forall(e,f)\in\{(\bar{1},0),(\bar{1},1),(0,1)\})$

$((\Lambda s)(F)\in T_{e,f} \;\leftrightarrow\; (\exists ixj\in 2N_{e,f})(Z(ixj) = (\Lambda s)(F))\;\;\&$

$(\Lambda s)(F)\in T_{e,f} \;\leftrightarrow\; (\exists ixj\in 2N_{\bar{f},\bar{e}})(Y(ixj) = (\Lambda s)(F)))\;.$

$7,D355,D356 \succ\!\!-$ T1048.

T1048,D156,T652,Cit.6 $\succ\!\!-$ T1049,T1050.

TEOREM 1049

$(\forall ixk\in 2N_{ef})(Z(ixk) = RY(ixk) = \frac{1}{Y(ixk)}).$

TEOREM 1050

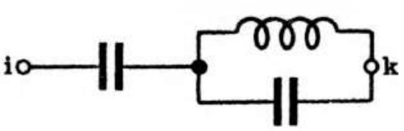
$(\forall ixk\in 2N_{ef})(Y(ixk) = RZ(ixk) = \frac{1}{Z(ixk)}).$

T1049,T1050,T721 $\succ\!\!-$

TEOREM 1051

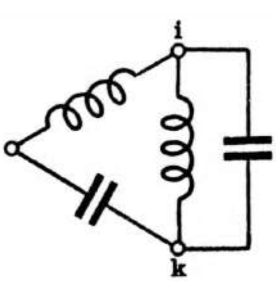
$(\forall ixk\in 2N_{ef})(\vartheta(Z(ixk)) = D\vartheta(Y(ixk))).$

PRIMJERI

546 $x\in$ 

$\longrightarrow \vartheta Z(ixk) = ABA\;\;\&$

$\vartheta Y(ixk) = BAB\;\;.$

547 $(x\in$ 

$\&\;\; i,k\in V_x) \longrightarrow \vartheta Z(ixk) = BABA.$

5.2.3. LJUSKA ELEKTRIČKIH DVOPOLA I ALGEBRA REZIDUALNIH KLASA LRC-DVOPOLA MODULO JEDAN

5.2.3.1. KORESPODENCIJA IZMEDU LJUSKE IMPEDANCIJE, RESP. ADMITANCIJE DVOGENERATORSKOG ELEKTRIČKOG DVOPOLA I LJUSKE NJEGOVOG m-GRAFA

TEOREM 1052

$(\forall jxk \in 2N_{ef})(q(Z(jxk)) = q(jO(x)k))$.

D o k a z

(1) $jxk \in 2N_{\bar{1},1}$ & $q(Z(jxk)) = aa$ Sup.

1,T610,D140,D141 >——

(2) $\lim\limits_{s \to 0}(Z(jxk)) = \lim\limits_{s \to 0}(k_1 s^{-1}) = \infty$ & $\lim\limits_{s \to \infty}(Z(jxk)) = \lim\limits_{s \to \infty}(k_2 s^{-1}) = 0$

2,D339,Cit.6 >——

(3)(U svakom putu u x, koji povezuje čvorišta "j" i "k" nalazi se bar jedan kapacitet) & (egzistira put u x koji veže čvorišta "j" i "k" i sve grane puta su kapaciteti).

3,D350-D353,D267,D268 >——

(4)$(\forall jxk \in 2N_{\bar{1},1})(q(Z(jxk)) = aa \rightarrow q(jO(x)k) = aa)$

(5) $jxk \in 2N_{\bar{1},1}$ & $q(jO(x)k) = aa$ Sup.

5,D350-D353,D267,D268 >——

(6)(U svakom putu u x, koji povezuje čvorišta "j" i "k" nalazi se bar jedan kapacitet) & (egzistira put u x koji povezuje čvorišta "j" i "k" i sve grane puta su kapaciteti).

6,D339,Cit.6 >——

(7) $\lim\limits_{s \to 0}(Z(jxk)) = \infty$ & $\lim\limits_{s \to \infty}(Z(jxk)) = 0$

5,7,T610,D140,D141 >——

(8)$(\forall jxk \in 2N_{\bar{1},1})(q(jO(x)k) = aa \rightarrow q(Z(jxk)) = aa)$

4,8 >——

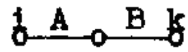
(9)$(\forall jxk \in N_{\bar{1},1})(q(Z(jxk)) = aa \leftrightarrow q(jO(x)k) = aa)$.

Iz 9 i korespodentnih lema za ostale tri moguće vrijednosti ljuske CL-dvopola, teorem 1052 vrijedi za svaki CL-dvopol,a da vrijedi i za svaki CR-dvopol i za svaki RL-dvopol, dokazuje se analogno.

PRIMJER 548

(1) $x \in$  Sup.

1,D350,D339 $\succ\!\!-\!\!-$

(2) $Z(ixk) = \dfrac{1 + LCs^2}{Cs}$ & $O(x) \in$ 

2,D269 $\succ\!\!-\!\!-$

(3) $\lim\limits_{s \to 0}(Z(ixk)) = \infty$ & $\lim\limits_{s \to \infty}(Z(ixk)) = \infty$ & $q(iO(x)k) = ab$

3,T610,D140,D141 $\succ\!\!-\!\!-$

(4) $q(Z(ixk)) = ab = q(iO(x)k)$.

T1052,T1051 $\succ\!\!-\!\!-$

TEOREM 1053

$(\forall jxk \in 2N_{ef})(q(Y(jxk)) = Dq(jO(x)k))$.

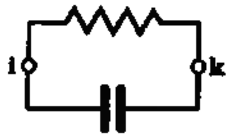
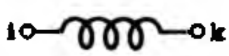
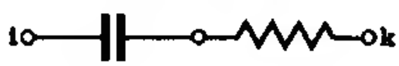
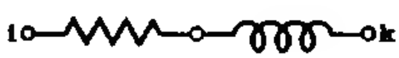
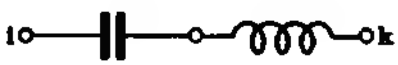
5.2.3.2. ALGEBRA REZIDUALNIH KLASA LRC-DVOPOLA MODULO JEDAN

Limes funkcije impedancije LRC-dvopola, kad "s" teži nuli, ili je jednak nuli, ili je jednak beskonačnom,ili nije niti prvo niti drugo. Isto vrijedi i za limes funkcije impedancije kad "s" teži beskonačnom.Na osnovi ovakvog ponašanja funkcija kompleksne varijable u nuli i u beskonačnosti, razvrstane su i funkcije impedancije LRC-dvopola u devet ekvivalencijskih klasa.Neka je svakoj od ovih klasa pridružen po jedan uređeni par iz skupa$\{\bar{1},0,1\}$ i to tako, da je prvi član para $\bar{1}$, resp. 1, resp. 0 jedino onda, ako impedancija reprezentanata ekvivalencijske klase ima u nuli pol, resp. nulu, resp. niti jedno niti drugo, dok je drugi član para jednak $\bar{1}$, resp. 1, resp. 0, jedino onda ako impedancija reprezentanta klase ima u beskonačnom nulu, resp. pol, resp. niti prvo niti drugo. Neka se ovako, svakom LRC-dvopolu,pridružen par brojeva zove "ljuska impedancije LRC-dvopola".

U analogiji sa m-grafom dvogeneratorske mreže, m-graf RLC-mreže karakteriziran je time, što se kod njega razlikuju tri tipa grana, koje neka se zovu C-, resp. R-, resp. L-grane. Pretpostavlja se da je odavde jasno, što se podrazumjeva pod C-putem, resp. C-rezom i sličnim terminima uz prefikse R-, resp. L-.

Skup ovakvih m-grafova, odnosno skup m-dvopola koji se iz njih dobiva, također je razvrstan u devet ekvivalencijskih klasa, već prema tome iz kojeg tipa grana se sastoji put, resp.rez u odnosu na čvorišta dvopola.Neka je svakoj od ovih ekvivalencijskih klasa pridružen po jedan uređeni par iz skupa simbola $\{C,R,L\}$ i to tako da je prvi član para jednak C, resp. L, resp. R, jedino ako reprezentant klase ima C-rez, resp. L-put, resp. nema niti prvo niti drugo, dok je drugi član para jednak C, resp. L, resp. R, samo onda ako reprezentant klase ima C-put, resp. L-rez, resp. nema niti prvo niti drugo.Neka se ovako svakom LRC-dvopolu pridružen par simbola zove "ljuska m-grafa LRC-dvopola".

Lako je uočiti da su obe particije LRC-dvopola u ekvivalencijske klase, međusobno jednake, odnosno da kao generalizacija teorema 1052 vrijedi i za svaka dva LRC-dvopola, da im je ljuska impedancije jednaka, ako i samo ako im je jednaka i ljuska pripadnih m-dvopola. Opisanu korespodenciju prikazuje tablica 1 .

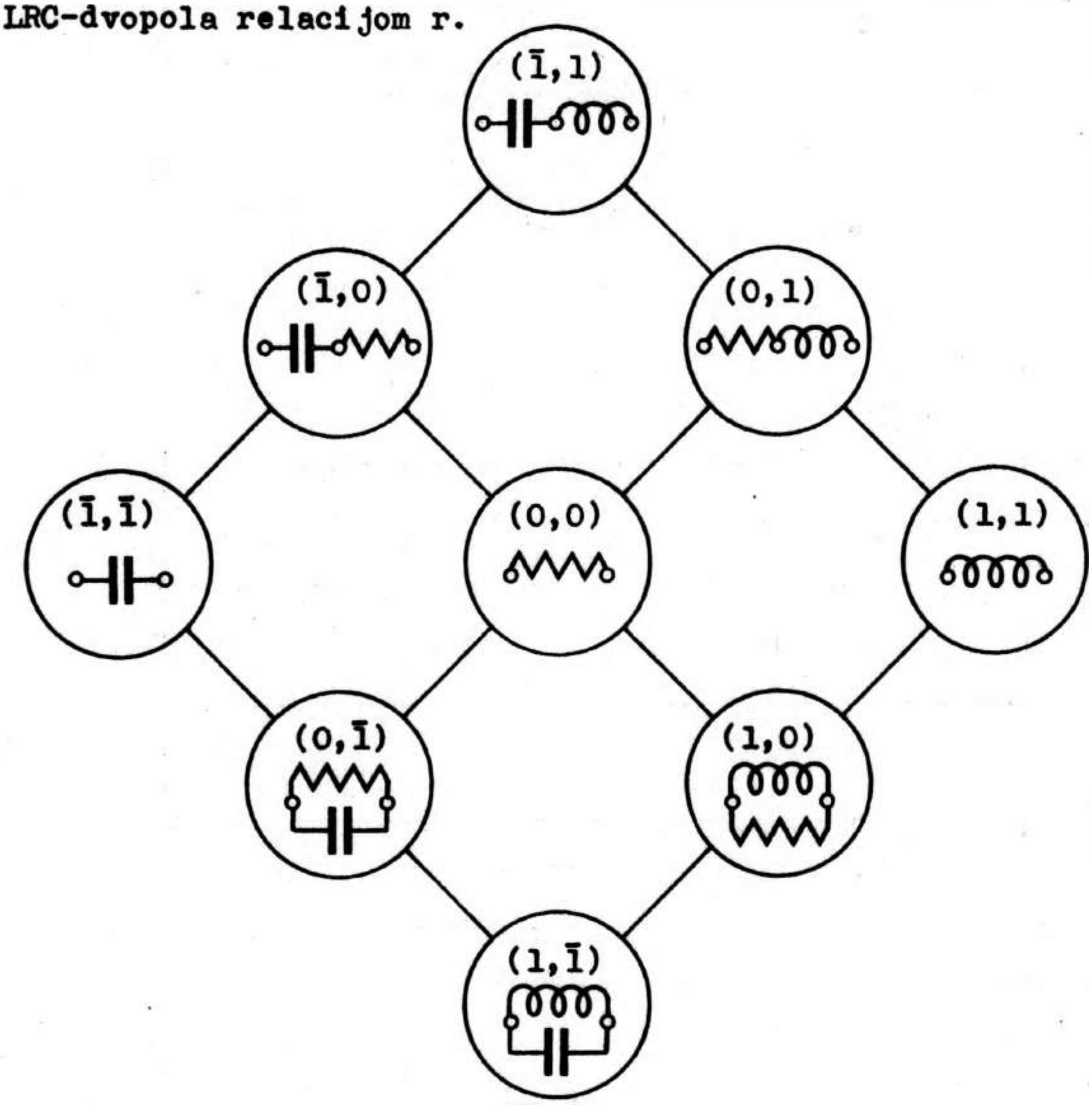
x	q(Z(ixk))	q(iO(x)k)
	$(1,\bar{1})$	(L,C)
	$(0,\bar{1})$	(R,C)
	(1,0)	(L,R)
	$(\bar{1},\bar{1})$	(C,C)
	(0,0)	(R,R)
	(1,1)	(L,L)
	$(\bar{1},0)$	(C,R)
	(0,1)	(R,L)
	$(\bar{1},1)$	(C,L)

Tb.1

Devet ekvivalencijskih klasa LRC-dvopola prikazano je u prvom stupcu preko reprezentanata klase koji imaju najmanji broj grana. U drugom, resp. trećem stupcu tablice naznačene su vrijednosti ljuske impedancije, resp. ljuske m-dvopola koje pripadaju svakom dvopolu iz korespodentne ekvivalencijske klase RLC-dvopola.

U analogiji sa kvazi uređenjem r u teoriji m-brojeva,definirano je u skupu ekvivalencijskih klasa LRC-dvopola parcijalno uređenje "r" tako, da je za svaka dva RLC-dvopola "x r y", onda i samo onda, ako prvi član ljuske impedancije od x nije manji od prvog člana ljuske impedancije od y i drugi član ljuske impedancije od x nije veći od drugog člana ljuske impedancije od y.

Ako se "x r y" tumači kao "x je niži od y", onda slika 1 je Hasseov dijagram parcijalnog uređenja skupa ekvivalencijskih klasa LRC-dvopola relacijom r.



Sl.1

Algebra rezidualnih klasa LRC-dvopola modulo jedan je algebra koja se dobiva uvođenjem, u skup ekvivalencijskih klasa LRC-dvopola, dviju binarnih operacija i to preko serijskog,resp. paralelnog spajanja reprezentanata ekvivalencijskih klasa.Ova je algebra izomorfna rešetci (lattice) koja se dobiva iz parcijalnog uređenja, prema slici 1, uz supremum i infimum kao binarnim operacijama, pri čemu "serijskom spajanju" korespondira operacija "sup", a "paralelnom spajanju" operacija "inf".Rešetka na slici 1 izomorfna je kardinalnom produktu antiizomorfno uređenih tročlanih lanaca, kao što su npr. $\{+1 < 0 < -1\}$ i $\{-1 < 0 < +1\}$.

5.2.4. KORESPODENCIJA IZMEDU IMENA IMPEDANCIJE RESP. ADMITANCIJE I IMENA m-GRAFA DVOGENERATORSKOG ELEKTRIČKOG DVOPOLA.

Citat 11

(1)8-1.Enumeration of natural frequencies. The zeros of the network determinant (either loop or node)are referred to as the natural frequencies of the network, for these are the frequencies of the transient response. One of the classical problems is to count the number of natural frequencies of a network by inspection. An early solution to this problem is an algorithm due to Guillemin(69), applicable to networks which contain no all-capacitor or all-inductor loops. More recently, Reza(145) gave the solution for networks containing only two types elements.The complete solution was obtained independently by Bryant(18), Bers (10), and Seshu (in unpublished notes,1958).
Definition 8-1. Order of complexity. The order of complexity of a network is the number of finite non zero zeros of the determinant (loop or node),with each R,L and C considered as a network element.

----

(2)It is convenient to use a formal method of computing the number of linearly independent cut-sets of G contained in a subgraph and thereby introduce some useful notation.Let G be, for instance, the L-subgraph, i.e.,the subgraph consisting of all the inductor.Consider short-circuiting all elements of G which are not inductors. From the vertex-partitioning interpretation, of a cutset given in Section 2-4 (immediately preceding Definition 2-12), it is clear that every all-inductor cut-set of G remains a cutset of the resultant graph.Since the new graph contains only inductors, the number of linearly independent L-cut-set of G is equal to the rank of the graph obtained by short-circuiting all other elements.Let this number be denoted by ρ_{Ls}, with a similar meaning for ρ_{Cs}. Similarly, the rank of the L-subgraph [= (number of inductors) - (nullity of L-subgraph)] can be found by open-circuiting all other elements and can be denoted by ρ_{Lo} , with a similar meaning for ρ_{Co}.Similarly, ρ_{Lo} is the nullity of the graph obtained by deleting (open-circuiting) all non-L-elements, etc.

----

(3)Theorem 8-1. The order of complexity of a passive network without mutual inductances is

$$N = \rho_{Co} + \rho_{Lo} - \rho_{Cs} - \rho_{Ls} , \qquad (8\text{-}8)$$

with the above notation".(Seshu-Reed, Linear graphs and electrical networks, 1961, str.197-200).

KONVENCIJA 15

Za svako $x \in N_{ef}$, "red kompleksnosti od x" je "order of complexity of a network" iz definicije 8-1 u citatu 11(1).

TEOREM 1054

$(\forall(e,f)\in\{(\bar{1},1),(\bar{1},0),(0,1)\})(\forall x\in N_{ef})$("Red kompleksnosti od x" jednak je $(f-e)\lambda_s(O(x))$).

D o k a z

D186,D188,D280-283,D353,(T025,T026),Cit.11(2) $\rightarrowtail$ (1),(2),(3).

(1)$(\forall x\in N_{\bar{1},1})(\rho_{L_s} = R_b(O(x))$ & $\rho_{C_s} = R_a(O(x))$ &

$\rho_{L_o} = \varepsilon_B(O(x)) - C_b(O(x)) = v(O(x)) - 1 - R_a(O(x))$ &

$\rho_{C_o} = \varepsilon_A(O(x)) - C_a(O(x)) = v(O(x)) - 1 - R_b(O(x)))$.

(2)$(\forall x\in N_{\bar{1},1})(\rho_{L_s} = 0$ & $\rho_{C_s} = R_a(O(x))$ & $\rho_{L_o} = 0$ &

$\rho_{C_o} = v(O(x)) - 1 - R_b(O(x))$.

(3)$(\forall x\in N_{0,1})(\rho_{L_s} = R_b(O(x))$ & $\rho_{C_s} = 0$ & $\rho_{C_o} = 0$ &

$\rho_{L_o} = v(O(x)) - 1 - R_a(O(x)))$.

1,2,3,T921 $\rightarrowtail$ (4),(5).

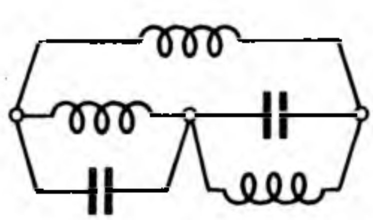
(4)$(\forall x\in N_{\bar{1},1})(\rho_{L_o} + \rho_{C_o} - \rho_{L_s} - \rho_{C_s} = 2(v(O(x))-1-R_a(O(x))-R_b(O(x)) =$

$= 2\lambda_s(O(x)))$.

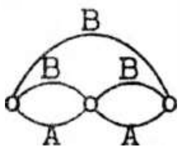
(5)$(\forall x\in(N_{\bar{1},0} \cup N_{0,1}))(\rho_{L_o} + \rho_{C_o} - \rho_{L_s} - \rho_{C_s} = \lambda_s(O(x)))$.

Cit.11,(4),(5) $\rightarrowtail$ T1054.

PRIMJERI

549 (1) $x\in$  Sup.

1,D353,D346 $\succ\!\!-$

(2) $O(x)\in$ 

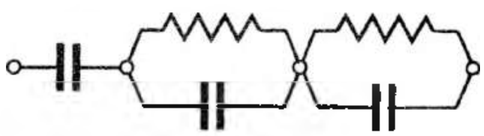
$\&$ $x\in N_{\bar{1},1}$

2,D284 $\succ\!\!-$

(3) $\lambda_s(O(x)) = 2$

2,3,T1054 $\succ\!\!-$

(4) red kompleksnosti od x jednak je $(1-(-1))2 = 4$.

550 $x\in$  $\longrightarrow$

red kompleksnosti od x jednak je $(0-(-1))2 = 2$.

TEOREM 1055

$(\forall ixk\in 2N_{ef})(\neg El_\Delta(ixk) = 0 \longleftrightarrow \lambda_p(Y(ixk)) = \lambda_s(iO(x)k))$.

D o k a z

(1) $ixk\in 2N_{ef}$ $\&$ $\neg El_\Delta(ixk) = 0$ Sup.

1,D342,T1044,Cit.11(D8-1),T1054,T628 $\succ\!\!-$

(2) $(f-e)\lambda_p(Y(ixk)) = (f-e)\lambda_s(iO(x)k)$

1,2 $\succ\!\!-$

(3)$(\forall ixj\in 2N_{ef})(\neg El_\Delta(ixk) = 0 \rightarrow \lambda_p(Y(ixk)) = \lambda_s(iO(x)k)$

3,D324,T1044,T1054,T628 $\succ\!\!-$ T1055.

T1055,T1051,T625 $\succ\!\!-$

TEOREM 1056

$(\forall ixk\in 2N_{ef})(\neg El_\Delta(ixk) = 0 \longleftrightarrow \lambda_s(Z(ixk)) = \lambda_s(iO(x)k)$.

T1054-T1056,T1044 $\succ\!\!-$ T1057,T1058.

TEOREM 1057

$(\forall ixk\in 2N_{ef})(\lambda_s(Z(ixk)) \leq \lambda_s(iO(x)k))$.

TEOREM 1058

$(\forall ixk\in 2N_{ef})(\lambda_p(Y(ixk)) \leq \lambda_p(iO(x)k))$.

T1054-T1056,T1052 $\succ\!\!-$ T1059,T1060.

TEOREM 1059

$(\forall ixk\in 2N_{ef})(\vartheta(Z(ixk)) = \vartheta(iO(x)k) \longleftrightarrow \neg El_\Delta(ixk) = 0)$.

TEOREM 1060

$(\forall ixk\in 2N_{ef})(\vartheta(Y(ixk)) = D\vartheta(iO(x)k) \longleftrightarrow \neg El_{\Delta}(ixk) = 0).$

T1057,T1058,T1052,T361 ≻—— T1061,T1062.

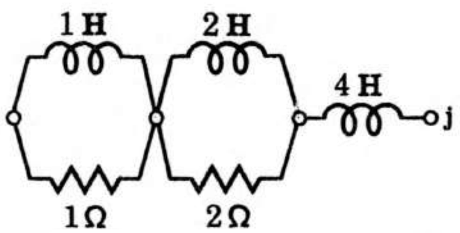
TEOREM 1061

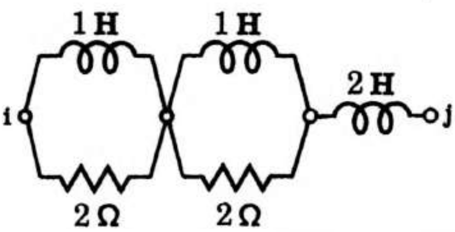
$(\forall ixk\in 2N_{ef})(\vartheta(Z(ixk)) \leq \vartheta(iO(x)k)).$

TEOREM 1062

$(\forall ixk\in 2N_{ef})(\vartheta(Y(ixk)) \leq D\vartheta(iO(x)k)).$

PRIMJER 551

(1) $x\in$ 

& $y\in$ 

Sup.

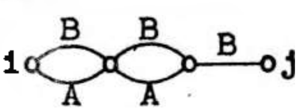
1,D339,K9 ≻——

(2) $Z(ixj) = s\,\dfrac{21 + 22s + 8s^2}{3 + 7s + 2s^2}$ & $Z(iyj) = s\,\dfrac{4 + s}{1 + s}$

2,T1048,D180,D140,D141,D147 ≻——

(3) $\vartheta(Z(ixj)) = BABAB$ & $\vartheta(Z(iyj)) = BAB$

1,D353,T1046 ≻——

(4) $O(x)$, $O(y)\in$ 

4,D286 ≻——

(5) $\vartheta(iO(x)j) = \vartheta(iO(y)j) = BABAB$

3,5,T361 ≻——

(6) $\vartheta(Z(ixj)) = \vartheta(iO(x)j)$ & $\vartheta(Z(iyj) \leq \vartheta(iO(y)j).$

DEFINICIJA 357

$(\forall(e,f))(2N_{ef(nedeg)} = \{ixk\colon ixk\in 2N_{ef}$ & $\neg El_{\Delta}(ixk) = 0\}).$

D357,T1059,T1060 ≻—— T1063,T1064.

TEOREM 1063

$(\forall ixk\in 2N_{ef(nedeg)})(\vartheta(Z(ixk)) = \vartheta(iO(x)k)).$

TEOREM 1064

$(\forall ixk\in 2N_{ef(nedeg)})(\vartheta(Y(ixk)) = D\vartheta(iO(x)k)).$

5.3. INTERPRETACIJA RELACIJE KONGRUENCIJE MODULO NULA m_t-STRUKTURE U MODELU TEORIJE ELEKTRIČKIH MREŽA

5.3.1. IZOMORFIZAM DVOGENERATORSKIH ELEKTRIČKIH DVOPOLA

DEFINICIJA 358

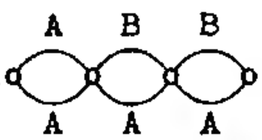
$(\forall(e,f)\in\{(\bar{1},1),(\bar{1},0),(0,1)\})(\forall ixk\in 2G_N)$

$(O^{-1}_{ef}(ixk) = \{myn:\ myn\in 2N_{ef(nedeg)}\ \ \&\ \ mO(y)n \approx ixk\})$.

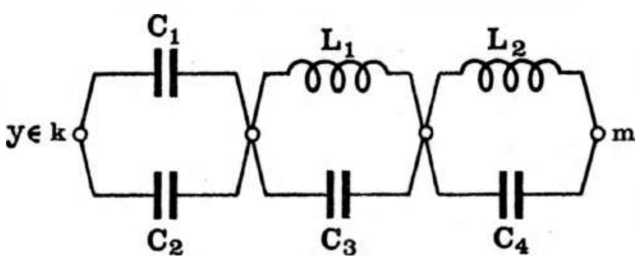
PRIMJERI

552 $x\in i\circ\!\!\xrightarrow{A}\!\!\circ j \ \rightarrow\ (kym\in O^{-1}_{0,1}(ixj) \ \longleftrightarrow\ (R\in Re_{poz}\ \ \&$

$y\in k$  m)).

553 $x\in i$  $j \ \rightarrow\ (kym\in O^{-1}_{\bar{1},1}(ixj) \ \longleftrightarrow$

$(C_1,C_2,C_3,C_4,L_1,L_2\in Re_{poz}\ \ \&\ \ L_1C_3 \neq L_2C_4\ \ \&$



)).

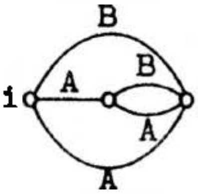
Heuristički evidentan je

TEOREM 1065

$(\forall z\in T_{ef})(\forall ixj\in 2G_N)(\theta(z) = \theta(ixj) \ \rightarrow$

$(\exists kym\in O^{-1}_{ef}(ixj))(Z(kym) = z))$.

PRIMJERI

554 (1) $z = s\,\dfrac{0{,}7s^2 + 0{,}05}{1{,}7s^4 + s^2 + 0{,}05}$ & x∈i 

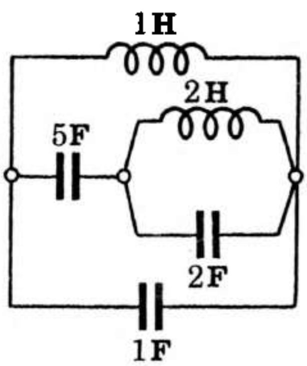
j Sup.

1,D166,T674,T602,D180,D286 >——

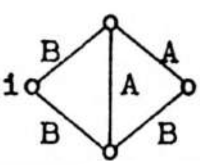
(2) $z \in T_{\bar{1},1}$ & $\vartheta(z) = \vartheta(ixj)$ = BABA

1,2,T1065 >——

(3) Postoji LC-dvopol sa impedancijom jednakom z i m-grafom izomorfnim sa x, kao npr.

y∈i 

j ⟶ $(iyj \in O^{-1}_{\bar{1},1}(ixj)$ & $Z(iyj) = z)$.

555 (1) $z = s\,\dfrac{6s^2 + 30s + 8}{9s^2 + 12s + 2}$ & x∈i 

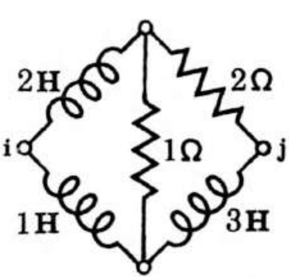
j Sup.

1,D166,T647,T602,D180,D286 >——

(2) $z \in T_{0,1}$ & $\vartheta(z) = \vartheta(ixj)$ = ABABA

1,2,T1065 >——

(4) Postoji RL-dvopol sa impedancijom jednakom z i m-grafom izomorfnim sa x, kao npr.

y∈i 

j ⟶ $(iyj \in O^{-1}_{0,1}(ixj))$ & $Z(iyj) = z$.

5.3.2. PROSTOR IMPEDANCIJE I PROSTOR ADMITANCIJE NAD SKUPOM IZOMORFNIH DVOGENERATORSKIH ELEKTRIČKIH DVOPOLA

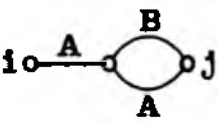
DEFINICIJA 359

$(\forall ixj\in 2G_N)(ZO^{-1}_{ef}(ixj) = \{z:\ kym\in O^{-1}_{ef}(ixj)\ \ \&\ \ Z(kym) = z\})$.

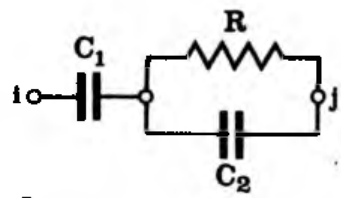
DEFINICIJA 360

$(\forall ixj\in 2G_N)(YO^{-1}_{ef}(ixj) = \{z:\ kym\in O^{-1}_{ef}(ixj)\ \ \&\ \ Y(kym) = z\})$.

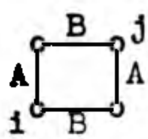
PRIMJERI

556 $x\in$ 

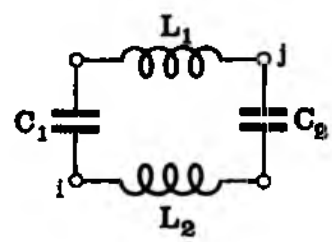
$\rightarrow\ (z\in ZO^{-1}_{1,0}(ixj)\ \longleftrightarrow$

$(C_1, C_2, R\in Re_{poz}\ \ \&\ \ y\in$ 

$\&\ \ Z(iyj) = z))$.

557 $x\in$ 

$\rightarrow\ (z\in YO^{-1}_{1,0}(ixj)\ \longleftrightarrow\ (C_1, C_2, L_1, L_2\in Re_{poz}\ \ \&$

$C_1L_1 \neq C_2L_2\ \ \&\ \ y\in$ 

$\&\ \ Y(iyj) = z))$.

D359,D360,D358,D357,T1059,T1060 $\succ\!\!-$ T1066,T1067.

TEOREM 1066

$(\forall z\in T_{ef})(\forall ixj\in 2G_N)(z\in ZO^{-1}_{ef}(ixj)\ \longleftrightarrow\ \theta(z) = \theta(ixj))$.

TEOREM 1067

$(\forall z\in T_{ef})(\forall ixj\in 2G_N)(z\in YO^{-1}_{ef}(ixj)\ \longleftrightarrow\ \theta(z) = D\theta(ixj))$.

TEOREM 1068

$(\forall ixj, kym\in 2G_N)(\theta(ixj) = \theta(kym)\ \longleftrightarrow$

$(\exists(e,f))(ZO^{-1}_{ef}(ixj) = ZO^{-1}_{ef}(kym)))$.

D o k a z

(1) $ixj, kym\in 2G_N\ \ \&\ \ \theta(ixj) = \theta(kym)$ Sup.

1,T1065 >——

(2) $z \in ZO^{-1}_{ef}(ixj) \rightarrow z \in ZO^{-1}_{ef}(kym)$ &

$w \in ZO^{-1}_{ef}(kym) \rightarrow w \in ZO^{-1}_{ef}(ixj)$.

1,2, >——

(3)$(\forall ixj, kym \in 2G_N)(\theta(ixj) = \theta(kym) \rightarrow ZO^{-1}_{ef}(ixj) = ZO^{-1}_{ef}(kym))$

(4)$(\exists ixj, kym \in 2G_N)(ZO^{-1}_{ef}(ixj) = ZO^{-1}_{ef}(kym) = W)$ Sup.

4,T1065 >——

(5)$(\exists z \in W)(\theta(z) = \theta(ixj)$ & $(\exists w \in W)(\theta(w) = \theta(kym))$

5,T1061 >——

(6) $\theta(z) \leq \theta(kym)$ & $\theta(w) \leq \theta(ixj)$

5,6 >——

(7) $\theta(ixj) \leq \theta(kym)$ & $\theta(kym) \leq \theta(ixj)$

7,T358 >——

(8)$(\forall ixj, kym \in 2G_N)(ZO^{-1}_{ef}(kym) \rightarrow ixj\ \theta\ kym)$.

3,8 >—— T1068.

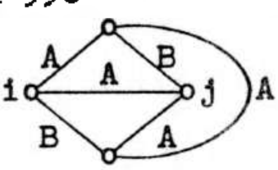
T1065,T1061,T358 >——

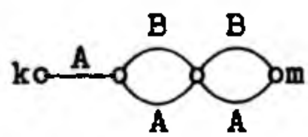
TEOREM 1069

$(\forall ixj, kym \in 2G_N)(\theta(ixj) = \theta(kym) \leftrightarrow$

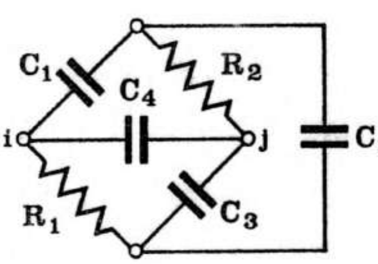
$(\exists(e,f))(YO^{-1}_{ef}(ixj) = YO^{-1}_{ef}(kym)))$.

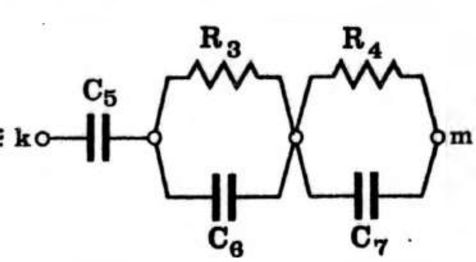
PRIMJER 558

(1) $x \in$ 

& $y \in$ 

&

$z \in$ 

& $w \in$ 

Sup.

1,D286 >——

(2) $\theta(ixj) = \theta(kym) = ABABA$

1,2,T1068 >——

(3)$(C_1, C_2, C_3, C_4, R_1, R_2 \in Re_{poz} \rightarrow$

$(\exists C_5, C_6, C_7, R_3, R_4 \in Re_{poz})(Z(izj) = Z(kwm)$ &

$C_5, C_6, C_7, R_3, R_4 \in Re_{poz} \rightarrow$

$(\exists C_1, C_2, C_3, C_4, R_1, R_2 \in Re_{poz})(Z(kwm) = Z(izj))$.

5.3.3. PROSTOR m-DVOPOLA NAD SKUPOM DVOGENERATORSKIH ELEKTRIČKIH DVOPOLA JEDNAKE IMPEDANCIJE, RESP. ADMITANCIJE

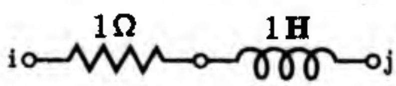
DEFINICIJA 361

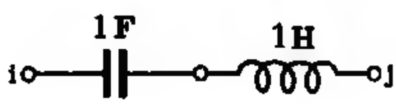
$(\forall z\in T_{ef})(Z^{-1}(z) = \{ixj\colon ixj\in 2N_{ef(nedeg)} \ \&\ Z(ixj) = z\})$.

DEFINICIJA 362

$(\forall z\in T_{ef})(Y^{-1}(z) = \{ixj\colon ixj\in 2N_{ef(nedeg)} \ \&\ Y(ixj) = z\})$.

PRIMJERI

559 $x\in$  $\longrightarrow$ $ixj\in Z^{-1}(1+s)$.

560 $x\in$  $\longrightarrow$ $ixj\in Y^{-1}\left(\frac{s}{1+s^2}\right)$.

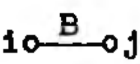
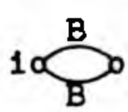
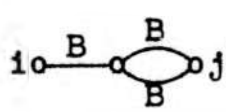
DEFINICIJA 363

$(\forall z\in T_{ef})(OZ^{-1}(z) = \{ixj\colon ixj\in 2G_N \ \&\ (\exists iyj\in Z^{-1}(z))(iO(y)j = ixj)\})$.

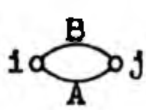
DEFINICIJA 364

$(\forall z\in T_{ef})(OY^{-1}(z) = \{ixj\colon ixj\in 2G_N \ \&\ (\exists iyj\in Y^{-1}(z))(iO(y)j = ixj)\})$.

PRIMJERI

561 $x\in\{$ , ,  $\}$ $\rightarrow$

$(\forall k\in Re_{poz})(ixj\in OZ^{-1}(ks))$

562 $x\in$  $\rightarrow$ $(\forall k_1,k_2\in Re_{poz})(ixj\in OY^{-1}\left(\frac{1+k_1s^2}{k_2s}\right))$.

D358-D364,D172,T1066-T1069,T1049,T1050 $\succ\!\!-\!\!-$ T1070-T1075.

TEOREM 1070

$(\forall z\in T_{ef})(\forall ixk\in 2G_N)(ixk\in OZ^{-1}(z) \longleftrightarrow \theta(z) = \theta(ixk))$.

TEOREM 1071

$(\forall z\in T_{ef})(\forall ixk\in 2G_N)(ixk\in OY^{-1}(z) \longleftrightarrow \theta(z) = D\theta(ixk))$.

TEOREM 1072
$(\forall z \in T_{ef})(OZ^{-1}(z) = OY^{-1}(\Re z))$.

TEOREM 1073
$(\forall z \in T_{ef})(OY^{-1}(z) = OZ^{-1}(\Re z))$.

TEOREM 1074
$(\forall x,y \in T_{ef})(x \;\theta\; y \;\longleftrightarrow\; OZ^{-1}(x) = OZ^{-1}(y))$.

TEOREM 1075
$(\forall x,y \in T_{ef})(x \;\theta\; y \;\longleftrightarrow\; OY^{-1}(x) = OY^{-1}(y))$.

5.4. KANONSKI DVOGENERATORSKI ELEKTRIČKI DVOPOLI

DEFINICIJA 365
$(\forall x \in N_{ef})(\varepsilon(x) = \varepsilon(O(x)) \;\&\; v(x) = v(O(x)))$.

D365,D334,D332,D186,D189 $\succ\!\!-\!\!-$
TEOREM 1076
$(\forall x \in N_{ef})(F_{\text{mreža}}(x) = y \;\rightarrow\; (\varepsilon(x) = \kappa(E_y') \;\&\; v(x) = \kappa(V_y')))$.

DEFINICIJA 366
$(\forall (e,f) \in \{(\bar{1},1),(\bar{1},0),(0,1)\})(2N_{ef(kan)} = \{ixj\colon ixj \in 2N_{ef} \;\&$
$(\exists kym \in 2N_{ef})(Z(kym) = Z(ixj) \;\rightarrow\; \varepsilon(x) \leq \varepsilon(y))\})$.

D366,T1048 $\succ\!\!-\!\!-$
TEOREM 1077
$(\forall z \in T_{ef})(\exists ixj \in 2N_{ef(kan)})(Z(ixj) = z)$.

TEOREM 1078
$(\forall ixj \in 2N_{ef})(ixj \in 2N_{ef(kan)} \;\longleftrightarrow\; \varepsilon(x) = \lambda(\vartheta(Z(ixj))))$.

D o k a z
T1057,T281 $\succ\!\!-\!\!-$
(1) $(\forall ixj \in 2N_{ef})(\lambda(Z(ixj)) \leq \lambda(iO(x)j))$.

1,T1046,T1006 $\succ\!\!-\!\!-$
(2) $(\forall ixj \in 2N_{ef})(\lambda(Z(ixj)) \leq \varepsilon(O(x)))$.

2,T1077,T1065,T1009 $\succ\!\!-\!\!-$

(3) $(\forall z\in T_{ef})(\exists ixj\in Z^{-1}(z))(\lambda(Z(ixj)) = \varepsilon(O(x)))$.

2,3,D365,D366,T726 $\succ\!\!-\!\!-$ T1078.

T1078,T290,T1051 $\succ\!\!-\!\!-$

TEOREM 1079

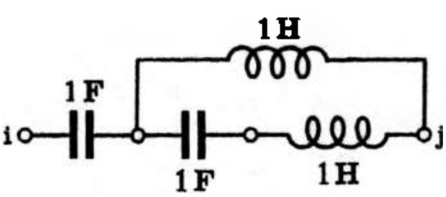
$(\forall ixj\in 2N_{ef})(ixj\in 2N_{ef(kan)} \longleftrightarrow \varepsilon(x) = \lambda(\theta(Y(ixj))))$.

T1078,T1059,T1010 $\succ\!\!-\!\!-$

TEOREM 1080

$(\forall ixj\in 2N_{ef})(ixj\in 2N_{ef(kan)} \longleftrightarrow \neg\, El_{\Delta}(ixj) = 0 \;\;\&\;\; iO(x)j\in 2G_{tkan})$.

PRIMJER 563

(1) x∈  Sup.

1,D339 $\succ\!\!-\!\!-$

(2) $Z(ixj) = \frac{1}{s}\;\frac{1 + 3s^2 + s^4}{1 + 2s^2}$

2,T1048,D180 $\succ\!\!-\!\!-$

(3) $\theta(Z(ixj)) = ABAB$

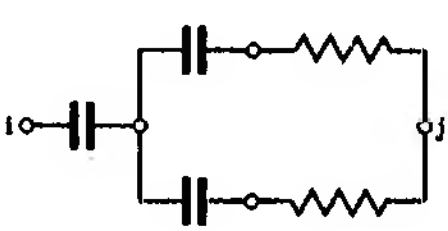
3,D029,D365 $\succ\!\!-\!\!-$

(4) $\lambda(\theta(Z(ixj)) = \lambda(ABAB) = 4 = \varepsilon(x)$

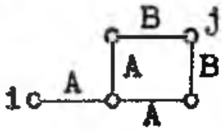
1,4,T1078,D346 $\succ\!\!-\!\!-$

(5) $ixj\in 2N_{\bar{1},1(kan)}$.

PRIMJER 564

(1) x∈  Sup.

1,D353 $\succ\!\!-$

(2) $O(x) \in$ 

2,D286,D189 $\succ\!\!-$

(3) $\theta(iO(x)j) = ABAB \quad \& \quad \varepsilon(O(x)) = 5$

3 $\succ\!\!-$

(4) $\lambda(ABAB) = 4 \neq \varepsilon(O(x))$

4,T1010 $\succ\!\!-$

(5) $\neg\, iO(x)j \in 2G_{tkan}$

1,5,T1080,D347 $\succ\!\!-$

(6) $\neg\, ixj \in N_{\bar{1},O(kan)}$.

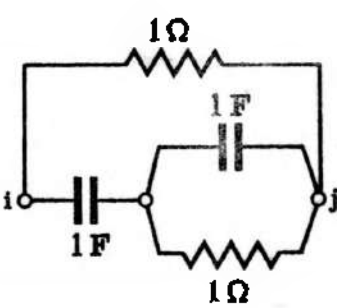
DEFINICIJA 367

$(\forall z \in T_{ef})(Z^{-1}_{kan}(z) = \{ixj\colon\ ixj \in 2N_{ef(kan)} \quad \& \quad Z(ixj) = z\})$.

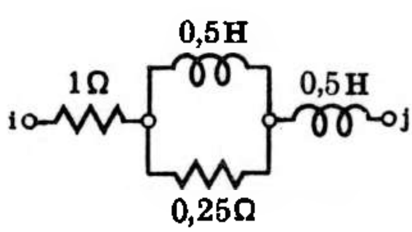
DEFINICIJA 368

$(\forall z \in T_{ef})(Y^{-1}_{kan}(z) = \{ixj\colon\ ixj \in 2N_{ef(kan)} \quad \& \quad Y(ixj) = z\})$.

PRIMJERI

565 $x \in$ 

$\longrightarrow \quad ixj \in Z^{-1}_{kan}\left(\dfrac{2s + 1}{s^2 + 3s + 1} \right)$.

566 $x \in$ 

$\longrightarrow \quad ixj \in Y^{-1}_{kan}\left(\dfrac{2s + 1}{s^2 + 3s + 1} \right)$.

DEFINICIJA 369

$(\forall z\in T_{ef})(OZ^{-1}_{kan}(z) = \{w:\ ixj\in Z^{-1}_{kan}(z)\ \ \&\ \ w = i0(x)j\})$.

DEFINICIJA 370

$(\forall z\in T_{ef})(OY^{-1}_{kan}(z) = \{w:\ ixj\in Y^{-1}_{kan}(z)\ \ \&\ \ w = i0(x)j\})$.

D367-D370, T1079, T1080 $\succ\!\!\!-\!\!\!-$

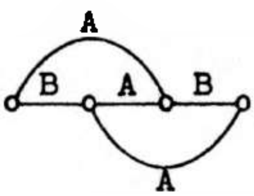
TEOREM 1081

$(\forall z\in T_{ef})(\forall ixk\in 2G_N)(ixk\in OZ^{-1}_{kan}(z) \ \longleftrightarrow\ ixk\in 2G_{tkan}\ \&\ \theta(ixk)=\theta(z))$.

TEOREM 1082

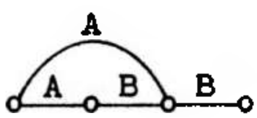
$(\forall z\in T_{ef})(\forall ixk\in 2G_N)(ixj\in OY^{-1}_{kan}(z) \ \longleftrightarrow\ ixk\in 2G_{tkan}\ \&\ \theta(ixk)=D\theta(z))$.

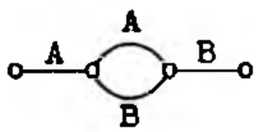
PRIMJERI

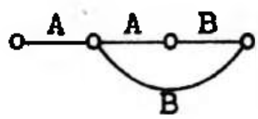
567 x∈i 

j $\rightarrow$ $ixj\in OZ^{-1}_{kan}\left(\frac{1}{s}\,\frac{s^2+3s+1}{s^2+4s+2}\right)$.

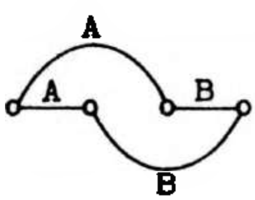
568 $ixj\in OY^{-1}_{kan}\left(s\,\frac{s+1}{s^2+2s+1}\right) \longleftrightarrow$

x∈ U{ 

, 

, 

,



}.

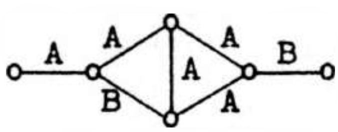
5.5. m-RIJEČI KAO PRIRODNI REPREZENTANTI STRUKTURE DVOGENERATORSKOG ELEKTRIČKOG DVOPOLA

PRIMJER 569

(1) ixj je električki dvopol i impedancija, resp. admitancija od ixj je z i z je (e,f)-imitancija i ime od z je ABABAB. Sup.

Ime od z je ABABAB, što povlači 2 i 3.

(2) dijametar m-grafa od x nije manji (u smislu D74) od ABABAB i ixj je nedegenerirani električki dvopol ($El_\Delta(ixj) \neq 0$), ako i samo ako je dijametar m-grafa od x jednak ABABAB. To jest, imenom od z karakterizirana su topološka svojstva svih dvogeneratorskih električkih dvopola impedancije jednake z. Tako npr., ako je

x∈ 

, onda je dijametar od x jednak ABABAB, što implicira egzistenciju električkog (e,f)-dvopola impedancije jednake z i m-grafa izomorfnog sa x.

(3) $s^e \dfrac{A_0+A_1 s^{f-e}+A_2 s^{2(f-e)}+s^{3(f-e)}}{B_0+B_1 s^{f-e}+B_2 s^{2(f-e)}}$ je kanonska s-forma od z, ako i samo ako svaka početna glavna subdeterminanta matrice

$$\begin{bmatrix} A_0 & A_1 & A_2 & 1 & 0 & 0 \\ 0 & B_0 & B_1 & B_2 & 0 & 0 \\ 0 & A_0 & A_1 & A_2 & 1 & 0 \\ 0 & 0 & B_0 & B_1 & B_2 & 0 \\ 0 & 0 & A_0 & A_1 & A_2 & 1 \\ 0 & 0 & 0 & B_0 & B_1 & B_2 \end{bmatrix}$$

je pozitivna. Imenom imitancije z, odredena je korespodencija izmedu parametara kanonske s-forme od z i glavne dijagonale matrice od z.

Ljuska imena od z je ab, što povlači 4 i 5.

(4) Postoje pozitivni realni brojevi k_1 i k_2, takvi da je

$$\lim_{s\to 0}(z) = k_1 s^e \quad \& \quad \lim_{s\to \infty}(z) = k_2 s^f$$

(5) (i) Ako je z impedancija od ixj, onda ixj ima ulazni e-rez (cut-set) i ulazni f-rez. Tako npr. ako je z $(\bar{1},0)$-imitancija, onda u x postoji skup kapaciteta uklanjanjem kojih se postiže razdvajanje čvorišta "i" i "j", i postoji skup otpora uklanjanjem kojih se postiže razdvajanje čvorišta "i" i "j".

(ii) Ako je z admitancija od ixj, onda ixj ima ulazni $\bar{f}$-put i ulazni $\bar{e}$-put. Tako npr. ako je z $(\bar{1},0)$-imitancija, onda čvorišta "i" i "j" povezana su u x putem kojem su sve grane otpori i putem kojem su sve grane induktiviteti.

Broj slova imena od z je šest, što povlači 6 i 7.

(6) Stupanj karakterističnog polinoma od z je šest i red matrice od z je šest.

(7) ixj je kanonski električki dvopol, ako i samo ako x sadrži šest elemenata (grana).

Ime od z sadrži dvije s-jedinice (hiperslova BA) i tri p-jedinice (hiperslova AB), što povlači 8, 9 i 10.

(8) Broj "unutarnjih polova" od z je 2(f-e) i broj "unutarnjih nula" od z je 3(f-e).

(9) Red kompleksnosti (order of complexity, transient response, natural frequencies) od ixj je 2(f-e), resp. 3(f-e).
Tako npr. ako je z $(\bar{1},0)$-imitancija, tj. ako se radi o CR-, resp. RL-dvopolu, onda je red kompleksnosti od ixj jednak dva, resp. jednak tri. Ako se radi o CL-dvopolu, tj. ako je z $(\bar{1},1)$-imitancija, onda je red kompleksnosti od ixj jednak četiri, resp. jednak šest.

(10) ixj je kanonski (e,f)-dvopol, ako i samo ako broj temeljnih krugova (broj suvislosti, prvi Betti broj) je dva i broj temeljnih rezova(fundamental cut-set) u grafu od x nakon identifikacije čvorišta i, j je tri, resp. ixj je kanonski

$(\bar{f},\bar{e})$-dvopol, ako i samo ako broj temeljnih krugova u grafu od x je tri i broj temeljnih rezova u grafu od x nakon identifikacije čvorišta i, j je dva.

(11) Postoje dvadeset dva i samo dvadeset dva po " algebarskoj formi" (u smislu poglavlja 2.7) različita kanonska m_t-polinoma u koje se ime od z može dekomponirati i to

$$
\begin{array}{llll}
ABABAB & = A \cup (B \cap (A \cup (B \cap AB))) & = A \cup (B \cap AB \cap AB) & = \\
 & = A \cup (B \cap (AB \cup BA)) & = A \cup (B \cap ((AB \cap A) \cup B)) & = \\
 & = A \cup (BA \cup (B \cap AB)) & = A \cup ((BA \cup B) \cap AB) & = \\
 & = A \cup (BA \cup BA \cup B) & = A \cup (((BA \cup B) \cap A) \cup B) & = \\
 & = A \cup ((B \cap (A \cup BA)) \cup B) & = A \cup ((BA \cap AB) \cup B) & = \\
 & = AB \cap (A \cup (B \cap AB)) & = AB \cap (AB \cup BA) & = \\
 & = AB \cap AB \cap AB & = AB \cap ((AB \cap A) \cup B) & = \\
 & = (AB \cap A) \cup (B \cap AB) & = (AB \cap A) \cup BA \cup B & = \\
 & = (AB \cap AB) \cup BA & = (AB \cap (A \cup BA)) \cup B & = \\
 & = (AB \cap AB \cap A) \cup B & = ((A \cup BA \cup B) \cap A) \cup B & = \\
 & = ((A \cup (B \cap AB)) \cap A) \cup B & = (((AB \cap A) \cup B) \cap A) \cup B &
\end{array}
$$

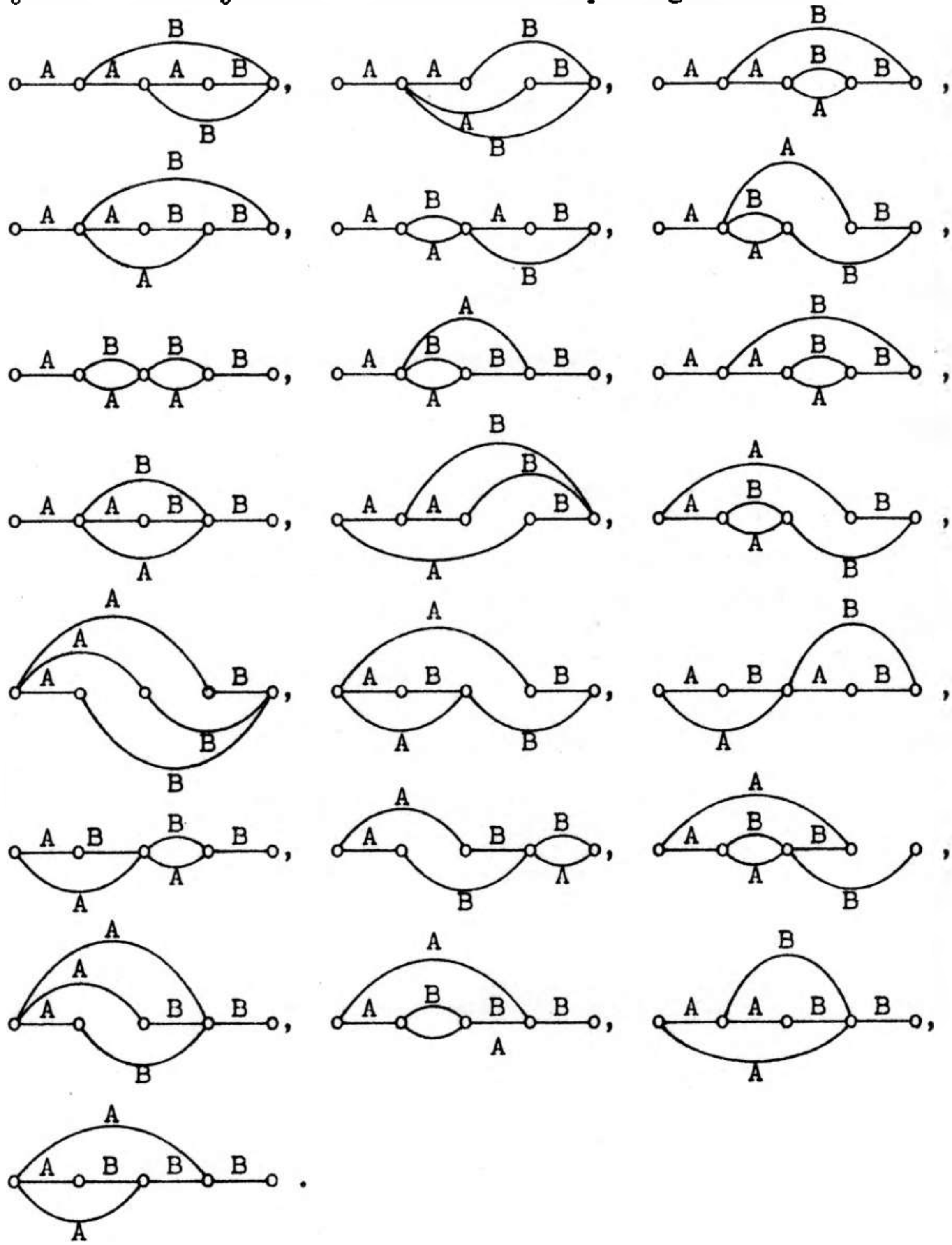
što implicira 12 i 13.

(12) z se može rastaviti, točno na dvadeset dva, netrivijalno razna načina, u serijsko-paralelnu sumu od šest generatorskih imitancija i to u korespodenciji sa m_t-polinomima iz (11). Tako npr. u vezi sa prvim m_t-polinomom iz (11), vrijedi: postoje pozitivni, realni brojevi k_1, k_2, k_3, k_4, k_5 i k_6 takvi da je

$$
z = k_1 s^e + (k_2 s^f \mathbin{\hat{+}} (k_3 s^e + (k_4 s^f \mathbin{\hat{+}} (k_5 s^e + k_6 s^f)))) =
$$

$$
= k_1 s^e + \cfrac{1}{\cfrac{1}{k_2 s^f} + \cfrac{1}{k_3 s^e + \cfrac{1}{\cfrac{1}{k_4 s^f} + \cfrac{1}{k_5 s^e + k_6 s^f}}}} .
$$

(13) Ako je z impedancija od ixj, onda ixj je kanonski, serijsko-paralelni (e,f)-dvopol, ako i samo ako m-graf od x pripada jednom od slijedeća dvadeset dva skupa m-grafova:



GLOSARIJ m-SIMBOLA

1	a	(i) subatom (primitivni simbol) sistema M. (ii) $j\ a_x\ k$ :: j i k povezani su a-putem u m-grafu x.	D266
2	A	m_t-atom. m_t-slovo.	D6
3	$A_{e,f}$	skup A-generatora t_{ef}-strukture	D133
4	A_t	skup m_t-dvopola dijametra A	D310
5	$(ab)_\zeta$	skup m_ζ-dvopola dijametra ab	D328
6	$(AB)_{e,f}$	skup p-jedinica t_{ef}-strukture	D136
7	b	(i) subatom (primitivni simbol) sistema M (ii) $j\ b_x\ k$ :: j i k povezani su b-putem u m-grafu x.	D265
8	B	m_t-atom. m_t-slovo.	D7
9	$B_{e,f}$	skup B-generatora t_{ef}-strukture	D134
10	B_t	skup m_t-dvopola dijametra B	D311
11	$(ba)_\zeta$	skup m_ζ-dvopola dijametra ba	D327
12	$(BA)_{ef}$	skup s-jedinica t_{ef}-strukture	D135
13	c_p	$x\ c_p\ y$ :: x je neposredni p-prethodnik od y	D71
14	c_s	$x\ c_s\ y$ :: x je neposredni s-prethodnik od y	D70
15	C	suvislost, resp. broj temeljnih petlji grafa m-grafa.	D278
16	C_a	a-suvislost, resp. broj temeljnih a-petlji m-grafa.	D282
17	C_b	b-suvislost, resp. broj temeljnih b-petlji m-grafa.	D283

18	d	d(x) :: desni završetak, resp. svršetak od x	
		(i) svršetak m-broja	D11
		(ii) m-valuacija svršetka imitancije	D141
		(iii) d(jxk) = a :: j i k povezani su a-putem u m-grafu x.	
		d(jxk) = b :: j i k povezani su b-rezom u m-grafu x.	D268
19	$\bar{d}$	dual svršetka m-broja	D13
20	d_t	(i) t-svršetak m_t-polinoma	D116
		(ii) t-svršetak imitancije	D167
21	$\bar{d}_t$	dual t-svršetka m_t-polinoma	D118
22	D	(i) operator dualnosti sistema M	D29
		(ii) operator dualnosti m_t-strukture	D126
		(iii) autoizomorfizam dualnosti t_{ef}-struktura	D155
23	D_T	permutacija dualiteta dvogeneratorskih imitancija	D159
24	$\mathcal{E},\bar{\mathcal{E}}$	skupovi dualnih simbola za prirodne brojeve u dekadskom brojevnom sistemu	D101
25	E	(i) primitivni simbol m-topologije	D183
		(ii) E_x :: skup grana m-grafa x	D185
		(iii) E_x :: skup imena grana imenovane električke mreže x.	D335
26	E_A	$E_A(x)$ :: skup grana karaktera A, resp. A-grana u m-grafu x.	D187
27	E_{ab}	$E_{ab}(x)$ :: skup grana karaktera ab, resp. ab-grana, resp. praznih grana u m-grafu x.	D187
28	E_{ba}	$E_{ba}(x)$ :: skup grana karaktera ba, resp. ba-grana, resp. kratkospojnih grana, resp. geometrijskih grana u m-grafu x.	D187
29	E_B	$E_B(x)$ :: skup grana karaktera B, resp. B-grana u m-grafu x.	D187

30	E'	E'_x :: skup grana u električkoj mreži x.	D332
31	E'_c	$E'_c(x)$ :: skup grana sa kapacitetom u pasivnoj električkoj mreži x.	D343
32	E'_L	$E'_L(x)$ :: skup grana sa induktivitetom u pasivnoj električkoj mreži x.	D343
33	E'_R	$E'_R(x)$ :: skup grana sa otporom u pasivnoj električkoj mreži x.	D344
34	El_p	$El_p(x,y)$ :: p-eliminanta imitancija x i y.	D150
35	El_s	$El_s(x,y)$ :: s-eliminanta imitancija x i y.	D149
36	El_Δ	$El_\Delta(ixk)$ :: eliminanta admitancije električkog dvopola ixk	D342
37	f_k	(i) $f_k(x,y)$ :: nadoveza m_t-polinoma x i y	D125
		(ii) $f_k(x,y)$ :: nadoveza m-grafova x i y	D193
38	$F_\cup$	pomoćna relacija za opravdanje uvođenja operacije s-zbrajanja m-brojeva	D35
39	$F_\cap$	pomoćna relacija za opravdanje uvođenja operacije p-zbrajanja m-brojeva	D37
40	$F_{\text{ime čv.}}$	$F_{\text{ime čv.}}(x)$ :: drugi član imenovane električke mreže x	D334
41	$F_{\text{ime gr.}}$	$F_{\text{ime gr.}}(x)$ :: treći član imenovane električke mreže x	D334
42	$F_{\text{mreža}}$	$F_{\text{mreža}}(x)$ :: prvi član imenovane električke mreže x	D334
43	F_{per}	$F_{per}(X)$ :: perioda (torzija) m-skupa X.	D85
44	g	g(x) :: graf m-grafa x.	D190
45	G	skup m-grafova	D184
46	G_t	skup m_t-grafova	D305
47	G_ζ	skup m_ζ-grafova	D314
48	2G	skup m-dvopola	D260

49	$2G_N$	skup neseparabilnih m_t-dvopola	D354
50	$2G_{s-p}$	skup serijsko-paralelnih m-dvopola	D264
51	$2G\theta$	skup klasa ostataka m-dvopola modulo nula	D295
52	$2G_t$	skup m_t-dvopola	D306
53	$2G_{tkan}$	skup kanonskih m_t-dvopola	D307
54	$2G_t\theta$	skup klase ostataka m_t-dvopola modulo nula	D309
55	$2G_\zeta$	skup m_ζ-dvopola	D315
56	$2G_\zeta\theta_1$	skup klasa ostataka m_ζ-dvopola modulo jedan	D317
57	$2G_\zeta\theta$	skup klasa ostataka m_ζ-dvopola modulo nula	D326
58	h	$h(x)$ :: defekt (spol, rod) od x	
		(i) defekt m-broja	D55
		(ii) defekt imitancije	D144
		(iii) defekt m-dvopola	D285
59	$\bar{h}$	dual defekta m-broja	D56
60	I	(i) operator inverznosti m-brojeva	D32
		(ii) $I_x(e) = \{j,k\}$ :: e je incidentno sa j i k u m-grafu x.	D185
		(iii) I_x :: funkcija incidencije u imenovanoj električkoj mreži x	D336
61	I'	I'_x :: funkcija incidencije u električkoj mreži	D332
62	J_p	p-jezgra m-broja	D44
63	J_s	s-jezgra m-broja	D43
64	K	operator komplementarnosti m-brojeva	D31
65	l	$l(x)$ :: lijevi završetak, resp. početak od x	
		(i) početak m-broja	D10
		(ii) m-valuacija početka imitancije	D140
		(iii) $l(jxk) = a$ :: j i k povezani su a-rezom u m-grafu x.	
		$l(jxk) = b$ :: j i k povezani su b-putem u m-grafu x.	D267

66	$\bar{1}$	$\bar{1}(x)$ :: dual početka od x	D12
67	1_t	(i) t-početak m_t-polinoma (ii) t-početak imitancije	D115 D167
68	$\bar{1}_t$	dual t-početka m_t-polinoma	D117
69	L_π	skup m-istina i m-neistina kongruentnih modulo π	D97
70	$\mathscr{L}_\pi$	m_ζ-logika periode (reda) π	D98
71	M	(i) skup m-brojeva. Skup m-riječi (ii) M_π :: m-skup periode π, resp. reda π	D2 D86
72	M^*	skup riječi formiran alfabetom $\{a,b\}$	D2
73	$\mathscr{M}$	m-struktura	D39
74	M_t	skup m_t-polinoma	D104
75	$\mathscr{M}_t$	m_t-struktura	D107
76	M_ζ	m-Zeroid	D99
77	$\mathscr{M}_\zeta$	m_ζ-struktura	D100
78	$M(\Theta)_n$	skup klasa ostataka m-brojeva modulo n	D78
79	$\mathscr{M}_n$	m-sistem klasa ostataka m-brojeva modulo n	D80
80	M_s^+	skup s-nenegativnih m-brojeva	D58
81	M_p^+	skup p-nenegativnih m-brojeva	D59
82	M_s^-	skup s-negativnih m-brojeva	D60
83	M_p^-	skup p-negativnih m-brojeva	D61
84	M_{poz}	skup pozitivnih m-brojeva. Skup m_t-brojeva	D62
85	M_{neg}	skup negativnih m-brojeva	D63
86	M_{nepoz}	skup nepozitivnih m-brojeva	D64

87	M_{neneg}	skup nenegativnih m-brojeva	D65
88	$\mathcal{M}_{2G\theta}$	m-struktura klasa ostataka m-dvopola modulo nula	D304
89	Mod	$Mod_u(M_\pi)$ :: modul nad u, m-sistema reda π	D87
90	N_ζ	skup brojki dualizirane aditivne polugrupe prirodnih brojeva	D102
91	$\mathcal{N}_\zeta$	dualizirana aditivna polugrupa prirodnih brojeva	D102
92	N	skup imenovanih pasivnih električkih mreža	D333
93	2N	skup pasivnih električkih dvopola	D338
94	$N_{\bar{1},1}$	skup električkih CL-mreža	D346
95	$N_{\bar{1},0}$	skup električkih CR-mreža	D347
96	$N_{0,1}$	skup električkih RL-mreža	D348
97	$2N_{\bar{1},1}$	skup neseparabilnih električkih CL-dvopola	D349
98	$2N_{\bar{1},0}$	skup neseparabilnih električkih CR-dvopola	D349
99	$2N_{0,1}$	skup neseparabilnih električkih RL-dvopola	D349
100	$2N_{ef(kan)}$	skup kanonskih (e,f)-dvopola	D366
101	$2N_{ef(nedeg)}$	skup nedegeneriranih (e,f)-dvopola	D357
102	O	$O_{e,f}(x)$ :: m-graf električke (e,f)-mreže x	D353
103	O^{-1}	$O^{-1}_{e,f}(ixk)$ :: skup izomorfnih nedegeneriranih električkih (e,f)-dvopola iznad m-grafa ixk	D358
104	OY^{-1}	$OY^{-1}(z)$ :: prostor m-dvopola nad skupom nedegeneriranih električkih dvopola admitancije z	D364

105	OY^{-1}_{kan}	$OY^{-1}_{kan}(z)$ :: prostor m-dvopola nad skupom kanonskih električkih dvopola admitancije z	D370
106	OZ^{-1}	$OZ^{-1}(z)$ :: prostor m-dvopola nad skupom nedegeneriranih električkih dvopola impedancije z	D363
107	OZ^{-1}_{kan}	$OZ^{-1}_{kan}(z)$ :: prostor m-dvopola nad skupom kanonskih električkih dvopola impedancije z	D369
108	P_d	$P_d(x)$, resp. xP :: desni sljedbenik od x	
		(i) d-sljedbenik m-broja	D16
		(ii) d-sljedbenik m_t-polinoma	D120
109	P'_d	$P'_d(x)$, resp. xP' :: desni prethodnik od x	
		(i) d-prethodnik m-broja	D22
		(ii) d-prethodnik negeneratorske imitancije	D169
110	P^n_d	(i) d-sljedbenik n-tog reda m-broja	D18
		(ii) d-prethodnik n-tog reda m-broja	D20
		(iii) d-sljedbenik n-tog reda m_t-polinoma	D122
		(iv) d-prethodnik n-tog reda imitancije	D171
111	p.Id	$p.Id_u(M_\pi)$ :: p-ideal nad u m-sistema reda π	D89
112	$p.Id_A(M_t)$	p.ideal m_t-strukture nad A	D111
113	$p.Id_B(M_t)$	p.ideal m_t-strukture nad B	D114
114	P_l	$P_l(x)$, resp. Px :: lijevi sljedbenik od x	
		(i) l-sljedbenik m-broja	D15
		(ii) l-sljedbenik m_t-polinoma	D119
115	P'_l	$P'_l(x)$, resp. $P'x$ :: lijevi prethodnik od x	
		(i) l-prethodnik m-broja	D21
		(ii) l-prethodnik negeneratorske imitancije	D168
116	P^n_l	(i) l-sljedbenik n-tog reda m-broja	D17
		(ii) l-prethodnik n-tog reda m-broja	D19
		(iii) l-sljedbenik n-tog reda m_t-polinoma	D121
		(iv) l-prethodnik n-tog reda imitancije	D170

117	P^o	izomorfizam t_{ef}-strukture	D154
118	P_T^o	permutacija identiteta dvogeneratorskih imitancija	D158
119	q	(i) ljuska m-broja	D14
		(ii) ljuska imitancije	D142
		(iii) ljuska m-dvopola	D269
		q_x :: ljuska m-grafa x	
120	$\bar{q}$	$\bar{q}(x)$:: dual ljuske m-broja x	D30
121	q_p	(i) p-ljuska m-broja	D46
		(ii) p-ljuska m-dvopola	D271
122	q_s	(i) s-ljuska m-broja	D45
		(ii) s-ljuska m-dvopola	D270
123	q_t^+	$q_t^+(x)$ :: ljuska m_t-polinoma x	D110
124	Q	(i) q-sljedbenik m-broja	D25,D26
		(ii) q-sljedbenik m_t-polinoma	D108
125	Q'	q-prethodnik m-broja	D27,D28
126	Q^n	(i) q-sljedbenik, resp. q-prethodnik n-tog reda m-broja	D23,D24
		(ii) q-sljedbenik n-tog reda m_t-polinoma	D109
127	r	kvazi uređenje m-brojeva	D41
128	$\bar{r}$	dualno kvazi uređenje m-brojeva	D42
129	R	(i) recipročni izomorfizam t-struktura	D156
		(ii) rang (broj temeljnih rezova) grafa m-grafa	D279
130	$\bar{R}$	dualno recipročni izomorfizam t-struktura	D157
131	R_T	permutacija reciprociteta dvogeneratorskih imitancija	D160
132	$\bar{R}_T$	permutacija dualnog reciprociteta dvogeneratorskih imitancija	D161

133	R_a	$R_a(x)$ :: broj temeljnih a-rezova (a-rang) m-grafa x	D280
134	R_b	$R_b(x)$ :: broj temeljnih b-rezova (b-rang) m-grafa x	D281
135	$\mathcal{S}, \bar{\mathcal{S}}$	sljedbeničke funkcije dualnih Peanovih sistema	D101
136	$S_{el.mreža}$	$x \in S_{el.mreža}$ :: x je pasivna električka mreža	D331
137	s.Id	$s.Id_u(M_\pi)$ :: s-ideal nad u m-sistema reda π	D88
138	$s.Id_A(M_t)$	s-ideal m_t-strukture nad A	D113
139	$s.Id_B(M_t)$	s-ideal m_t-strukture nad B	D112
140	Slog	Slog(x) :: slog parametara kanonske s-forme imitancije x	D145
141	S_m	skup m-skupova	D81
142	$S_{m\text{-sist.}\{a,b\}}$	skup m-sistema nad $\{a,b\}$	D82
143	$S_{m\text{-subgr.}}$	$y \in S_{m\text{-subgr.}}(x)$ :: y je m-subgraf od x	D191
144	T	skup dvogeneratorskih imitancija	D131
145	$T_{e,f}$	skup (e,f)-imitancija	D130
146	$\mathcal{T}_{e,f}$	t_{ef}-struktura	D132
147	$T_{ef}(\theta)$	skup klasa ostataka (e,f)-imitancija modulo nula	D174
148	v	(i) broj čvorišta m-grafa	D186
		(ii) broj čvorišta imenovane električke mreže	D365
149	V	(i) primitivni simbol (skup čvorišta) m-topologije	D183
		(ii) V_x :: skup čvorišta m-grafa x	D185
		(iii) V_x :: skup imena čvorišta imenovane električke mreže x.	D335

150	V'	V'_x :: skup čvorišta u električkoj mreži x	D332
151	w	valuacija ljuske dvogeneratorske imitancije	D143
152	w_d	valuacija svršetka imitancije	D139
153	w_1	valuacija početka imitancije	D139
154	$\mathscr{W}$	skup riječi m-topologije	D258
155	Y	Y(ixk) :: admitancija električkog dvopola ixk	D340
156	Y^{-1}	$Y^{-1}(z)$ :: skup nedegeneriranih električkih dvopola admitancije z	D362
157	$Y_{\bar{1},1}$	funkcija admitancije CL-dvopola	D356
158	$Y_{\bar{1},0}$	funkcija admitancije CR-dvopola	D356
159	$Y_{0,1}$	funkcija admitancije RL-dvopola	D356
160	Y^{-1}_{kan}	$Y^{-1}_{kan}(z)$ :: skup kanonskih električkih dvopola admitancije z	D368
161	$YO^{-1}_{e,f}$	prostor admitancije nad skupom izomorfnih nedegeneriranih električkih (e,f)-dvopola	D360
162	Z	Z(ixk) :: impedancija električkog dvopola ixk	D339
163	$Z_{e,f}$	funkcija impedancije (e,f)-dvopola, tj. CL- , resp. CR- , resp. RL-dvopola	D355
164	Z^{-1}	$Z^{-1}(z)$ :: skup nedegeneriranih električkih dvopola impedancije z	D361
165	Z^{-1}_{kan}	$Z^{-1}_{kan}(z)$:: skup kanonskih električkih dvopola impedancije z	D367
166	$ZO^{-1}_{e,f}$	prostor impedancije nad skupom izomorfnih nedegeneriranih električkih (e,f)-dvopola	D359
167	Γ	skup četiriju osnovnih permutacija m-brojeva	D33
168	Γ_T	skup četiriju osnovnih permutacija dvogeneratorskih imitancija	D162
169	δ	$\delta(x)$ :: dijametar m-prostora nad m-grafom x	D289

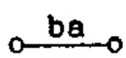
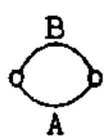
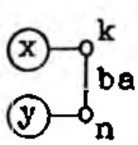
170	Δ	(i) permutacija dualiteta u dualiziranoj aditivnoj grupi prirodnih brojeva	D102
		(ii) $\Delta(ixk)$ :: determinanta matrice admitancije čvorišta električke mreže x sa referentnim čvorištem i	D341
		(iii) $\Delta_{11}(ixk)$ :: kofaktor (1,1) determinante matrice admitancije čvorišta električke mreže x sa referentnim čvorištem "i"	D341
171	ε	(i) broj nepraznih grana m-grafa	D189
		(ii) broj grana imenovane električke mreže	D365
172	ε_A	broj A-grana m-grafa	D188
173	ε_{ab}	broj ab-grana (praznih grana) m-grafa x	D188
174	ε_{ba}	broj ba-grana (kratkospojnih grana) m-grafa x	D188
175	ε_B	broj B-grana m-grafa x	D188
176	θ	(i) $\theta(x)$ :: ime (e,f)-imitancije x	D180
		(ii) $\theta\|x\|_o$ :: ime klase ostataka (e,f)-imitancija modulo nula	D181
		(iii) $\theta(jxk)$ :: razmak čvorišta "j" i "k" u m-grafu x	D286
		θ_x :: m-prostor nad m-grafom x	D286
		(iv) $\theta\|ixk\|$ :: ime klase ostataka m-dvopola modulo nula	D296
177	$\theta^{-1}_{e,f}$	$\theta^{-1}_{e,f}(x)$ :: skup (e,f)-imitancija imena x	D182
178	θ^{-1}_{2G}	$\theta^{-1}_{2G}(x)$ :: skup m-dvopola imena x	D297
179	θ	$x\ \theta\ y$ :: x i y su kongruentni modulo nula	
		$x\ \theta_n y$ :: x i y su kongruentni modulo n	
		(i) relacija kongruencije modulo n sistema M	D76
		(ii) relacija kongruencije modulo nula t_{ef}-strukture	D172
		(iii) relacija kongruencije modulo nula m-topologije	D293
180	θ_1	relacija kongruencije modulo jedan m-topologije	D292

181	λ	(i) o-duljina m-broja	D57
		(ii) broj pribrojnika u C_l- i C_d-formi m_t-polinoma	D124
		(iii) red matrice imitancije	D147
182	λ_p	(i) p-duljina m-broja	D50
		(ii) "stupanj brojnika" kanonske s-forme imitancije	D148
		(iii) broj "temeljnih rezova" m-grafa	D287
183	λ_s	(i) s-duljina m-broja	D49
		(ii) "stupanj nazivnika" kanonske s-forme imitancije	D148
		(iii) suvislost (broj "temeljnih petlji") m-grafa	D284
184	λ_t	(i) broj simbola za operacije zbrajanja u kanonskoj formi m_t-polinoma	D123
		(ii) broj "unutarnjih polova i nula" imitancije	D146
185	λ_t^+	broj neophodnih pari zagrada u F_s-, i F_p-formi m_t-polinoma	D110
186	μ	$\mu(x)$ :: matrica imitancije x	D166
187	π	$\pi(x)$ :: broj maksimalno povezanih subgrafova grafa m-grafa x	D273
188	π_a	$\pi_a(x)$ :: broj maksimalno povezanih a-subgrafova u m-grafu x	D275
189	π_b	$\pi_b(x)$ :: broj maksimalno povezanih b-subgrafova u m-grafu x	D277
190_1	Π	$\Pi(x)$ :: karakteristični polinom imitancije x	D151
190_2	ΠG	partitivni skup, skupa m-grafova	D291
191	ρ_p	$\rho_p(x)$ :: p-rastav m-broja x	D52
192	ρ_s	$\rho_s(x)$ :: s-rastav m-broja x	D51
193	Σ	sljedbenička funkcija dualizirane aditivne grupe prirodnih brojeva	D102

194	Φ_0	operator o-formiranja riječi m-topologije	D209
195	Φ_1	operator 1-formiranja riječi m-topologije	D210
196	Φ_2	operator 2-formiranja riječi m-topologije	D211
197	Ψ	$\Psi(W)$ :: skup m-sistema nad skupom simbola W	D1
198	Ψ_t	$\Psi_t\{A,B\}$:: skup m_t-struktura generiran m_t-atomima	D103
199	ω	$\omega_x(e)$ :: karakter grane e u x	
		(i) karakter grane u m-grafu	D185
		(ii) karakter grane u CL-mreži	D350
		(iii) karakter grane u CR-mreži	D351
		(iv) karakter grane u RL-mreži	D352
200	Ω	baza sistema M. Skup m-atoma, resp. m-slova	D4
201	$\Omega_{e,f}$	skup generatorskih (e,f)-imitancija	D127
202	Ω_o	skup m_o-atoma (m_o-slova) sistema M. Skup nula (s-nula i p-nula) m-strukture	D9
203	Ω_p	p-baza m-strukture	D48
204	$\Omega_{p(e,f)}$	skup p-ireducibilnih (e,f)-imitancija	D165
205	Ω_s	s-baza m-strukture	D47
206	$\Omega_{s(e,f)}$	skup s-ireducibilnih (e,f)-imitancija	D164
207	Ω_t	(i) skup m_t-atoma (m_t-slova) sistema M	D8
		(ii) skup generatora m_t-strukture	D105
208	Ω_t^+	skup primitivnih m_t-polinoma	D106
209	$\Omega(\theta)$	$\Omega(\theta_n)$ :: baza m-sistema klasa ostataka m-brojeva modulo n	D79
210	$\Omega_{2G\theta}$	baza m-strukture klasa ostataka m-dvopola modulo nula	D301
211	$\wedge$	$x \wedge y = xy$ :: y nadovezano na x	D2,D3

212	$\bar{}$	$\bar{s}$ :: dualno s	D5
213	$\star$	(i) kompozicija (relativni produkt) osnovnih permutacija m-brojeva	D34
		(ii) kompozicija osnovnih permutacija dvogeneratorskih imitancija	D163
214	$\smile$	(i) operacija serijskog zbrajanja m-brojeva	D36
		(ii) s-operacija generiranja m_t-strukture	D104
		(iii) s-zbrajanje u skupu klasa ostataka (e,f)-imitancije modulo nula	D178
		(iv) s-nadovezivanje u skupu klasa ostataka m-dvopola modulo nula	D302
		(v) s-nadovezivanje u skupu klasa ostataka m_t-dvopola modulo nula	D312
		(vi) s-nadovezivanje u skupu klasa ostataka m_ζ-dvopola modulo nula	D329
215	$\frown$	(i) operacija paralelnog zbrajanja m-brojeva	D38
		(ii) p-operacija generiranja m_t-strukture	D104
		(iii) p-zbrajanje u skupu klasa ostataka (e,f)-imitancija modulo nula	D179
		(iv) p-nadovezivanje u skupu klasa ostataka m-dvopola modulo nula	D303
		(v) p-nadovezivanje u skupu klasa ostataka m_t-dvopola modulo nula	D313
		(vi) p-nadovezivanje u skupu klasa ostataka m_ζ-dvopola modulo nula	D330
216	$<_{alf}$	uređajna relacija subalfabeta sistema M	D40
217	$+_s$, $+_p$	operacije neprave dekompozicije m-strukture u tenzorski produkt rešetke (lattice) i cikličke grupe	D53,D54
218	$<_s$	$x <_s y$ :: x je s-manje od y, resp. y je s-veće od x	D66
219	$<_p$	$x <_p y$ :: x je p-manje od y, resp. y je p-veće od x	D67
220	$\leq_s$	$x \leq_s y$ :: x je s-manje ili jednako y	D68
221	$\leq_p$	$x \leq_p y$ :: x je p-manje ili jednako y	D69

222	$[\ ]_s$	$[x,y]_s$ :: zatvoreni s-interval m-brojeva od x do y	D72
223	$[\ ]_p$	$[x,y]_p$ :: zatvoreni p-interval m-brojeva od x do y	D73
224	$<$	$x < y$ :: x je manje od y, resp. y je veće od x	D74
225	$\leq$	$x \leq y$ :: x je manje ili jednako y	D75
226	$\|\ \|$	$\|x\|_n$ :: klasa ostataka m-brojeva modulo n, sa reprezentantom klase x	D77
227	$\top_o$	m-istina beskonačnog reda (periode nula)	D90
228	$\perp_o$	m-neistina beskonačnog reda (periode nula)	D91
229	$\neg_m$	m-negacija	D92
230	$\&_m$	m-konjukcija	D93
231	v_m	m-disjunkcija	D94
232	$\rightarrow_m$	m-implikacija	D95
233	$\leftrightarrow_m$	m-ekvivalencija	D96
234	$0,\vec{0}$	inicijalni elementi dualnih Peanovih sistema	D101
235	$+$	(i) operacija s-zbrajanja u dualiziranoj aditivnoj grupi prirodnih brojeva	D102
		(ii) operacija s-zbrajanja u skupu funkcija kompleksne varijable	D128
236	$\hat{+}$	(i) operacija p-zbrajanja u dualiziranoj aditivnoj grupi prirodnih brojeva	D102
		(ii) operacija p-zbrajanja u skupu funkcija kompleksne varijable	D129
237	$\bigvee$	$\bigvee_{i=o}^{n} x_i$ :: $x_o + x_1 + \dots + x_n$	D137
238	$\bigwedge$	$\bigwedge_{i=o}^{n} x_i$ :: $x_o \hat{+} x_1 \hat{+} \dots \hat{+} x_n$	D138

239	$\bigvee$	${}_{e,f}\bigvee^{g,h}$ :: izomorfizam t_{ef}-strukture i t_{gh}-strukture	D152
240	$\bigwedge$	${}_{e,f}\bigwedge^{g,h}$ :: dualni izomorfizam t_{ef}-strukture i t_{gh}-strukture	D153
241	$\lvert\ \rvert_o$	$\lvert x\rvert_o$ :: klasa ostataka (e,f)-imitancija modulo nula	D173
242	$\approx$	$x \approx y$ :: x i y su izomorfni	
		(i) izomorfizam m-grafova	D192
		(ii) izomorfizam m-dvopola	D261
243	o	skup m-grafova koji sadrže samo jedno čvorište i nemaju niti jedne grane	D194
244		skup m-grafova koji sadrže samo jedno čvoriste i samo jednu singularnu A-granu	D198
245		skup m-grafova koji sadrze dva čvorišta i jednu nesingularnu ba-granu	D205
246		skup m-grafova koji se sastoje od paralelne kombinacije jedne B- i jedne A-grane	D243
247		serijska kombinacija disjunktnih m-grafova x i y, povezivanjem kratkospojnom granom čvorišta "k" u x i čvorišta "n" u y.	D256
248	( )	i(x)k, resp. ixk	
		(i) m-dvopol	D259
		(ii) električki dvopol	D337
249	$\rightarrow$	$x^{i\rightarrow j}$ :: "identifikacija" čvorišta "i" m-grafa x sa čvorištem "j"	D263
250	'	x' :: operacija uklanjanja praznih grana iz m-grafa x	D272
251	$\lvert\ \rvert_a$	$\lvert j\rvert_{a_x}$ :: skup čvorišta u m-grafu x povezanih a-putem sa "j"	D274
252	$\lvert\ \rvert_b$	$\lvert j\rvert_{b_x}$ :: skup čvorišta u m-grafu x povezanih b-putem sa "j"	D276

253	$\mid\;\mid$	$\lvert jxk \rvert$ :: skup m-dvopola kongruentnih modulo nula sa jxk	D294
254	$\mid\;\mid_t$	$\lvert jxk \rvert_t$ :: skup m_t-dvopola kongruentnih modulo nula sa jxk	D308
255	$\mid\;\mid_{\zeta 1}$	$\lvert jxk \rvert_{\zeta 1}$ :: skup m_ζ-dvopola kongruentnih modulo jedan sa jxk	D316
256	$\top_1$	skup m_ζ-dvopola sa ljuskom jednakom ba	D318
257	$\bot_1$	skup m_ζ-dvopola sa ljuskom jednakom ab	D319
258	$\neg_1$	negacija u $2G_\zeta\theta_1$, tj. u skupu klasa ostataka m_ζ-dvopola modulo jedan	D320
259	$\&_1$	konjukcija u $2G_\zeta\theta_1$	D321
260	$\vee_1$	disjunkcija u $2G_\zeta\theta_1$	D322
261	$\rightarrow_1$	implikacija u $2G_\zeta\theta_1$	D323
262	$\leftrightarrow_1$	ekvivalencija u $2G_\zeta\theta_1$	D324
263	$\mid\;\mid_\zeta$	$\lvert jxk \rvert_\zeta$ :: skup m_ζ-dvopola kongruentnih modulo nula sa jxk	D325

P o d a c i   o   p i s c u   k n j i g e

Miro Šare, inženjer elektrotehnike, rođen je u Šibeniku 1918. Srednju školu je završio u rodnom mjestu, a studirao na Tehničkom fakultetu u Zagrebu, na kojem je i diplomirao 1951. godine. Od 1948. zaposlen je na Zavodu za osnove i mjerenja u slaboj struji, Tehničkog, kasnije Elektrotehničkog fakulteta u Zagrebu, na kojem i sad obučava iz predmeta Elektronička mjerna tehnika. Radeći u tom području, otkrio je u proljeću 1957. godine mogućnost reprezentacije strukture električkih mreža linearnim kombinacijama triju simbola, koje je nazvao m-brojevima. Ograničivši se samo na dvogeneratorske električke mreže, ubrzo je našao glavne zakonitosti m_t-strukture, iz kojih je razvio Algebru dvogeneratorskih imitancija, Sistem M i m-Topologiju, onako kako su i prikazane u ovoj knjizi.

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m-BROJEVI

Izdavač:
AUTOR

Strojopis:
MAJA ŠARE

Radovi u tušu:
DUBRAVKO PAVLOVIĆ

Tisak:
KUĆNA TISKARA SVEUČILIŠTA U ZAGREBU

Oprema i uvez:
GRAFIČKI ŠKOLSKI CENTAR – ZAGREB

Tisak dovršen:
LIPNJA 1970.

Naklada:
200 PRIMJERAKA